INFORMATION AND COMPUTATION 94, 48–61 (1991)

Universal Homogeneous Event Structures and Domains

MANFRED DROSTE

Fachbereich 6—Mathematik und Informatik, Universität GHS Essen, D-4300 Essen 1, Germany

In the theory of denotational semantics of programming languages, several authors established the existence of particular kinds of "universal" domains. Here, we use a general model-theoretic result to show that there exists a unique countable universal homogeneous event structure. From this, we deduce that the category of all event domains, with stable embedding–projection pairs as morphisms, contains a universal object. Similarly, we also obtain a universal dI-domain. We also show that the category of all event domains is closed under inverse limits. Similar results are derived for Kahn and Plotkin's concrete data structures and concrete domains. © 1991 Academic Press, Inc.

1. INTRODUCTION

In the theory of denotational semantics of programming languages, several authors have established the existence of particular kinds of "universal" domains. Scott (1976) provided a universal domain for the class of ω-algebraic lattices and showed that in this domain computations can be handled by a calculus of retracts. Universal domains for the classes of all coherent, respectively all bounded-complete, ω-algebraic cpo's were given by Plotkin (1978) and Scott (1981). Gunter (1987) established a family of universal domains for the class of profinite domains. Recently, Gunter and Jung (1990) described a systematic way of constructing universal—even saturated—domains.

In this paper, we will deal with concrete domains and event domains. Concrete data structures and concrete domains were introduced by Kahn and Plotkin (1978) in order to allow a fairly general semantics definition of sequentiality. see also (Berry, 1978; Berry and Curien, 1982). Winskel (1981, 1987) studied a generalization, event structures and event domains. Here, our main goal is to use a general model-theoretic result to show that various categories of event domains and also the category of all concrete domains (in each case with stable embedding–projection pairs as morphisms) contain universal objects. We also show that some of these categories, in particular those comprising all event domains, respectively all concrete domains, are closed under inverse limits.

Let us introduce some notation. An *event structure* $\mathscr{E}$ consists of a count-

0890-5401/91 $3.00

able set E of tokens together with a consistency relation for finite subsets of E and an enabling relation between consistent subsets and elements of E satisfying certain natural axioms. The elements of E can be thought of, e.g., as the units of information which can in principle be computed by a machine, whereas the enabling relation describes the computation possibilities themselves. A *state* of $\mathscr{E}$ is a subset X of E such that each finite subset of X is consistent and each element of X can be deduced through finitely many successive applications of the enabling relation from a finite number of elements of X which are "a priori true," i.e., enabled by the empty set. The set of all such states of $\mathscr{E}$, partially ordered by inclusion, is denoted by $(D(\mathscr{E}), \subseteq)$. An *event domain* is here defined to be any partial order $(D, \leqslant)$ isomorphic to $(D(\mathscr{E}), \subseteq)$ for some event structure $\mathscr{E}$. We also consider *k-recognizable* event structures in which finite sets are consistent iff all their k-element subsets are consistent ($k \in \mathbb{N}$).

Our argument proceeds as follows (for some unexplained terminology, see Section 2). We will first use a model-theoretic result due to Fraissé (1954, 1986), which shows how to construct countable homogeneous relational structures with prescribed isomorphism-types of finite substructures, to obtain a universal homogeneous event structure. (This structure seems to be also of independent interest.) The same argument almost automatically yields universal homogeneous stable (k-recognizable, stable and k-recognizable) event structures. We then use Winskel's observation (1987) that if $\mathscr{E}$ is a substructure of $\mathscr{E}'$, then there exists a stable embedding–projection pair from $(D(\mathscr{E}), \subseteq)$ to $(D(\mathscr{E}'), \subseteq)$. Hence the various classes of event domains contain a universal object. Similarly, we obtain a universal concrete domain.

Stable event structures have been studied in detail by Winskel (1987). Their domains are precisely the dI-domains introduced by Berry (1978); these are distributive domains in which each compact element dominates only finitely many elements. With stable functions as morphisms, the dI-domains form a cartesian closed category (Berry, 1978). It follows that the universal dI-domain constructed here forms a model of the untyped λ-calculus (cf. (Barendregt, 1981; Koymans, 1982). We note that the category of dI-domains is also important for studies of models of the polymorphic λ-calculus; see Coquand, Gunter, and Winskel (1988).

Finally, we use the order-theoretic characterization of event domains obtained in (Droste, 1989) to show that the category of event domains is closed under inverse limits. Consequently, recursive domain equations of the form $D \cong F(D)$ can be solved (for continuous functors F) within the category of event domains (cf. (Curien, 1986; Smyth and Plotkin, 1982)). We note that while there are many similar results in the literature for categories of cpo's with embedding–projection pairs as morphisms, categories with stable embedding–projection pairs have been considered

less often. The argument here shows that for the category of event domains as well as several full subcategories, the use of stable embedding–projection pairs is essential. We note that for the category of distributive event domains (= dI-domains), the corresponding result was already obtained in (Kahn and Plotkin, 1978).

2. Universal Homogeneous Event Structures

In this section we wish to prove the existence of various kinds of universal event domains and of universal concrete domains. In fact, we will first prove a stronger result for event structures and concrete data structures. For any set E, let $\mathrm{Fin}(E)$ denote the set of all finite subsets of E.

Definition 2.1 (cf. Winskel (1987)). An *event structure* is a triple $\mathscr{E} = (E, \mathrm{Cons}, \vdash)$ satisfying the following conditions:

(a) E is a countable set (comprising the *events* or units of information);

(b) $\mathrm{Cons} \subseteq \mathrm{Fin}(E)$ is non-empty (the *consistent* sets);

(c) $\vdash \subseteq \mathrm{Cons} \times E$ (the *enabling relation* between consistent subsets and elements of E);

(d) whenever $A \subseteq B$ and $B \in \mathrm{Cons}$, then $A \in \mathrm{Cons}$; whenever $A \vdash e$, $A \subseteq B$ and $B \in \mathrm{Cons}$, then $B \vdash e$.

An event structure $\mathscr{E}$ is called *k-recognizable* (where $k \in \mathbb{N}$) if

$$X \in \mathrm{Cons} \Leftrightarrow \forall A \subseteq X\colon |A| = k \Rightarrow A \in \mathrm{Cons}$$

for any $X \in \mathrm{Fin}(E)$, and $\mathscr{E}$ is called *stable*, if

$$A \vdash e,\ B \vdash e \text{ and } A \cup B \cup \{e\} \in \mathrm{Cons} \quad \Rightarrow \quad A \cap B \vdash e.$$

Let $\mathfrak{E}$ be the class of all event structures $\mathscr{E}$, and let $\mathfrak{E}_k$, $\mathfrak{E}_{\mathrm{stab}}$, $\mathfrak{E}_{k,\mathrm{stab}}$ be the subclasses comprising all members of $\mathfrak{E}$ which are k-recognizable, stable, k-recognizable and stable, respectively.

