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Gauge hierarchy from a topological viewpoint?

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Abstract

In this Letter we explore an alternative to the central point of the Randall–Sundrum brane world scenario, namely, the particular non-factorizable metric, in order to solve the hierarchy problem. From a topological viewpoint, we show that the exponential factor, crucial in the Randall–Sundrum model, appears in our approach, only due to the brane existence instead of a special metric background. Our results are based in a topological gravity theory via a non-standard interaction between scalar and non-Abelian degrees of freedom and in calculations about localized modes of matter fields on the brane. We point out that we obtain the same results of the Randall–Sundrum model using only one 3-brane, since a specific choice of a background metric is no longer required.

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May the standard model be placed in form of the recent insights coming from string theories, where several dimensions appear so naturally? The standard model for strong, weak and electromagnetic interactions, described by the gauge group $SU(3) \times SU(2) \times U(1)$, has its success strongly based on experimental evidences. However, it has several serious theoretical drawbacks suggesting the existence of new and unexpected physical facts beyond those discussed in the last years. One of these problems is the so-called *gauge hierarchy problem* which is related to the weak and Planck scales, the fundamental scales of the model. The central idea of this problem is to explain the smallness and radiative stability of the hierarchy $M_{\mathrm{ew}}/M_{\mathrm{pl}} \sim 10^{-17}$. In the context of the minimal standard model, this hierarchy of scales is unnatural since it requires a fine-tuning order by order in the perturbation theory. The first attempts to solve this problem were the technicolor scenario [1] and the low energy supersymmetry [2].

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With the string theories, the search of many-dimensional theories became important. The basic idea is that extra dimensions can be used to solve the hierarchy problem: the fields of the standard model must be confined to a $(3+1)$-dimensional subspace, embedded in a n-dimensional manifold. In the seminal works of Arkani-Hamed, Dimopoulos, Dvali and Antoniadis [3], the 4-dimensional Planck mass is related to M, the fundamental scale of the theory, by the extra-dimensions geometry. Through the Gauss law, they have found $M_{\rm pl}^2 = M^{n+2}V_n$, where V_n is the extra dimensions volume. If V_n is large enough, M can be of the order of the weak scale. However, unless there are many extra dimensions, a new hierarchy is introduced between the compactification scale, $\mu_{\rm c} = V^{-1/n}$, and M. An important feature of this model is that the space–time metric is factorizable, i.e., the n-dimensional space–time manifold is approximately a product of a 3-dimensional space by a compact $(n-3)$-dimensional manifold.

Because of this new hierarchy, Randall and Sundrum [4] have proposed a higher-dimensional scenario that does not require large extra dimensions, neither the supposition of a metric factorizable manifold. Working with a single S^1/Z_2 orbifold extra dimension, with three-branes of opposite tensions localized on the fixed points of the orbifold and with adequate cosmological constants as 5-dimensional sources of gravity, they have shown that the space–time metric of this model contains a redshift factor which depends exponentially on the radius $r_{\rm c}$ of the compactified dimension:

$$ds^2 = e^{-2kr_{\rm c}|\phi|}\eta_{\mu\nu}\,dx^\mu\,dx^\nu - r_{\rm c}\,d\phi^2, \qquad (1)$$

where k is a parameter of the order of M, x^μ are Lorentz coordinates on the surfaces of constant ϕ, and $-\pi \leqslant \phi \leqslant \pi$ with (x,ϕ) and $(x,-\phi)$ identified. The two 3-branes are localized on $\phi = \pi$ and $\phi = 0$. In fact, this scenario is well known in the context of string theory [5]. The non-factorizable geometry showed in Eq. (1) has at least two important consequences that will be discussed here. The first one is that the 4-dimensional Planck mass is given in terms of the fundamental scale M by

$$M_{\rm pl}^2 = \frac{M^3}{k}\left[1 - e^{-2kr_{\rm c}\pi}\right], \qquad (2)$$

in such a way that, even for large $kr_{\rm c}$, $M_{\rm pl}$ is of the order of M. The second one is that because of the exponential factor on the space–time metric, a field confined to a 3-brane at $\phi = \pi$ with mass parameter m_0 will have physical mass $m_0 e^{-kr_{\rm c}\pi}$ and for $kr_{\rm c}$ near of 12, the weak scale is dynamically generated by the fundamental scale M which is of the order of the Planck mass.

