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Seesaw options for three neutrinos

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ABSTRACT

The seesaw mechanism for three neutrinos is discussed, clarifying the situation where the seesaw texture results in three approximately zero mass eigenvalues. The true underlying mechanism is shown to be just the *inverse* (or *linear*) seesaw, which explains why there could be large mixing. However, these zeroes cannot occur naturally, unless there is a conserved symmetry, i.e. lepton number L, either global or gauged, which is softly or spontaneously broken at the TeV scale. We discuss in particular the case where the three heavy singlet neutrinos have $L = 3, -2, -1$.

In the famous canonical seesaw mechanism, the Standard Model (SM) of particle interactions is implemented with a heavy singlet "right-handed" neutrino N_R per family, so that the otherwise massless left-handed neutrino ν_L gets a mass from diagonalizing the 2×2 mass matrix spanning $(\bar{\nu}_L, N_R)$:

$$\mathcal{M}_{\nu,N} = \begin{pmatrix} 0 & m_D \\ m_D & m_N \end{pmatrix}, \tag{1}$$

resulting in

$$m_\nu \simeq \frac{-m_D^2}{m_N}, \tag{2}$$

with mixing between ν_L and N_R given by

$$\tan\theta \simeq \frac{m_D}{m_N} \simeq \sqrt{|m_\nu/m_N|}. \tag{3}$$

As a result, the 3×3 mixing matrix linking the 3 light neutrinos to the 3 charged leptons cannot be exactly unitary. However, for $m_\nu \sim 1$ eV and $m_N \sim 1$ TeV, this violation of unitarity is of order 10^{-6}, which is much too small to be observed.

Suppose the 6×6 mass matrix spanning $\nu_{1,2,3}$ and $N_{1,2,3}$ has three zero mass eigenvalues, without requiring $m_D = 0$ identically [1], then it has been pointed out that the addition of small perturbations to this texture will result in acceptably small neutrino masses as well as possible large mixing [2–5] between $\nu_{1,2,3}$ and $N_{1,2,3}$, in contrast to the case of only one family. It this Letter, we will discuss what this really means, and show that the underlying mechanism for the origin of this large mixing is just the *inverse* seesaw [6–8] with a conserved symmetry, i.e. lepton number L, which may be global (and softly or spontaneously broken) or gauged (and spontaneously broken). We will implement this idea with a specific model with $L = 3, -2, -1$ for $N_{1,2,3}$.

For simplicity, consider first two families. It has been argued that large mixing between (ν_1, ν_2) and (N_1, N_2) may occur if the Dirac mass matrix linking them is of the form

$$\mathcal{M}_D = \begin{pmatrix} a_1 b_1 & a_1 b_2 \\ a_2 b_1 & a_2 b_2 \end{pmatrix}, \tag{4}$$

in the basis where

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$$\mathcal{M}_N = \begin{pmatrix} M_1' & 0 \\ 0 & M_2' \end{pmatrix}. \tag{5}$$

In that case, the arbitrary imposed condition

$$\frac{b_1^2}{M_1'} + \frac{b_2^2}{M_2'} = 0 \tag{6}$$

renders all two light neutrinos massless, without requiring $\mathcal{M}_D = 0$. To understand what this really means, first note that the determinant of $\mathcal{M}_D$ is zero, hence there is only one nonzero eigenvalue. Then consider the most general 4×4 mass matrix spanning (ν_1, ν_2, N_1, N_2) in the basis where $\mathcal{M}_D$ is diagonal, i.e.

$$\mathcal{M}_{\nu,N} = \begin{pmatrix} 0 & 0 & m_1 & 0 \\ 0 & 0 & 0 & m_2 \\ m_1 & 0 & M_1 & M_3 \\ 0 & m_2 & M_3 & M_2 \end{pmatrix}. \tag{7}$$