Let $\mathscr{E}$ be an event structure. A subset X of E is a *state* of $\mathscr{E}$, if the following two conditions are satisfied:

(1) $A \subseteq X$, A finite $\Rightarrow A \in \mathrm{Cons}$ (consistency)

(2) $e \in X \Rightarrow \exists e_1, \ldots, e_n \in X$ such that $e_n = e$ and

$$\forall j \leqslant n\colon \{e_i\colon i < j\} \vdash e_j \text{ (deducibility).}$$

The set of all states of $\mathscr{E}$, partially ordered by inclusion, is denoted by $(D(\mathscr{E}), \subseteq)$. A partially ordered set $(D, \leqslant)$ will be called an *event domain*,

if $(D, \leqslant)$ is isomorphic to $(D(\mathscr{E}), \subseteq)$ for some event structure $\mathscr{E}$, and a *k-recognizable event domain*, if here $\mathscr{E}$ can be chosen to be k-recognizable.

Order-theoretic characterizations of event domains and of 2-recognizable event domains will be given in Section 3. We just note here that these partial orders are indeed domains, i.e., bounded-complete ω-algebraic cpo's, and that the partial orders $(D, \leqslant)$ isomorphic to $(D(\mathscr{E}), \subseteq)$ for some stable event structure $\mathscr{E}$ are precisely the distributive event domains.

DEFINITION 2.2. Let $\mathscr{E} = (E, \mathrm{Cons}, \vdash)$ and $\mathscr{E}' = (E', \mathrm{Cons}', \vdash')$ be two event structures. A one-to-one function $f\colon E \to E'$ is called an *embedding* of $\mathscr{E}$ into $\mathscr{E}'$, if

$$X \in \mathrm{Cons} \Leftrightarrow f(X) \in \mathrm{Cons}' \qquad \text{and} \qquad X \vdash e \Leftrightarrow f(X) \vdash' f(e),$$

for any $X \subseteq E$ and $e \in E$. An embedding $f\colon \mathscr{E} \to \mathscr{E}'$ which maps E onto E' is called an *isomorphism*. An isomorphism of $\mathscr{E}$ onto itself is called an *automorphism* of $\mathscr{E}$. When $E \subseteq E'$ and the identity mapping id: $E \to E'$ is an embedding, we say that $\mathscr{E}$ is a *substructure* of $\mathscr{E}'$, denoted $\mathscr{E} \subseteq \mathscr{E}'$.

An event structure $\mathscr{E}$ is called *homogeneous* if whenever $\mathscr{E}_1, \mathscr{E}_2 \subseteq \mathscr{E}$ are two finite substructures and $f\colon \mathscr{E}_1 \to \mathscr{E}_2$ is an isomorphism, then there exists an automorphism g of $\mathscr{E}$ which extends f. Let $\mathfrak{E}^*$ be a class of event structures. An element $\mathscr{E} \in \mathfrak{E}^*$ is *universal* for $\mathfrak{E}^*$, if each member $\mathscr{A} \in \mathfrak{E}^*$ can be embedded into $\mathscr{E}$. We first wish to prove the following result.

THEOREM 2.3. *Each of the classes* $\mathfrak{E}$, $\mathfrak{E}_k (k \in \mathbb{N})$, $\mathfrak{E}_{\mathrm{stab}}$, $\mathfrak{E}_{k,\mathrm{stab}}$ $(2 \leqslant k \in \mathbb{N})$ *contains a universal homogeneous object. Moreover, it is unique up to isomorphism.*

In our argument for Theorem 2.3, we will use a general model-theoretic result due to Fraissé (1954, 1986) which allows us to construct countable homogeneous relational structures. Let L be a relational language and $\mathscr{S} = \langle S; R_1, R_2, \ldots \rangle$ a structure for L. (We allow $S = \varnothing$.) We say that $\mathscr{S}$ is *homogeneous*, if whenever $\mathscr{A}$, $\mathscr{B}$ are two finite substructures of $\mathscr{S}$, then any isomorphism from $\mathscr{A}$ onto $\mathscr{B}$ extends to an automorphism of $\mathscr{S}$. The *age* of $\mathscr{S}$ is the class of all finite L-structures embeddable into $\mathscr{S}$. A class $\mathscr{C}$ of L-structures is called *isomorphism-closed*, if $\mathscr{A} \cong \mathscr{B}$ and $\mathscr{A} \in \mathscr{C}$ imply $\mathscr{B} \in \mathscr{C}$, and *hereditary*, if $\mathscr{C}$ is closed under taking substructures. $\mathscr{C}$ has the *amalgamation property*, if whenever $\mathscr{A}_1, \mathscr{A}_2, \mathscr{B} \in \mathscr{C}$ and $f_i\colon \mathscr{B} \to \mathscr{A}_i$ $(i = 1, 2)$ are embeddings, there exist $\mathscr{A} \in \mathscr{C}$ and embeddings $g_i\colon \mathscr{A}_i \to \mathscr{A}$ $(i = 1, 2)$ such that $g_1 \circ f_1 = g_2 \circ f_2$. Note that if $\mathscr{C}$ is isomorphism-closed, then $\mathscr{C}$ has the amalgamation property if whenever $\mathscr{A}_1$, $\mathscr{A}_2$, $\mathscr{B} \in \mathscr{C}$ such that $\mathscr{A}_1 \cap \mathscr{A}_2 = \mathscr{B}$ (i.e., $\mathscr{B} \subseteq \mathscr{A}_1$, $\mathscr{B} \subseteq \mathscr{A}_2$ and $A_1 \cap A_2 = B$), then there exists $\mathscr{A} \in \mathscr{C}$ with $\mathscr{A}_1 \subseteq \mathscr{A}$ and $\mathscr{A}_2 \subseteq \mathscr{A}$.