On the other hand, background independent theories are welcome. As an example it is worth mentioning the quantum loop gravity, developed mainly by Asthekar et al. [6,7]. Also the problem of background dependence of string field theory has not been successfully addressed. The string field theory has a theoretical problem: it is only consistently quantized in particular backgrounds, which means that we have to specify a metric background in order to write down the field equations of the theory. This problem is fundamental because a unified description of all string backgrounds would make possible to answer questions about the selection of particular string vacua and in general to give us a more complete understanding of geometrical aspects of string theory [8].

In this Letter we explore an alternative to the central point of the Randall–Sundrum model, namely, the particular non-factorizable metric. Using a topological theory, we show that the exponential factor, crucial in the Randall–Sundrum model, appears in our approach, only due to the brane existence instead of a special metric background.

Some searches have been made trying to implement branes as topological defects in order to solve the hierarchy problem [9]. Here the brane is simulated by a 3-dimensional domain wall embedded in a 5-dimensional space–time. Domain walls are simple solitons, objects whose great stability is due to the non-trivial topology of the parameter space of the theory [10]. They only appear after phase transitions, specifically, when discrete symmetries are broken.

In order to study the hierarchy problem we choose to work with topological gravity. Motivated by current searches in the quantum gravity context [6,11], we study topological gravity of $B \wedge F$ type [12,13]. Then, we can affirm that our model is purely topological because (1) the brane exists due to the topology of the parameter space of the model and (2) gravity is metric independent. We will see that these features give us interesting results when compared to the Randall–Sundrum model.

The model is based on the following action:

$$S=\int d^5x\left[\frac{1}{2}\partial_\mu\theta\partial^\mu\theta+k\varepsilon_{\mu\nu\alpha\rho\lambda}\theta H^a_{\mu\nu\alpha}F^a_{\rho\lambda}-V(\theta)\right]. \tag{3}$$

In this action the θ field is a real scalar field that is related to the domain wall. In this context, the presence of a kinetic term for the θ field (together with the symmetry breaking potential), is required to construct a topological defect (the domain wall). We remark that the θ field acts as a background field in order to provide a brane where we have an effective BF-type theory. The fields $H^a_{\mu\nu\alpha}$ and $F^a_{\rho\lambda}$ are non-Abelian gauge fields strengths and will be related to the gravitational degrees of freedom. Namely, in pure gauge theory, $H^a_{\mu\nu\alpha}=\partial_\mu B^a_{\nu\alpha}-\partial_\nu B^a_{\alpha\mu}-\partial_\alpha B^a_{\mu\nu}+gf^{abc}A^b_\mu B^c_{\nu\alpha}$ and $F^a_{\mu\nu}=\partial_\mu A^a_\nu-\partial_\nu A^a_\mu+g'f^{abc}A^b_\mu A^c_\nu$. The second term of this action is a topological term that generalizes to $D=5$ the theta-term of QCD. To see this, it is enough to do a simple dimensional reduction, namely, define $B^a_{\mu 5}=-B^a_{5\mu}=V^\mu_a$, $A^a_5=\varphi$, $\varepsilon_{5\nu\alpha\rho\lambda}\equiv\varepsilon_{\nu\alpha\rho\lambda}$ and $\partial_5 G(x^\mu)=0$, where G is any field of this model. Then, the theta-term arises as a result from the compactification procedure defined above, as

$$\int d^5x\,k\varepsilon_{\mu\nu\alpha\rho\lambda}\theta H^a_{\mu\nu\alpha}F^a_{\rho\lambda}\to\int d^4x\,k'\varepsilon_{\nu\alpha\rho\lambda}\theta V^a_{\nu\alpha}F^a_{\rho\lambda}, \tag{4}$$

where $V^a_{\nu\alpha}=\partial_\nu V^a_\alpha-\partial_\alpha V^a_\nu+gf^{abc}V^b_\nu V^c_\alpha$. Identifying V^a_α with A^a_α we obtain the term discussed. Because of this fact, the θ field can be thought as the axionic field. The axion has appeared as a proposal to solve the strong CP problem [14]. The presence of instantons in the theory results in an effective term added to the QCD action, namely, $\sim\int d^4x\,\varepsilon^{\nu\alpha\rho\lambda}\theta F^a_{\nu\alpha}F^a_{\rho\lambda}$, which violates CP symmetry. The problem is solved when we add to the theory the axionic field with the imposition of a new symmetry, the Peccei–Quinn symmetry, that is $\theta\to\theta+a$ (a is a constant which contains the CP violating quantities of the theory). The action (3) is invariant under the Peccei–Quinn symmetry transformation