Changing to this basis does not say anything about the basis of the charged-lepton mass matrix which is still arbitrary. Neither approach has fixed it to be diagonal. Rotating the nondiagonal $\mathcal{M}_D$ of Eq. (4) on the left with $\tan\theta_L = a_1/a_2$ by the matrix

$$\mathcal{U}_L^\dagger = \begin{pmatrix} \cos\theta_L & -\sin\theta_L \\ \sin\theta_L & \cos\theta_L \end{pmatrix}, \tag{8}$$

and on the right with $\tan\theta_R = b_1/b_2$ by the matrix

$$\mathcal{U}_R = \begin{pmatrix} \cos\theta_R & \sin\theta_R \\ -\sin\theta_R & \cos\theta_R \end{pmatrix}, \tag{9}$$

the texture hypothesis is equivalent to setting

$$m_1 = 0, \qquad m_2 = \sqrt{a_1^2 + a_2^2}\sqrt{b_1^2 + b_2^2}, \tag{10}$$

$$M_1 = \cos^2\theta_R M_1' + \sin^2\theta_R M_2' = 0, \tag{11}$$

$$M_2 = \sin^2\theta_R M_1' + \cos^2\theta_R M_2' = \left(1 - \tan^2\theta_R\right)M_2', \tag{12}$$

$$M_3 = \sin\theta_R \cos\theta_R \left(M_1' - M_2'\right) = -\tan\theta_R M_2'. \tag{13}$$

It is then clear that ν_1 and the linear combination $\nu_2' = (M_3\nu_2 - m_2 N_1)/\sqrt{M_3^2 + m_2^2}$ are massless. Once small perturbations are added, i.e. $0 \neq m_1 \ll m_2$ and $0 \neq M_1 \ll M_{2,3}$, ν_2' gets a small mass proportional to M_1 given by $(m_2^2/M_3^2)M_1$ through the inverse seesaw, and the possibly large ν_2–N_1 mixing remains. The complete reduced 2×2 mass matrix spanning ν_1 and ν_2' is given by

$$\mathcal{M}_\nu \simeq \begin{pmatrix} m_1^2 M_2/M_3^2 & -m_1 m_2/M_3 \\ -m_1 m_2/M_3 & m_2^2 M_1/M_3^2 \end{pmatrix}. \tag{14}$$

Since $M_2 \sim M_3$ in this hypothesis, the $(1,1)$ entry is a canonical seesaw, whereas the $(2,2)$ entry is an inverse seesaw. The $(1,2)$ or $(2,1)$ entry is known as the linear seesaw [9], but it is equivalent to the inverse seesaw, as explained in Ref. [10]. Note first that if $m_1 = 0$, then only ν_2' gets a small mass (because M_1 is small) through the inverse seesaw. If $M_1 = 0$, then since $m_1 M_2/M_3 \ll m_2$ is assumed in such a texture scenario, the two neutrinos are pseudo-Dirac partners and are nearly degenerate in mass. If $m_1 \neq 0$ and $M_1 \neq 0$, then it is possible to have a solution where the $(1,1)$ entry is negligible and the other entries are comparable.

It has been argued that such a texture (resulting in two massless fermions, i.e. ν_1 and ν_2') is protected by chiral symmetry. Whereas this may be correct for ν_1, it is obviously not true for ν_2' because ν_2 couples to N_2, and N_2 has a nonzero Majorana mass, i.e. M_2. The one-loop diagram connecting ν_2 to itself through N_2 and the SM Higgs boson is infinite and there is no corresponding diagram from N_1 to cancel it. Thus the Majorana mass of ν_2' has an infinite correction and cannot be zero naturally. The texture idea alone has no support in terms of a symmetry.

On the other hand, if $M_2 = 0$, then a conserved lepton number L can be defined, with $L = 1$ for N_1 and $L = -1$ for N_2. If small $M_{1,2}$ and m_1 are now added, thus breaking L to $(-1)^L$, Eq. (14) will be obtained with a very small $(1,1)$ entry. The difference between the texture hypothesis and that supported by lepton number is thus M_2. It is nonzero in the former but zero in the latter. The infinite diagram for the correction to the zero mass of ν_2' in the former is absent in the latter, precisely because $M_2 = 0$.