THEOREM 2.4 (Fraissé (1954, 1986)). *Let $\mathscr{C}$ be a class of finite L-structures. Then the following are equivalent*:

(1) *There exists a countable homogeneous L-structure $\mathscr{S}$ with age $\mathscr{C}$.*

(2) *$\mathscr{C}$ is isomorphism-closed, hereditary, has the amalgamation property and has, up to isomorphism, only countable many elements.*

Moreover, if the structure $\mathscr{S}$ in (1) *exists, it is unique up to isomorphism.*

For generalizations of Theorem 2.4, see (Bell and Slomson, 1974) or (Maier, 1987). A categorical version will be given in (Droste and Göbel, 1990). For the convenience of the reader, we include a

Proof of the Implication (2) $\to$ (1) (sketch). Let $\{\mathscr{C}_i\colon i \in \mathbb{N}\}$ be an enumeration, up to isomorphism, of $\mathscr{C}$. By the amalgamation property and induction, we obtain a sequence of structures $\mathscr{S}_i \in \mathscr{C}$ $(i \in \mathbb{N})$ such that for each $i \in \mathbb{N}$, $\mathscr{S}_i \subseteq \mathscr{S}_{i+1}$, there exists an embedding of $\mathscr{C}_i$ into $\mathscr{S}_i$, and whenever $\mathscr{A} \subseteq \mathscr{S}_i$ and $f\colon \mathscr{A} \to \mathscr{S}_i$ is an embedding, then there exists an embedding $g\colon \mathscr{S}_i \to \mathscr{S}_{i+1}$ which extends f. Then $\mathscr{S} = \bigcup_{i \in \mathbb{N}} \mathscr{S}_i$ satisfies the requirements.

Now let $\mathscr{C}^*$ be a class of L-structures and $\mathscr{S} \in \mathscr{C}^*$. We say that $\mathscr{S}$ is *universal* in $\mathscr{C}^*$, if each $\mathscr{A} \in \mathscr{C}^*$ can be embedded into $\mathscr{S}$. Next assume $\mathscr{C}^*$ is hereditary and $\mathscr{S}$ is homogeneous. If each finite element of $\mathscr{C}^*$ can be embedded into $\mathscr{S}$, then, as is well-known, even each countable structure in $\mathscr{C}^*$ can be embedded into $\mathscr{S}$. (Indeed, let $\mathscr{A} \in \mathscr{C}^*$. Write $\mathscr{A} = \bigcup_{i \in \mathbb{N}} \mathscr{A}_i$ with finite structures $\mathscr{A}_i \in \mathscr{C}^*$ such that $\mathscr{A}_i \subseteq \mathscr{A}_{i+1}$ for each $i \in \mathbb{N}$. As $\mathscr{S}$ is homogeneous, we inductively obtain embeddings $f_i\colon \mathscr{A}_i \to \mathscr{S}$ such that $f_{i+1}|_{\mathscr{A}_i} = f_i$ for each $i \in \mathbb{N}$. Then $f\colon \mathscr{A} \to \mathscr{S}$, defined such that f extends each f_i, is an embedding.)

Next we show that event structures correspond to a particular type of relational structures. We assume from now on that L contains relation symbols C_n, E_n of arity n for each $n \in \mathbb{N}$.

DEFINITION 2.5. Let $\mathscr{A} = \langle A; C_n, E_n\ (n \in \mathbb{N})\rangle$ be a relational structure. We say that $\mathscr{A}$ is a *relational event structure*, if the following conditions are satisfied:

(1) A is a countable set and C_n and E_n are n-ary relations on A, for each $n \in \mathbb{N}$.

(2) Whenever $(a_1, ..., a_n) \in C_n$ $(n \in \mathbb{N})$, then:

— the elements $a_1, ..., a_n$ are pairwise distinct;

— $(a_{\pi(1)}, ..., a_{\pi(n)}) \in C_n$ for any permutation π of $\{1, ..., n\}$;

— $(a_1, ..., a_m) \in C_m$ for any $m \leqslant n$.

(3) Whenever $(a_1, ..., a_n, a) \in E_{n+1}$ $(n \in \mathbb{N})$, then $(a_1, ..., a_n) \in C_n$ and $(a_{\pi(1)}, ..., a_{\pi(n)}, a) \in E_{n+1}$ for any permutation π of $\{1, ..., n\}$.

(4) Whenever $(a_1, ..., a_n) \in C_n$, $0 \leqslant m \leqslant n \in \mathbb{N}$, and $(a_1, ..., a_m, a) \in E_{m+1}$ then $(a_1, ..., a_n, a) \in E_{n+1}$.

Let $\mathscr{E} = (E, \mathrm{Cons}, \vdash)$ be an event structure and $\mathscr{A} = \langle A;\ C_n,\ E_n\ (n \in \mathbb{N})\rangle$ a relational event structure. We say that $\mathscr{E}$ and $\mathscr{A}$ *correspond* to each other, if $E = A$ and for any pairwise distinct elements $a_1, ..., a_n \in A$ and any $a \in A$, we have

$$\{a_1, ..., a_n\} \in \mathrm{Cons} \quad \text{iff} \quad (a_1, ..., a_n) \in C_n \quad (n \in \mathbb{N}), \quad \text{and}$$

$$\{a_1, ..., a_n\} \vdash a \quad \text{iff} \quad (a_1, ..., a_n, a) \in E_{n+1} \quad (n \in \mathbb{N} \cup \{0\}).$$

Clearly, to each event structure there corresponds a unique relational event structure, and conversely. Moreover, this correspondence preserves the properties of being "homogeneous," "universal," "a substructure," in the natural way.

Now we can give the

Proof of Theorem 2.3. Let L be a first order language for relational event structures. Let $\mathscr{C}$, $\mathscr{C}_k$, $\mathscr{C}_{\mathrm{stab}}$, $\mathscr{C}_{k,\mathrm{stab}}$ denote the classes of all finite relational event structures corresponding to elements of $\mathfrak{E}$, $\mathfrak{E}_k$, $\mathfrak{E}_{\mathrm{stab}}$, $\mathfrak{E}_{k,\mathrm{stab}}$, respectively. It is clear that each of the classes $\mathscr{C}$, $\mathscr{C}_k$, $\mathscr{C}_{\mathrm{stab}}$, $\mathscr{C}_{k,\mathrm{stab}}$ is isomorphism-closed and hereditary and has, up to isomorphism, only countably many elements. Next we show, assuming $k \geqslant 2$ for $\mathscr{C}_{k,\mathrm{stab}}$, that each of these four classes has the amalgamation property.