$$\theta\to\theta+2\pi n. \tag{5}$$

The axionic potential is

$$V(\theta)=\lambda(1-\cos\theta), \tag{6}$$

which preserves the Peccei–Quinn symmetry. Nevertheless, it is spontaneously broken in scales of the order of $M_{\rm PQ}\sim 10^{10}$–10^{12} GeV. This value is obtained from cosmological and experimental constraints [15]. The potential (6) is not interesting for our purposes. The fact is that domain walls appeared for the first time in the universe in the QCD phase transition era, i.e., when $T_{\rm QCD}\sim 100$ MeV [16], a scale relatively close to the weak scale $M_{\rm ew}\sim 10^3$ GeV. In this situation, the Peccei–Quinn symmetry is explicitly broken ($U_{\rm PQ}(1)\to Z(N)$) by instanton effects. It is possible to simulate this explicit break by a simple theoretical field toy model. For such, we write $V(\theta)$ as a polinomial potential in powers of θ, what is equivalent to take terms only up to the second order in the expansion of the Eq. (6). We propose the following potential

$$V(\theta)=\frac{\lambda}{4}\left(\theta^2-v^2\right)^2, \tag{7}$$

which explicitly breaks the $U_{\rm PQ}(1)$ Peccei–Quinn symmetry, in order to generate a brane in an energy close to the weak scale. With this particular choice of the potential, the existence of the brane is put on more consistent grounds. In other words, the brane appears almost exactly in an energy scale of the universe near the symmetry breaking scale of the electroweak theory. This feature was assumed in previous works without a careful justification. However, this mechanism leads to a large disparity between the Planck mass $M_{\rm pl}\sim 10^{18}$ GeV and the scale of explicit breaking of $U_{\rm PQ}(1)$ which is relatively close to the weak scale, $M_{\rm ew}\sim 10^3$ GeV: we assume this disparity as a new version of the hierarchy problem.

The equation of motion of the θ field considering the potential (7) is the following:

$$\theta+\lambda\theta^3-\lambda v^2\theta=k\varepsilon_{\mu\nu\alpha\rho\lambda}H^a_{\mu\nu\alpha}F^a_{\rho\lambda}. \tag{8}$$

This equation is easily solved. Supposing a static configuration and that $\theta\equiv\theta(x_4)$, the solution is:

$$\theta(x_4)=v\tanh\left(\sqrt{\frac{\lambda}{2}}vx_4\right). \tag{9}$$

This solution defines a 3-brane embedded in a $(4+1)$-dimensional space–time. The mass scale of this model is $m=\sqrt{\lambda}v$ and the domain wall-brane thickness is

m^{-1}. With this information we can now discuss the effective theory on the domain wall-brane. An integration by parts of the topological term in the action (3) will result in

$$\varepsilon_{\mu\nu\alpha\rho\lambda}\theta(x_4)H^a_{\mu\nu\alpha}F^a_{\rho\lambda} = -3\varepsilon_{\mu\nu\alpha\rho\lambda}\partial_\mu\theta B^a_{\nu\alpha}F^a_{\rho\lambda} + \cdots, \tag{10}$$

where we do not consider complicated interactions and linear terms on θ (the function (9) is odd). Because of $\theta \equiv \theta(x_4)$ the summation on the μ index will result only in a derivative term of the x_4 coordinate. Then, the Levi-Civita tensor $\varepsilon_{\mu\nu\alpha\rho\lambda}$ will be an authentic four-dimensional tensor: $\varepsilon_{4\nu\alpha\rho\lambda} \equiv \varepsilon_{\nu\alpha\rho\lambda}$. We have assumed that the tensors $B^a_{\mu\nu}$ and $A_{\rho a}$ are weakly dependent on the x_4 coordinate. Then, the second term of the action (3) is rewritten as