To maintain Eq. (7) with $m_1 = M_1 = 0$ and $M_2 \sim M_3$, the lepton-number global symmetry has to be redefined, with for example $L = 3, -1$ for $N_{1,2}$. In that case, the addition of the standard Higgs doublet $\Phi_1 = (\phi_1^+, \phi_1^0)$ with $L = 0$ will link ν_2 with N_2 to obtain m_2, whereas a Higgs singlet χ_2 with $L = 2$ will supply N_2 with the Majorana mass M_2, and its complex conjugate $\chi_2^\dagger$ will link N_1 with N_2 to obtain M_3. The absence of a Higgs singlet with $L = 6$ will forbid a Majorana mass M_1 for N_1 at tree level, but it will be induced by the mass splitting of $\mathrm{Re}(\chi_2)$ and $\mathrm{Im}(\chi_2)$ in one loop after the breaking of $U(1)_L$, as shown in Fig. 1. This diagram is finite because of the cancellation between $\mathrm{Re}(\chi_2)$ and $\mathrm{Im}(\chi_2)$. If $U(1)_L$ is spontaneously broken, then $\mathrm{Im}(\chi_2)$ is a massless Goldstone boson, resulting in a majoron which is dominated by $\mathrm{Im}(\chi_2)$ but also picks up a small doublet component. If $U(1)_L$ is explicitly broken but only softly, with the addition of the term $\mu^2\chi_2^2 + \mathrm{H.c.}$ for example, then $\mathrm{Im}(\chi_2)$ is massive.

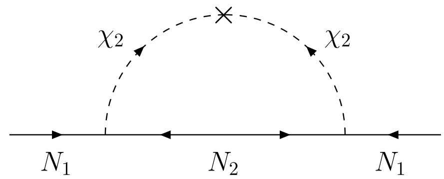

Fig. 1. One-loop generation of M_1.

Consider now the most general 6×6 mass matrix spanning $(\nu_{1,2,3}, N_{1,2,3})$:

$$\mathcal{M}_{\nu,N} = \begin{pmatrix} 0 & 0 & 0 & m_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & m_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & m_3 \\ m_1 & 0 & 0 & M_1 & M_4 & M_5 \\ 0 & m_2 & 0 & M_4 & M_2 & M_6 \\ 0 & 0 & m_3 & M_5 & M_6 & M_3 \end{pmatrix}. \tag{15}$$

The texture hypothesis is equivalent to $m_1 = m_2 = 0$ and $M_1 = M_4 = 0$. Again it is clear that there could be large mixing between ν_3 and N_1, but there is no symmetry which enforces it. Consider now lepton number with $L = 1$ for $N_{1,2}$ and $L = -1$ for N_3, then $M_1 = M_2 = M_3 = M_4 = 0$ and the linear combination $(M_6 N_1 - M_5 N_2)/\sqrt{M_5^2 + M_6^2}$ is massless. Once small perturbations are added, this becomes a scenario for four light neutrinos (three active and one sterile). Suppose N_1 has $L = 1$, N_2 has $L = 0$, N_3 has $L = -1$, then $M_1 = M_3 = M_4 = M_6 = 0$. In this case, N_2 has mass M_2, and there are exactly three massless neutrinos.

To maintain the seesaw texture $m_{1,2} = 0$, $m_3 \neq 0$, $M_{1,4} = 0$, and $M_{2,3,5,6} \neq 0$, the lepton-number global symmetry must again be redefined. Let $\nu_{1,2,3}$ have $L = 1$ as usual, and $N_{1,2,3}$ have $L = 3, -2, -1$ respectively. Let there again be a Higgs doublet Φ_1 with $L = 0$ and now three Higgs singlets $\chi_{2,3,4}$ with $L = 2, 3, 4$. Then M_3 comes from $\langle\chi_2\rangle$, M_5 from $\langle\chi_2^\dagger\rangle$, M_6 from $\langle\chi_3\rangle$, and M_2 from $\langle\chi_4\rangle$. The three massless eigenstates are