Let $\mathscr{A}_1, \mathscr{A}_2, \mathscr{B} \in \mathscr{C}$ with $\mathscr{A}_1 \cap \mathscr{A}_2 = \mathscr{B}$ and $\mathscr{A}_i = \langle A_i;\ C_n^i,\ E_n^i\ (n \in \mathbb{N})\rangle$ for $i = 1, 2$. We define an L-structure $\mathscr{A} = \langle A;\ C_n,\ E_n\ (n \in \mathbb{N})\rangle$ as follows. Put $A = A_1 \cup A_2$, $C_n = C_n^1 \cup C_n^2$ (for the class $\mathscr{C}_1$, let here $C_n = \{(a_1, ..., a_n)$: $(a_i) \in C_1^1 \cup C_1^2$, $a_i \neq a_j$ for all $i \neq j\}$) and $E_n^* = E_n^1 \cup E_n^2$ (set unions) for each $n \in \mathbb{N}$. We define $E_n \supseteq E_n^*$ by letting $(b_1, ..., b_n, e) \in E_{n+1}$ iff $(b_1, ..., b_n) \in C_n$ and $(b_1, ..., b_m, e) \in E_{m+1}^*$ for some $m \leqslant n$. Then clearly $\mathscr{A} \in \mathscr{C}$ and $\mathscr{A}_1, \mathscr{A}_2 \subseteq \mathscr{A}$. It is easy to check that if here $\mathscr{A}_1$, $\mathscr{A}_2$, $\mathscr{B}$ all belong to $\mathscr{C}_k(\mathscr{C}_{\mathrm{stab}}, \mathscr{C}_{k,\mathrm{stab}}$ and $k \geqslant 2)$, then $\mathscr{A}$ also belongs to $\mathscr{C}_k$ $(\mathscr{C}_{\mathrm{stab}}, \mathscr{C}_{k,\mathrm{stab}})$, respectively.

By Theorem 2.4, there exist four unique countable homogeneous relational L-structures $\mathscr{S}_1$, $\mathscr{S}_2$, $\mathscr{S}_3$, $\mathscr{S}_4$ whose ages are $\mathscr{C}$, $\mathscr{C}_k$, $\mathscr{C}_{\mathrm{stab}}$, $\mathscr{C}_{k,\mathrm{stab}}$, respectively. It follows that $\mathscr{S}_1$, $\mathscr{S}_2$, $\mathscr{S}_3$, $\mathscr{S}_4$ are relational event structures and that they embed each relational event structure corresponding to an element of $\mathfrak{E}$, $\mathfrak{E}_k$, $\mathfrak{E}_{\mathrm{stab}}$, $\mathfrak{E}_{k,\mathrm{stab}}$, respectively. The result follows by taking the corresponding event structures.

We just remark here that the above argument shows that each of the classes $\mathscr{C}$, $\mathscr{C}_k$, $\mathscr{C}_{\mathrm{stab}}$, $\mathscr{C}_{k,\mathrm{stab}}$ even has the strong amalgamation property: Whenever $\mathscr{A}_1$, $\mathscr{A}_2$, $\mathscr{B}$ belong to one of these classes and $f_i\colon \mathscr{B} \to \mathscr{A}_i$ $(i = 1, 2)$ are embeddings, then the amalgam $\mathscr{A}$ and the embeddings $g_i\colon \mathscr{A}_i \to \mathscr{A}$ can

be chosen such that $g_1 \circ f_1 = g_2 \circ f_2$ and if $a_i \in A_i$ with $g_1(a_1) = g_2(a_2)$, then there exists $b \in B$ with $f_i(b) = a_i$ $(i = 1, 2)$. (In other words, $\mathscr{A}_1$ and $\mathscr{A}_2$ can be amalgamated over $\mathscr{B}$ without making any identifications of points outside B.) It follows that the universal homogeneous objects of the classes $\mathfrak{E}$, $\mathfrak{E}_k$, $\mathfrak{E}_{\text{stab}}$, $\mathfrak{E}_{k,\text{stab}}$ have mathematically interesting symmetry properties: By Cameron (1989, Proposition 2.3, Theorem 4.1), in their automorphism groups no stabilizer of a finite tuple has additional fixed points, and the automorphism groups contain, for instance, the free group of countable rank as a subgroup.

In contrast, we note that the class $\mathscr{C}_{1,\text{stab}}$ does *not* have the amalgamation property. Hence, by Theorem 2.4, the class $\mathfrak{C}_{1,\text{stab}}$ does not contain a universal homogeneous object.

Next we turn to concrete data structures and wish to derive a result similar to Theorem 2.3 for them.

DEFINITION 2.6 (cf., e.g., (Kahn and Plotkin, 1978; Berry and Currien, 1982; Currien, 1988). A *concrete data structure* is a quadruple $\mathscr{M} = (C, V, E, \vdash)$ such that

(a) C, V, E are countable sets (of *cells*, *values*, and *events*, respectively) with $E \subseteq C \times V$;

(b) $\vdash \subseteq \text{Fin}(E) \times C$, i.e., $\vdash$ is a relation (the *enabling* relation) between finite sets of events and cells.

Let Cons be the system of all finite subsets X of E such that whenever (c, v_1), $(c, v_2) \in X$, then $v_1 = v_2$. We say that $\mathscr{M}$ is *stable*, if whenever A, $B \subseteq E$ are finite subsets, $e = (c, v) \in E$, $A \vdash c$, $B \vdash c$, and $A \cup B \cup \{e\} \in$ Cons, then $A \cap B \vdash c$. $\mathscr{M}$ is *finite*, if $C \cup V$ is finite. Let $\mathscr{CDS}$ ($\mathscr{CDS}_{\text{stab}}$) denote the class of all (stable) concrete data structures, respectively.

We remark that often, when concrete data structures are dealt with, the assumption is made that for each $c \in C$ there exists $v \in V$ with $(c, v) \in E$ ("any cell may be filled with a value"). We do not make this assumption here since otherwise the class of all finite concrete data structures would not be hereditary (cf. the argument for Theorem 2.8).

Let $\mathscr{M}$ be a concrete data structure. A subset X of E is a *state* of $\mathscr{M}$ if the following two conditions are satisfied:

(1) $(c, v_1), (c, v_2) \in X \Rightarrow v_1 = v_2$ (consistency);

(2) $\forall e \in X\ \exists e_j = (c_j, v_j) \in X$ $(j = 1, ..., n)$ such that $e_n = e$ and $\forall j \leqslant n$ $\exists X_j \subseteq \{e_i : i < j\}$: $X_j \vdash c_j$ (deducibility).

The set of all states of $\mathscr{M}$, partially ordered by inclusion, is denoted by

$(D(\mathscr{M}), \subseteq)$. A partially ordered set $(D, \leqslant)$ which is isomorphic to $(D(\mathscr{M}), \subseteq)$ for some concrete data structure $\mathscr{M}$ is called a *concrete domain.*

An order-theoretic characterization of concrete domains will be given in Section 3. We just note here that all concrete domains are 2-recognizable event domains.