$$S \sim \int d^4x\, \varepsilon_{\nu\alpha\rho\lambda} B^a_{\nu\alpha}F^a_{\rho\lambda}\left[\lim_{r_c\to+\infty} k' \int_0^{r_c} dx_4\, \partial_4\theta(x_4)\right], \tag{11}$$

where r_c represents the extra dimension. This last conclusion denotes the domain wall-brane contribution to the effective four-dimensional theory. We can see that, effectively on the domain wall-brane, the theory is purely 4-dimensional (this is important) and is described by a non-Abelian topological $B \wedge F$ term. The importance of this fact is that there are several approaches to topological gravity by means of $B \wedge F$ type models in $D = 4$ and by Chern–Simons models in $D = 3$. In Ref. [12], the authors construct a $SU(2)$, $D = 4$ BF gravity in a basis independent formulation. The point we would like to comment on that article is that the tensorial field B is a 1-form gauge valued field. We stress that the structure of the BF term in our work is the same as in Ref. [12], i.e., our BF gravity on the brane is of the type $SU(2)$, $D = 4$.

Note that this approach opens the possibility to implement topological gravity on the brane. In these models, the fundamental fields are known. For example, the tetrad fields in $D = 4$: the metric is, by itself, a secondary object. The gauge symmetries of these theories are, actually, the symmetries of the general relativity [13]. It can be shown that, under parameterizations by tetrad fields, a $B \wedge F$ type action gives us

$$\int d^4x\, k\varepsilon^{\nu\alpha\rho\lambda} B^a_{\nu\alpha}F^a_{\rho\lambda} \to k\int d^4x\, \sqrt{g}R, \tag{12}$$

which is the Einstein–Hilbert action for the gravitational field, where R is the scalar curvature and g stands for the space–time metric [12]. It is not well understood if Eq. (11) can really describe the dynamics of the gravitational field [17]. In a model like this, the constant k has a direct relation with the Planck mass. From Eqs. (11) and (12), we can see the relation between the Planck mass k_4 in $D = 4$ and the extra dimension:

$$k_4 = \lim_{r_c\to+\infty} k' \int_0^{r_c} dx_4\, \partial_4\theta(x_4). \tag{13}$$

The limit $r_c \to +\infty$ ensures the topological stability of the domain wall-brane. By the substitution of Eq. (9) in Eq. (13), considering a finite r_c (which means that the domain wall-brane is a finite object), we can show that

$$k_4 = k'v\left(1 - e^{-2y}\right)\left(1 + e^{-2y}\right)^{-1}, \tag{14}$$

where $y = \sqrt{\lambda/2}\,v r_c$ is the scaled extra dimension. This result is very interesting: as our model is a topological one, the exponential factor must not appear from any special metric. Here, the exponential factor appears only due to the domain wall-brane existence. As in the Randall–Sundrum model, even for the large limit $r_c \to +\infty$, the 4-dimensional Planck mass has a specific value. This is the reason why we believe that our model can be used to treat the hierarchy problem.

We can make an estimative of the order of the extra dimension considering that the domain wall-brane thickness is of the order of $M_{ew} \sim 10^3$ GeV. This means that the fields confined to the domain wall-brane do not perceive the extra dimension, unless they interact with energies greater than M_{ew}. In this case, they can escape out of the brane, living in the higher-dimensional space–time [18]. By the calculation of domain wall-brane energy per unity volume σ we can find a simple polynomial equation of third degree in the $z = \theta(r_c)$ variable, containing all phase transition information:

$$m^{-1}\sigma = \sqrt{2}vz - \sqrt{2}m^{-1}z^3. \tag{15}$$

For the case of the Randall–Sundrum model, the extra dimension is calculated through the normalized radial oscillation field (referred by some authors as radion field [19]), i.e., it is stabilized by a mechanism of symmetry breaking involving bulk fields [20].