$$\nu_1, \nu_2, \nu_3' = \frac{M_5 \nu_3 - m_3 N_1}{\sqrt{M_5^2 + m_3^2}}, \tag{16}$$

showing explicitly how $\nu_3 - N_1$ mixing can be large even if all neutrinos are massless. The analog of Fig. 1 now applies to M_1 and M_4, both of which obtain one-loop finite masses, resulting in an inverse seesaw mass for ν_3', i.e. $M_1 m_3^2/M_5^2$. As for $\nu_{1,2}$ masses, we need extra Higgs doublets. Consider the minimal case of a second Higgs doublet $\Phi_2 = (\phi_2^+, \phi_2^0)$ with $L = 1$. It couples $\nu_{1,2,3}$ to N_2. By redefining $\nu_{1,2}$, we consider only the couplings to $\nu_{2,3}$, resulting in the masses m_{22} and m_{32}. Thus ν_1 remains massless and the reduced 2×2 mass matrix spanning ν_2 and ν_3' is given by

$$\begin{pmatrix} -m_{22}^2/M_2 & -(m_{22}/M_2)[m_{32} + m_3(M_1 M_6 - M_4 M_5)/M_5^2] \\ -(m_{22}/M_2)[m_{32} + m_3(M_1 M_6 - M_4 M_5)/M_5^2] & M_1 m_3^2/M_5^2 \end{pmatrix}. \tag{17}$$

In the above, let $m_{22} \sim m_{32} \sim M_1 \sim M_4 \sim 1$ MeV, $m_3 \sim 1$ GeV, and $M_2 \sim M_3 \sim M_5 \sim M_6 \sim 1$ TeV, then all entries are of order 1 eV, and suitable for a realistic neutrino mass matrix, allowing for both normal and inverse hierarchies.

Another minimal case is to add a second Higgs doublet $\Phi_2 = (\phi_2^+, \phi_2^0)$ with $L = -4$ instead. Now we have m_{21} and m_{31} instead, and the reduced 2×2 mass matrix $\mathcal{M}_\nu$ spanning ν_2 and ν_3' is given by

$$\begin{pmatrix} -m_{21}^2(M_6^2 - M_2 M_3)/M_2 M_5^2 & -m_{21} m_3/M_5 \\ -m_{21} m_3/M_5 & M_1 m_3^2/M_5^2 - 2m_{31} m_3/M_5 \end{pmatrix}. \tag{18}$$

This structure is different from Eq. (17) but similar to Eq. (14). The off-diagonal entries could be much bigger than the diagonal ones (if $m_{31} \ll m_{21}$), thereby allowing for two nearly degenerate neutrino masses, which is perfect for understanding an inverse hierarchy, where the mass splitting responsible for solar neutrino oscillations is small compared to the neutrino masses themselves. On the other hand, if $m_{21} \ll m_{31}$, normal hierarchy is also possible. Once $\nu_{2,3}$ are massive, ν_1 will acquire a nonzero mass through the exchange of two W bosons [11], but this contribution is negligible.

Consider now the Higgs potential of $\Phi_{1,2}$ (with $L = -4$ for Φ_2) and $\chi_{2,3,4}$, invariant under $U(1)_L$:

$$\begin{aligned} V = {} & \sum_{i=1,2} \mu_i^2 \Phi_i^\dagger \Phi_i + \sum_{i=2,3,4} m_i^2 \chi_i^\dagger \chi_i + \frac{1}{2} \sum_{i,j=1,2} \lambda_{ij} \big(\Phi_i^\dagger \Phi_i\big)\big(\Phi_j^\dagger \Phi_j\big) + \lambda_{12}' \big(\Phi_1^\dagger \Phi_2\big)\big(\Phi_2^\dagger \Phi_1\big) + \frac{1}{2} \sum_{i,j=2,3,4} f_{ij} \big(\chi_i^\dagger \chi_i\big)\big(\chi_j^\dagger \chi_j\big) \\ & + \sum_{i=1,2,\, j=2,3,4} h_{ij} \big(\Phi_i^\dagger \Phi_i\big)\big(\chi_j^\dagger \chi_j\big) + \big[\mu_{124} \Phi_1^\dagger \Phi_2 \chi_4 + m_{224} \chi_2^2 \chi_4^\dagger + h_{122} \Phi_1^\dagger \Phi_2 \chi_2^2 + f_{234} \chi_2^\dagger \chi_3^2 \chi_4^\dagger + \text{H.c.}\big]. \end{aligned} \tag{19}$$