DEFINITION 2.7. Let $\mathscr{M} = (C, V, E, \vdash)$ and $\mathscr{M}' = (C', V', E', \vdash')$ be two concrete data structures. A one-to-one function f: $C \cup V \to C' \cup V'$ is called an *embedding* of $\mathscr{M}$ into $\mathscr{M}'$, if the following conditions are satisfied:

(1) $f(C) \subseteq C'$ and $f(V) \subseteq V'$;

(2) $(c, v) \in E$ iff $(f(c), f(v)) \in E'$, for any $c \in C$, $v \in V$;

(3) whenever $A = \{(c_i, v_i)\colon i = 1, ..., n\} \subseteq E$ is finite and $c \in C$, then $A \vdash c$ iff $\{(f(c_i), f(v_i))\colon i = 1, ..., n\} \vdash' f(c)$.

An embedding f: $\mathscr{M} \to \mathscr{M}'$ with $f(C) = C'$ and $f(V) = V'$ is called an *isomorphism.* An isomorphism of $\mathscr{M}$ onto itself is also called an *automorphism* of $\mathscr{M}$. When $C \subseteq C'$, $V \subseteq V'$ and the identity mapping id: $C \cup V \to C' \cup V'$ is an embedding (then, in particular, $E \subseteq E'$), then $\mathscr{M}$ is a *substructure* of $\mathscr{M}'$, denoted $\mathscr{M} \subseteq \mathscr{M}'$.

A concrete data structure $\mathscr{M}$ is called *homogeneous*, if whenever $\mathscr{M}_1$, $\mathscr{M}_2 \subseteq \mathscr{M}$ are two finite substructures and f: $\mathscr{M}_1 \to \mathscr{M}_2$ is an isomorphism, then f extends to an automorphism g of $\mathscr{M}$. Let $\mathscr{C}^*$ be a class of concrete data structures. An element $\mathscr{M} \in \mathscr{C}^*$ is *universal* for $\mathscr{C}^*$, if each member $\mathscr{A} \in \mathscr{C}^*$ can be embedded into $\mathscr{M}$. Now we show:

THEOREM 2.8. *The class $\mathscr{CDS}$ contains a universal homogeneous object. Moreover, it is unique up to isomorphism.*

Proof. We proceed analogously to the argument for Theorem 2.3. We wish to apply Theorem 2.4 again. Formally, we would have to define "relational concrete data structures" (similarly to relational event structures previously), which here we leave to the reader. We just note that the underlying set of a relational concrete data structure corresponding to a concrete data structure $\mathscr{M} = (C, V, E, \vdash)$ would be $C \cup V$, and the sets C, V could be interpreted as unary relations and E as a binary relation on $C \cup V$.

Let CDS denote the class of all finite concrete data structures. Clearly, CDS is isomorphism-closed and hereditary and has, up to isomorphism, only countably many elements. To check the amalgamation property, let $\mathscr{M}_i = (C_i, V_i, E_i, \vdash_i) \in \mathrm{CDS}$ $(i = 0, 1, 2)$ such that $\mathscr{M}_1 \cap \mathscr{M}_2 = \mathscr{M}_0$, i.e., $\mathscr{M}_0 \subseteq \mathscr{M}_1$, $\mathscr{M}_0 \subseteq \mathscr{M}_2$, and $C_0 = C_1 \cap C_2$, $V_0 = V_1 \cap V_2$. Define $\mathscr{M} = (C, V, E, \vdash)$ by putting $C = C_1 \cup C_2$, $V = V_1 \cup V_2$, $E = E_1 \cup E_2$, $\vdash = \vdash_1 \cup \vdash_2$ (set unions). Clearly $\mathscr{M} \in \mathrm{CDS}$ and $\mathscr{M}_1$, $\mathscr{M}_2 \subseteq \mathscr{M}$.

Now by Theorem 2.4 there exists a unique homogeneous concrete data structure $\mathscr{M}$ with age CDS. Clearly $\mathscr{M}$ is universal for $\mathscr{CDS}$.

Next we wish to establish the existence of universal event domains and universal concrete domains.

We first introduce some notation. Let $(P, \leqslant)$, $(Q, \leqslant)$ be two partially ordered sets. A non-empty subset $A \subseteq P$ is called *directed*, if for any $a, b \in A$ there exists $c \in A$ with $a \leqslant c$ and $b \leqslant c$. A function $f\colon P \to Q$ is *continuous*, if it preserves suprema of directed subsets of P (i.e., if $A \subseteq P$ is directed and $\sup A \in P$ exists, then $f(\sup A) = \sup f(A)$ in $(Q, \leqslant)$). Now let us recall the notion of stable embedding–projection pairs, which were introduced by Kahn and Plotkin (1978).

DEFINITION 2.9 (cf. Curien (1986)). Let $(P, \leqslant)$, $(Q, \leqslant)$ be two partially ordred sets and $f\colon P \to Q$, $g\colon Q \to P$ two continuous functions. Then (f, g) is called an embedding–projection pair, if $g \circ f = \mathrm{id}_P$ and $f \circ g \leqslant \mathrm{id}_Q$. If, moreover, $(f \circ g)(y) = y$ for each $y \in Q$ with $y \leqslant f(x)$ for some $x \in P$, then (f, g) is called a *stable embedding–projection pair* (*SEPP*) from $(P, \leqslant)$ into $(Q, \leqslant)$.

Substructures of event structures and SEPPs between event domains are closely related:

PROPOSITION 2.10 (a) (Winskel (1987, Proposition 1.6.11)). *Let $\mathscr{E}$, $\mathscr{E}'$ be two event structures with $\mathscr{E} \subseteq \mathscr{E}'$. Then there exists a SEPP (f, g) from $(D(\mathscr{E}), \subseteq)$ into $(D(\mathscr{E}'), \subseteq)$.*

(b) (Cf. Berry and Curien (1982, Lemma 6.1.2)). *Let $\mathscr{M}$, $\mathscr{M}'$ be two concrete data structures with $\mathscr{M} \subseteq \mathscr{M}'$. Then there exists a SEPP (f, g) from $(D(\mathscr{M}), \subseteq)$ into $(D(\mathscr{M}'), \subseteq)$.*

Indeed, here in both (a) and (b) we may simply put $f = \mathrm{id}$ and $g(Y) = \bigcup \{X \in D(\mathscr{E})\colon X \subseteq Y\}$ for each state Y of $\mathscr{E}'$ ($\mathscr{M}'$), respectively. We note that in (Droste, 1989) a sharpening and a partial converse of Proposition 2.10 were obtained.