We will now discuss about matter confined to the brane. It is a well-known fact that domain walls may have bound states of fields attached to them [18]. For the case of scalar fields, it was shown using WKB approximation that a particular zero-mode living in the domain wall-brane is given by the following field:

$$\varphi'\left(x^0, \boldsymbol{x}, x^4\right) = \frac{d\varphi(x^4)}{dx^4}\exp\left(-i\boldsymbol{k}\cdot\boldsymbol{x} + iEx^0\right); \quad E^2 = (\boldsymbol{k}\cdot\boldsymbol{k})^2. \tag{16}$$

In the last equation, $d\varphi(x^4)/dx^4 = Ce^{-2Ax_4}(1 + e^{-Ax_4})^{-2}$, C and A are constant parameters. In particular, a similar result is true for fermions. Then the zero-modes, bosonic or fermionic ones, are scaled by an exponential factor, just like in the Randall–Sundrum scenario. Despite the fact that they are non-massive fields, there are mechanisms involving several interacting fields [21] that generate spontaneous symmetry breaking in the defect core. In this way, the confined fields can acquire non-zero masses. In order to show this for the case of scalar fields, we use two real scalar fields: $\phi(x^0, \boldsymbol{x}, x^4)$ and $\eta(x^0, \boldsymbol{x})$. We regard the first one as a 4-dimensional confined field, i.e., $\phi(x^0, \boldsymbol{x}, x^4) = f(x^4)\varphi(x^0, \boldsymbol{x})$, where $f(x^4)$ is just the warp factor that comes from the extra dimension. The second one is a massless and purely 4-dimensional field. We built the following Lagrangian density

$$L = \frac{1}{2}\partial_\mu\eta\partial^\mu\eta + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - g\phi^2\eta^2 - V(\phi), \tag{17}$$

where $V(\phi) = -m^2\phi^2 + \frac{\lambda}{4!}\phi^4$ is a potential that spontaneously breaks the $\phi \to -\phi$ symmetry. In this case, if the extra dimension is finite and constant then, during the phase transition, only the φ field will oscillate, i.e., $\phi = f(x^4)\varphi \to f(x^4)[v + \chi]$, where v is the vacuum expectation value of the ϕ field and χ is the fluctuation around the vacua. Working out this idea in the last Lagrangian we can show that, after the phase transition, the η field will acquire a mass of the order of $f(x^4)v \sim e^{-2Ax_4}(1 + e^{-Ax_4})^{-1}v$. This expression is analogous to the Randall–Sundrum result [4], which provides a physical mass for fields of the standard model corrected by the warp factor. Therefore, this simple mechanism allows us to generate scales from fields confined to a domain wall-brane, without the requirement of a particular metric.

There is a final remark about gravity in this context: the matter zero-modes live effectively in $D = 4$ and, then, they must contribute to the effective four-dimensional energy–momentum tensor. They are, in fact, gravitational sources in the domain wall-brane space–time. Consequently, we can construct a propagation term for the gravitational field in $D = 4$ (on the brane). However, as can be seen from Eq. (12) it is possible to build a propagation term for gravity from a topological term. Therefore it is interesting to discuss if we can use Eq. (11) as an authentic propagation term for these gravitational degrees of freedom. This will be discussed in a forthcoming paper [22].

Summarizing, we have shown that a simple topological model in field theory has the necessary features to solve the gauge hierarchy problem in a very similar way to the one found by L. Randall and R. Sundrum. With this model we have built a stable 3-brane (a domain wall-brane) that simulates our four-dimensional universe and we have argued the possibility of topological gravity localization. Because of these facts, the exponential factor appears only due to the existence of the domain wall-brane and not from a special metric. Then, we have calculated the effective Planck mass in $D = 4$, pointing out the great similarity between our result and that of the Randall–Sundrum model. We have calculated a polinomial equation for the size of the extra dimension using some features of models containing domain walls. Finally, we have made a commentary about the zero-modes bounded by the domain wall-brane, remarking the fact that they are scaled by an exponential factor. This information makes possible the emergence of the electroweak scale.

We did not comment about how to introduce the cosmological constant in this model. In fact, in the Randall–Sundrum model the cosmological constant is extremely important because it is responsible for the final form of the metric given by Eq. (1). Another interesting fact is that brane models can answer the following question: *why is the cosmological constant so small?* These are good problems for future investigations in this topological approach.

The analysis of models containing several domain walls is also interesting. In this case, the potential that implements the phase transition has various stable vacua. Domain walls will appear interpolating these vacua in well defined positions: the distance between two domain walls is constant due to the topological stability of the model. Can we see this as another possible way to solve the *moduli stabilization problem*?

By virtue of the simplicity of this model, we can extend it to include supersymmetry. Indeed, brane world models suggests alternative mechanisms to the breaking of supersymmetry in our universe. All of these subjects are interesting research objectives.

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