To have $\langle\phi_2^0\rangle \ll \langle\phi_1^0\rangle$, the couplings μ_{124} and h_{122} must be chosen to be very small, and μ_2^2 positive and large [12]. It may be argued that μ_{124} and h_{122} are naturally small because if they were zero, then V would have an extra global $U(1)$ symmetry, in addition to $U(1)_L$. As it is, there is no extra global $U(1)$, but the spontaneous breaking of $U(1)_L$ does result in a massless Goldstone boson, the singlet–doublet majoron mentioned already. To avoid this complication, soft explicit $U(1)_L$ breaking terms, such as $\mu_{12}^2 \Phi_1^\dagger \Phi_2 + \text{H.c.}$, could be added. If

$L=1$ is chosen for Φ_2, then the μ_{124} and h_{122} terms of Eq. (19) are replaced by $h_{1223}\Phi_1^\dagger\Phi_2\chi_2\chi_3^\dagger + h_{1234}\Phi_1^\dagger\Phi_2\chi_3\chi_4^\dagger +$ H.c. and everything works just as well.

Lepton number may also be considered as a discrete symmetry, in which case Z_7 works for the case $L=-4$ (which is equivalent to $L=3$) for Φ_2. Now χ_3 is equivalent to $\chi_4^\dagger$ and should be eliminated. The new term $\chi_2\chi_4^3$ would now appear, by which the massless majoron is eliminated.

An alternative is to gauge the global $U(1)_L$ symmetry, using either $U(1)_{B-L}$ or $U(1)_\chi$ from the decomposition of $SO(10) \to SU(5) \times U(1)_\chi$, where $Q_\chi = 5(B-L) - 4Y$. The same seesaw texture may be maintained using exactly the same lepton number assignments. [However, the N singlets with unconventional lepton numbers are not part of the $\underline{16}$ of $SO(10)$. They may come from a group larger than $SO(10)$ such as E_6 or larger representations of $SO(10)$.] The difference is that there can be no soft symmetry breaking terms and the extra anomalies generated by $N_{1,2}$ should be offset, for example, by three pairs of singlets with $L=1,-2$, belonging to a separate (odd Z_2) sector.

Consider now specifically $U(1)_\chi$ [13]. Since Φ_2 has nonzero Q_χ, its vacuum expectation value $\langle\phi_2^0\rangle$ contributes to $Z - Z'_\chi$ mixing which is known to be very small [14]. This fits perfectly into our scenario because m_{22}, m_{32} and m_{21}, m_{31} are also proportional to $\langle\phi_2^0\rangle$, and have been chosen to be small for neutrino masses. Constraints on Z'_χ then come mainly from its direct search at the Tevatron and the anomalous $g-2$ value of the muon. The present best direct lower limit for the mass $M_{Z'_\chi}$ is 822 GeV [15]. Using this bound, the muon $g-2$ constraint is easily satisfied as well.

If $M_{Z'_\chi}$ is not too much larger than the present lower limit, it can be produced at the Large Hadron Collider (LHC), due to start taking data soon this year. Since Z'_χ couples to SM particles with different $U(1)_\chi$ charges: 1, -1 and 3 for left-handed quark doublets, right-handed *up* and *down* quark singlets; -3 and -1 for left-handed and right-handed charged leptons, the forward-backward asymmetries in $b\bar{b}$ and charged lepton-pair production will deviate from pure Z exchange. This may provide a signal of new physics beyond the SM.