Recall that an object U in a category $\mathscr{C}$ is called *universal* if it is weakly terminal; i.e., for every object A of $\mathscr{C}$, there exists an arrow $f\colon A \to U$. Now let $\mathscr{D}$, $\mathscr{D}_k$, $\mathscr{D}_{\mathrm{d}}$, $\mathscr{D}_{k,\mathrm{d}}$, $\mathscr{CD}$, $\mathscr{CD}_{\mathrm{d}}$ be the categories of all partial orders $(D, \leqslant)$ isomorphic to $(D(\mathscr{E}), \subseteq)$ for some member $\mathscr{E}$ of $\mathfrak{E}$, $\mathfrak{E}_k$, $\mathfrak{E}_{\mathrm{stab}}$, $\mathfrak{E}_{k,\mathrm{stab}}$, $\mathscr{CDS}$, $\mathscr{CDS}_{\mathrm{stab}}$, respectively, in each case with SEPPs as morphisms. Then we have:

THEOREM 2.11. *Each of the categories $\mathscr{D}$, $\mathscr{D}_k$ $(k \in \mathbb{N})$, $\mathscr{D}_{\mathrm{d}}$, $\mathscr{D}_{k,\mathrm{d}}$ $(2 \leqslant k \in \mathbb{N})$, $\mathscr{CD}$ contains a universal object.*

Proof. First, by Theorem 2.3, let $\mathscr{E} \in \mathfrak{E}$ be a universal event structure. We claim that $(D(\mathscr{E}), \subseteq)$ is universal in $\mathscr{D}$. Indeed, let $(D, \leqslant)$ be any element of $\mathscr{D}$. By definition, $(D, \leqslant) \cong (D(\mathscr{E}'), \subseteq)$ for some $\mathscr{E}' \in \mathfrak{E}$. As $\mathscr{E}$ is universal, there exists a substructure $\mathscr{E}^* \subseteq \mathscr{E}$ with $\mathscr{E}' \cong \mathscr{E}^*$. By Proposition 2.10(a), there exists a SEPP from $(D, \leqslant) \cong (D(\mathscr{E}^*), \subseteq)$ into $(D(\mathscr{E}), \subseteq)$. For the categories $\mathscr{D}_k$, $\mathscr{D}_\mathrm{d}$, $\mathscr{D}_{k,\mathrm{d}}$, $\mathscr{CD}$ the argument is completely analogous; for $\mathscr{CD}$ just use Theorem 2.8 and Proposition 2.10(b).

The reader may wonder why we did not prove Theorem 2.11 also for the important category $\mathscr{CD}_\mathrm{d}$ (which comprises all distributive elements of $\mathscr{CD}$). The reason is that, in contrast to the situation for event structures, under the usual notions of stability the class of all finite stable concrete data structures does not seem to have the amalgamation property. But this property was essential in our argument for Theorem 2.8. At present, it remains open whether $\mathscr{CD}_\mathrm{d}$ itself satisfies the amalgamation property.

3. Inverse Limits of Event Domains

In this section we prove that each of the categories $\mathscr{D}$, $\mathscr{D}_2$, $\mathscr{CD}$, $\mathscr{D}_\mathrm{d}$, $\mathscr{D}_{2,\mathrm{d}}$, $\mathscr{CD}_\mathrm{d}$ is closed under inverse limits. First we give purely order-theoretic characterizations of the partial orders $(D, \leqslant)$ belonging to these categories. Our notation, which we now introduce, is mostly standard.

Let $(D, \leqslant)$ be a partially ordered set. For $x, y \in D$ we write $x \uparrow y$ if there is $z \in D$ with $x \leqslant z$ and $y \leqslant z$, and $x \not\uparrow y$, if not $x \uparrow y$. $(D, \leqslant)$ is called *bounded-complete* (or consistently complete) if each subset A of D which is bounded above in D has a supremum in D; equivalently, each subset A which is bounded below in D has an infimum in D. Furthermore, $(D, \leqslant)$ is *complete* or *cpo* if $(D, \leqslant)$ has a smallest element and any directed subset of D has a supremum in D. An element $x \in D$ is *compact* if for any directed subset A of D for which $\sup A$ exists and $x \leqslant \sup A$ there is $a \in A$ with $x \leqslant a$. The set of all compact elements of D is denoted by D^0. Then $(D, \leqslant)$ is *algebraic* if for each $x \in D$ the set $\{d \in D^0 \colon d \leqslant x\}$ is directed and has x as supremum. If $(D, \leqslant)$ is a bounded-complete algebraic cpo and D^0 is countable, then $(D, \leqslant)$ is a *domain.* A bounded-complete cpo D is *distributive* if $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$ for all $x, y, z \in D$ with $y \uparrow z$. For $x, y \in D$ we write $x \prec y$ if y covers x, i.e., if $x < y$ and there is no $z \in D$ with $x < z < y$. A *prime interval* of D is a pair (x, x') such that $x, x' \in D^0$ and $x \prec x'$; this pair is then denoted by $[x, x']$. For prime intervals we put $[x, x'] \prec [y, y']$ if $x \prec y$, $x' \prec y'$, and $y \neq x'$. Let $\succ\!\prec$ denote the smallest equivalence relation on the set of all prime intervals of D containing $\prec$. Now we have:

THEOREM 3.1 (a) (Droste, 1989). *$\mathscr{D}$ contains precisely all domains $(D, \leqslant)$ satisfying the following conditions for any $x, x', y, y', z \in D^0$:*

(E) *$\{d \in D: d \leqslant x\}$ is finite;*

(C) *if $x \prec y$, $x \prec z$, $y \neq z$ and $y \uparrow z$, then $y \prec y \vee z$ and $z \prec y \vee z$.*

(I) *$[x, x'] \succ\!\prec [y, y']$ and $x \leqslant y$ imply $x' \leqslant y'$.*

(b) (Winskel (1981), cf. (Curien, 1986, Sect. 2.2)). *$\mathscr{D}_2$ contains precisely all domains $(D, \leqslant)$ satisfying conditions* (E), (C), (I), *and* (V):

(V) *Whenever $x, x', x'', y, y', y'' \in D^0$ with $[x, x'] \succ\!\prec [y, y']$ and $[x, x''] \succ\!\prec [y, y'']$, then $x' \uparrow x''$ iff $y' \uparrow y''$.*

(c) (Kahn and Plotkin (1978), cf. (Curien, 1986, Sect. 2.2)). *$\mathscr{CD}$ contains precisely all domains $(D, \leqslant)$ satisfying conditions* (F), (C), (I), *and* (Q):

(Q) *Whenever $x, y, z \in D^0$ such that $z \prec x$, $z < y$ and $x \nuparrow y$, then there exists a unique element $x' \in D^0$ such that $z \prec x' \leqslant y$ and $x \nuparrow x'$.*

In this case, $(D, \leqslant)$ also satisfies condition (V).

(d) (Kahn and Plotkin, 1978; Winskel, 1981, 1987; cf. Curien, 1986, Sect. 2.2). *$\mathscr{D}_d$ $(\mathscr{D}_{2,d}, \mathscr{CD}_d)$ contains precisely all distributive members of $\mathscr{D}$ $(\mathscr{D}_2, \mathscr{CD})$, respectively.*

The objects of the category $\mathscr{D}_d$ are also known to be precisely the dI-domains considered in Berry (1978); these are defined to be distributive domains $(D, \leqslant)$ satisfying condition (F) for any $x \in D^0$ (cf. Berry, 1978, Winskel, 1987). For still another characterization of dI-domains, see Zhang (1989). We note that whereas order-theoretic characterizations of the domains in $\mathscr{D}_2$ and $\mathscr{D}$ are known, no such results seem to be available for the objects in $\mathscr{D}_k$ $(3 \leqslant k \in \mathbb{N})$; axiomatizing these latter classes was raised as an open problem in Winskel (1981, p. 282). (Clearly, the domains in $\mathscr{D}_1$ are precisely those domains of $\mathscr{D}$ which have a greatest element.)

The following notation will be useful.

DEFINITION 3.2. Let $(P, \leqslant)$ be a partially ordered set. A non-empty subset S of P is called a *complete ideal* of P, denoted $S \lhd P$, if the following two conditions are satisfied:

(1) Whenever $x \in P$, $s \in S$, and $x \leqslant s$, then $x \in S$.

(2) If $A \subseteq S$ and $z \in P$ with $a \leqslant z$ for each $a \in A$, then there exists $s \in S$ such that $a \leqslant s \leqslant z$ for each $a \in A$.

The following result shows that complete ideals and stable embedding–projection pairs are closely related. Its proof is essentially contained

in Kahn and Plotkin (1978) (see Droste and Göbel, 1990, Proposition 4.5, Lemma 4.6) for a proof of the present version):

PROPOSITION 3.3. (a) *Let* $(P, \leqslant)$, $(Q, \leqslant)$ *be two partially ordered sets and* (f, g) *a SEPP from* $(P, \leqslant)$ *into* $(Q, \leqslant)$. *Then* $f(P) \lhd Q$, $(f(P))^0 \subseteq Q^0$, *and* f *is an isomorphism from* $(P, \leqslant)$ *onto* $(f(P), \leqslant)$.

(b) *Let* $(P, \leqslant)$ *be an algebraic partial order, and let* $S \lhd P$. *Let* f: $S \to P$ *be the identity mapping, and let* g: $P \to S$ *be defined by* $g(x) = \sup\{s \in S: s \leqslant x\}$ $(x \in P)$. *Then* (f, g) *is a SEPP from* $(S, \leqslant)$ *into* $(P, \leqslant)$.

Now let $(D_n, \leqslant)$ be cpo's and (f_n, g_n) embedding–projection pairs from $(D_n, \leqslant)$ into $(D_{n+1}, \leqslant)$ $(n \in \mathbb{N})$. The inverse limit of this sequence is the partially ordered set $(D_\infty, \leqslant)$ where

$$D_\infty = \{\langle x_n \rangle_{n \in \mathbb{N}}: x_n \in D_n,\ g_n(x_{n+1}) = x_n \text{ for each } n \in \mathbb{N}\}$$

and $\leqslant$ is the coordinatewise ordering on D_∞. Clearly $(D_\infty, \leqslant)$ is a cpo. Define two mappings $f_{n\infty}$: $D_n \to D_\infty$, $g_{\infty n}$: $D_\infty \to D_n$ by $f_{n\infty}(x) = \langle g_1 \circ g_2 \circ \cdots \circ g_{n-1}(x), \ldots, g_{n-1}(x), x, f_n(x), f_{n+1} \circ f_n(x), \ldots \rangle$, $g_{\infty n}(\langle x_1, x_2, \ldots \rangle) = x_n$.

Then $(f_{n\infty}, g_{\infty n})$ is an embedding–projection pair from $(D_n, \leqslant)$ into $(D_\infty, \leqslant)$, and we have $f_{n\infty}(D_n) \subseteq f_{n+1,\infty}(D_{n+1})$ and $D_\infty^0 = \bigcup_{n \in \mathbb{N}} f_{n\infty}(D_n^0)$. If each $(D_n, \leqslant)$ is a domain, then so is $(D_\infty, \leqslant)$. If each pair (f_n, g_n) is a SEPP, then so is each pair $(f_{n\infty}, g_{\infty n})$ $(n \in \mathbb{N})$, as is easy to check; hence, in this case by Proposition 3.3 we have $f_{n\infty}(D_n) \lhd D_\infty$.

As shown in Kahn and Plotkin (1978), the category $\mathscr{D}_\mathrm{d}$ is closed under inverse limits. We prove this again as a part of the following result:

THEOREM 3.4. *Each of the categories* $\mathscr{D}$, $\mathscr{D}_2$, $\mathscr{CD}$, $\mathscr{D}_\mathrm{d}$, $\mathscr{D}$, $\mathscr{D}_{2,\mathrm{d}}$, $\mathscr{CD}_\mathrm{d}$ *is closed under inverse limits.*

Proof. By the preceding remarks it suffices to show the following: Let $(D, \leqslant)$ be a domain, $D_n \lhd D$ $(n \in \mathbb{N})$, such that $D_n \subseteq D_{n+1}$ for each $n \in \mathbb{N}$ and $D^0 = \bigcup_{n \in \mathbb{N}} D_n^0$. If all $(D_n, \leqslant)$ $(n \in \mathbb{N})$ belong to $\mathscr{D}$ $(\mathscr{D}_2, \mathscr{CD}, \mathscr{D}_\mathrm{d}, \mathscr{D}_{2,\mathrm{d}}, \mathscr{CD}_\mathrm{d})$ then $(D, \leqslant)$ also belongs to $\mathscr{D}$ $(\mathscr{D}_2, \mathscr{CD}, \mathscr{D}_\mathrm{d}, \mathscr{D}, \mathscr{D}_{2,\mathrm{d}}, \mathscr{CD}_\mathrm{d})$, respectively.