The Z'_χ boson can also decay into final states containing the heavy singlet neutrinos. If $M_{Z'_\chi} > 2m_N$, then Z'_χ will decay into $N\bar{N}$ with subsequent decays $N \to l^-W^+$, νZ and $\bar{N} \to l^+W^-$, $\bar{\nu}Z$, etc. Depending on which N is the lightest and which ones are produced, the signature may be different. If N is Majorana, which is possible for N_2, then the final decay products of Z'_χ can have both $e^\pm e^\mp W^\mp W^\pm$ and $e^\pm e^\pm W^\mp W^\mp$. If N is from one of the linear combinations of $N_{1,3}$, and M_3 is much smaller than M_5, the mass eigenstate can be a Dirac particle paired from N_1 and N_3^c. If so, then the final product will have just $l^\pm l^\mp W^\mp W^\pm$. If the mass eigenstates have large Majorana components, i.e. $M_3 \sim M_5$, the final products also have significant $l^\pm l^\pm W^\mp W^\mp$ event rates.

There is another potentially large decay channel involving a single heavy neutrino, i.e. $Z'_\chi \to \nu N$, because large mixing between light and heavy neutrinos is possible. This will be the dominant channel producing heavy neutrinos from Z'_χ decay for $M_{Z'_\chi}$ in the range $m_N < M_{Z'_\chi} < 2m_N$.

It is obvious that the best way to verify the seesaw mechanism is to produce the heavy singlet neutrinos. The presence of Z'_χ allows for this to happen much more easily than in models without it. In the latter type of models, the production of N is through the single production channel, $q\bar{q} \to Z \to \nu N$ and $q\bar{q}' \to W \to lN$ with the subsequent decay of N into lW. This mechanism is not completely negligible because the texture hypothesis allows for large mixing between light and heavy neutrinos. It has been shown [16] that m_N up to a hundred GeV may be probed at the LHC. The detection of such a single N can provide useful information on the texture hypothesis discussed in this work. By looking at the decaying vertex of N, one can also estimate the size of mixing between light and heavy neutrinos. In the canonical seesaw case, mixing of order $(m_\nu/m_N)^{1/2}$ leads to a very small decay width for N. Although it is not stable enough to escape the detector, it will produce a displaced vertex. This will not be the case for the large mixing being considered here.

With Z'_χ, it is possible to produce N in pairs through, $q\bar{q} \to Z' \to N\bar{N}$, if kinematically allowed. The final states to be analyzed are $l^\pm l^\mp W^\pm W^\mp$ and $l^\pm l^\pm W^\mp W^\mp$. The situation is similar to that of the Type III seesaw model [17] where the charged partner $E^\pm$ of the neutral heavy neutrino in the $SU(2)_L$ triplet is analyzed using $q\bar{q} \to Z \to E^+E^-$ with E subsequently decaying into lZ [18]. There m_E up to a TeV can be probed. In this model, however, the cross section will be smaller because the heavier Z'_χ is mediating the interaction, except of course if the production is at the Z'_χ resonance, which is the main advantage of having $U(1)_\chi$. A possible scenario is thus the discovery of Z'_χ at the LHC and from a detailed study of its decay products, the heavy neutrino states are also discovered with the information necessary to reconstruct the appropriate seesaw texture.

To conclude, we have studied the seesaw mechanism for three neutrinos, clarifying the situation where the texture of the 6×6 mass matrix results in three approximately zero mass eigenvalues. The true underlying mechanism is shown to be just the *inverse* (or *linear*) seesaw, which explains why there could be large mixing. However, these zeroes cannot occur naturally, unless there is a conserved symmetry, i.e. lepton number L, either global, discrete or gauged, which is softly or spontaneously broken at the TeV scale. We discuss in particular a case where the heavy singlet neutrinos have $L=3,-2,-1$. To support the texture hypothesis, Higgs singlets must be added, and the zeros of the 3×3 mass matrix of the heavy singlet neutrinos at tree level are shown to be nonzero in one loop. The lepton symmetry may also be gauged, thereby predicting a Z' boson which would facilitate the discovery of the heavy singlet neutrinos at the LHC.

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