To check this, we apply Theorem 3.1. First assume that $(D_n, \leqslant) \in \mathscr{D}$ for each $n \in \mathbb{N}$. Choose $x, x', y, y', z \in D^0$. These five elements belong to some D_n^0 $(n \in \mathbb{N})$. Hence $\{d \in D: d \leqslant x\} = \{d \in D_n: d \leqslant x\}$ is finite. If $x \prec y$, $x \prec z$, $y \neq z$, and $y \uparrow z$ in $(D, \leqslant)$, then $y \vee z \in D_n$ exists and $y \prec y \vee z$, $z \prec y \vee z$ in $(D_n, \leqslant)$, hence also in $(D, \leqslant)$. Finally, if $[x, x'] \succ\!\prec [y, y']$ and $x \leqslant y$ in $(D, \leqslant)$, then also $[x, x'] \succ\!\prec [y, y']$ in some $(D_m, \leqslant)$ $(m \geqslant n)$ and thus $x' \leqslant y'$. Hence $(D, \leqslant) \in \mathscr{D}$.

Similarly, it is easy to check that if each $(D_n, \leqslant)$ satisfies condition (V) (or (Q), respectively), then so does $(D, \leqslant)$. Finally, note that domains are distributive iff the distributivity condition is satisfied by all compact elements. By Theorem 3.1, the result follows.

We conclude with some remarks as to how Theorem 3.4 can be used to solve recursive domain equations. Let $\mathscr{C}$ be any one of the categories of Theorem 3.4, and let F be a continuous functor on $\mathscr{C}$ (i.e., F commutes with limits of ω-chains in $\mathscr{C}$). Suppose $D \in \mathscr{C}$ is such that there exists a SEPP $\varphi = (f, g)$ from D into $F(D)$. Then $F^n(\varphi)$ is a SEPP rom $F^n(D)$ into $F^{n+1}(D)$ $(n \in \mathbb{N})$, and letting D^* be the limit of the ω-chain $(F^n(D), F^n(\varphi))_{n \in \mathbb{N}}$ in $\mathscr{C}$, we obtain $D^* \cong F(D^*)$ by continuity of F. For example, the binary product functor $\times$ is a continuous functor on $\mathscr{D}$ (as can be seen from Theorem 3.1(a)). As for any event domain D there exist (in general many) SEPPs from D to $D \times D$, we obtain various non-trivial solutions of the domain equation $D^* \cong D^* \times D^*$ within $\mathscr{D}$.

RECEIVED February 23, 1989; FINAL MANUSCRIPT RECEIVED January 23, 1990

REFERENCES

BARENDREGT, H. P. (1981), "The Lambda Calculus: Its Syntax and Semantics," Studies in Logic and the Foundations of Mathematics, Vol. 103, North-Holland, Amsterdam.

BELL, J. L., AND SLOMSON, A. B. (1974), "Models and Ultraproducts: An Introduction," North-Holland, Amsterdam.

BERRY, G. (1978), Stable models of typed λ-calculi, *in* "Fifth International Colloquium on Automata, Languages and Programs," pp. 72–89, Lecture Notes in Computer Science, Vol. 62, Springer-Verlag, Berlin/New York.

BERRY, G., AND CURIEN, P. L. (1982), Sequential algorithms on concrete data structures, *Theoret. Comput. Sci.* **20**, 265–321.

CAMERON, P. (1989), Groups of order-automorphisms of the rationals with prescribed scale type, *J. Math. Psychol.* **33**, 163–171.

COQUAND, T., GUNTER, C., AND WINSKEL, G. (1988), DI-domains as a model of polymorphism, *in* "Mathematical Foundations of Programming Language Semantics" (M. Main, A. Melton, M. Mislove, and D. Schmidt, Eds.), pp. 344–363, Lecture Notes in Computer Science, Vol. 298, Springer-Verlag, Berlin/New York.

CURIEN, P. L. (1986), "Categorical Combinators, Sequential Algorithms and Functional Programming," Research Notes in Theoretical Computer Science, Pitman, London.

DROSTE, M. (1989), Event structures and domains, *Theoret. Comput. Sci.* **68**, 37–47.

DROSTE, M. (1989), Recursive domain equations for concrete data structures, *Inform. Comput.* **82**, 65–80.

DROSTE, M., AND GÖBEL, R. (1990), Non-deterministic information systems and their domains, *Theoret. Comput. Sci.* **75**, 289–309.

DROSTE, M., AND GÖBEL, R. (1990), Universal domains in the theory of denotational semantics of programming languages, *in* 5th IEEE Symposium on "Logic in Computer Science" (Proc., Philadelphia), pp. 19–34, IEEE Computer Society Press.

FRAISSÉ, R. (1954), Sur l'extension aux relations de quelques propriétés des ordres, *Annales E.N.S.* **71**, 363–388.

FRAISSÉ, R. (1986), "Theory of Relations," North-Holland, Amsterdam.

GUNTER, C. A. (1987), Universal profinite domains, *Inform. Comput.* **72**, 1–30.

GUNTER, C. A. AND JUNG, A. (1990), Coherence and consistency in domains, *J. Pure Appl. Algebra* **63**, 49–66.

KAHN, G., AND PLOTKIN, G. (1978), Domaines concretes, Rapport de Recherche No. 336, IRIA, Paris.

KOYMANS, C. P. J. (1982), Models of the lambda calculus, *Inform. Control* **52**, 306–332.

MAIER, B. (1987), On countable locally described structures, *Ann. Pure Appl. Logic* **35**, 205–246.

PLOTKIN, G. (1978), T^{ω} as a universal domain, *J. Comput. System Sci.* **17**, 209–236.

SCOTT, D. (1978), Data types as lattices, *SIAM J. Comput.* **5**, 522–587.

SCOTT, D. (1981), Some ordered sets in computer science, *in* "Ordered Sets" (I. Rival, Ed.), pp. 677–718, Reidel, Dordrecht.

SMYTH, M. B., AND PLOTKIN, G. (1982), The category-theoretic solution of recursive domain equations, *SIAM J. Comput.* **11**, 761–783.

WINSKEL, G. (1981), "Events in Computation," Ph.D. Thesis, Edinburgh.

WINSKEL, G. (1987), Event structures, *in* "Petri Nets: Applications and Relationships to Other Models of Concurrency" (W. Brauer, W. Reisig, and G. Rozenberg, Eds.), pp. 325–392, Lecture Notes in Computer Science, Vol. 255, Springer-Verlag, Berlin/New York.

ZHANG, G. (1989), DI-domains as information systems, *in* "16th International Colloquium on Automata, Languages and Programming," pp. 773–788, Lecture Notes in Computer Science, Vol. 372, Springer-Verlag, Berlin/New York.