3. *Concluding Remarks*

We have considered a positive dynamic system with linear structure and discrete time which is observable, and have obtained necessary conditions and/or sufficient conditions for the nonnegativity of the initial state. Also, we assume a single input and a single output. The study of observability in positive systems with these assumptions relaxed is an open problem and worthy of attention.

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LINEAR MODELS AND PROGRAMS FOR THE OPTIMIZATION OF THE MECHANICS OF ASSISTED BREATHING

by MARCEL STAROSWIECKI,[34] PIERRE VANPEENE,[34] and MARIE CHRISTINE CHAMBRIN[35]

1. *Introduction*

This study takes place in a biomedical context. A much discussed problem in mechanical ventilation is that of modifying the inspiratory flow pattern. Most of the existing breathing machines operate by blowing a constant, increasing, or decreasing output. No reference exists in order to justify the use of such a blowing curve during the inspiration cycle. The blowing output

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value can only be modified via manual intervention. The chest-lung system is characterized by two interacting parts: the first is purely mechanical, while the second is concerned with the gas exchanges between air and blood. We only treat here the mechanical part of the problem. The work which is reported here is mainly concerned with the study of the effect of the pattern of air flow on the mechanical characteristics of the lung. This theoretical study, using a linear bicompartmental model (RC) and linear programs, attempts to determine the inspiratory flow pattern optimizing the peak airway pressure.

2. *The Mechanical Model*

For an intubated patient, the mechanical parts of the chest-lung system perform as two associated models. The first model is a nonlinear one [1] and represents the upper airways (endotracheal tube and equipment). To describe the dynamic behavior of the lower airways (bronchi and alveoli) we use a parallel linear model with two resistor-capacitor compartments [2]. Each lung i is represented by a compartment i, with resistance R_i and capacitance C_i (or elastance $E_i = 1/C_i$). The model can be described via the following discrete-time equations:

$$X(n+1) = \mathbf{A}X(n) + \mathbf{B}u(n)$$

[volumes of both compartments 1 and 2 at time $(n+1)T$] and

$$y(n) = \mathbf{C}^T X(n) + \mathbf{D}u(n)$$

(pressure at time nT), where

$$\mathbf{A} = \begin{bmatrix} \dfrac{E_2 + E_1 e^{-T/\tau}}{E} & \dfrac{E_2(1 - e^{-T/\tau})}{E} \\ \dfrac{E_2(1 - e^{-T/\tau})}{E} & \dfrac{E_1 + E_2 e^{-T/\tau}}{E} \end{bmatrix},$$

$$\mathbf{B} = \begin{bmatrix} R_2 \dfrac{T/\tau_2 + (1 - \tau/\tau_2)(1 - e^{-T/\tau})}{E} \\ R_1 \dfrac{T/\tau_1 + (1 - \tau/\tau_1)(1 - e^{-T/\tau})}{E} \end{bmatrix},$$

$$\mathbf{C}^T = \left[\frac{R_2 E_1}{R_1 + R_2}, \frac{R_1 E_2}{R_1 + R_2} \right], \qquad \mathbf{D} = \frac{R_1 R_2}{R_1 + R_2}, \qquad E = E_1 + E_2,$$

$\tau_i = R_i/E_i$ represents the time constant of the compartment i, and $\tau = (R_1 + R_2)/(E_1 + E_2)$ represents the time constant of the system.

3. *The Problem of Linear Optimization*

The predominant mechanical criterion is barotraumatic aggression, that is, the maximum pressure level reached in the lungs. The predominant constraint is the passage of a tidal volume V_T during the inspiration period. Our objective is to minimize barotraumatic aggression at the level of the lower airways; this minimisation is carried out under various constraints. The duration of inspiration $[0, I]$ is divided into N elementary periods T. We denote by $X(0)$ the initial state of the system and by $y^+(n)$ the maximum pressure. We seek the elementary flows $u(n)$, $n \in [0, N-1]$, which are solutions of the following linear optimization problem of type n:

$$\min_{u(n)} \left[y^+(n) = \mathbf{C}^T\mathbf{A}^n X(0) + \sum_{k=0}^{n-1} \mathbf{C}^T\mathbf{A}^k\mathbf{B}u(u-1-k) + \mathbf{D}u(n) \right]$$

under the following constraints:

(A) The passage of a tidal volume V_T:

$$V_T = T \sum_{n=0}^{N-1} u(n).$$

(B) The equations of the system:

$$X(n+1) = \mathbf{A}X(n) + \mathbf{B}u(n), \qquad n \in [0, N-1].$$

(C) The limitation of the average pressure:

$$\frac{1}{N+1} \sum_{n=1}^{N} y(n) \leqslant \overline{Y}_L.$$

(D) The limitation of the air flows and volumes:

$$u(N) = u(N-1) \quad \text{and} \quad U_{\min} \leqslant u(n) \leqslant U_{\max}, \quad n \in [0, N-1],$$

$$x_{i\,\min} \leqslant x_i(n) \leqslant x_{i\,\max}, \qquad i \in [1,2], \quad n \in [0, N].$$

4. Solution

The maximal pressure may occur at any point during the inspiration period. The problem is solved by treating $N+1$ linear programs (LP) of type n, by supposing each time that one type of pressure is greater than the others:

LP(p): $\min y^{+}(p)$ under the constraints (A), (B), (C), (D), and $y(n) \leqslant y(p)$, $n \in [0, N] - \{p\}$.

Each problem LP(p) is solved, and we obtain $N+1$ solutions. The optimal solution is the most favorable solution.

5. Example

Model for obstructed lungs:

$$R_1 = 30, \qquad R_2 = 20 \text{ (cm H}_2\text{O)(sec)/liter},$$

$$E_1 = 20, \qquad E_2 = 30 \text{ (cm H}_2\text{O)/liter};$$

$$N = 5, \qquad \text{tidal volume } V_T = 0.81, \qquad \overline{Y}_L = 11 \text{ cm H}_2\text{O}.$$

Reference: Constant air flow; $\{u(n) = 0.4,\ n \in [0,4]\}$;

$$y^{+}(5) = 15.09 \text{ cm H}_2\text{O}, \qquad y = 10.03 \text{ cm H}_2\text{O}.$$

Solutions:

LP(0, 1, 2, 4): No solution.
LP(3): Optimized air flow; $\{u^*(n),\ n \in [0,4]\} = \{0.39; 0.3; 0.3; 0.7; 0.3\}$;

$$y^{+}(3) = 13.90 \text{ cm H}_2\text{O}, \qquad y = 0.65 \text{ cm H}_2\text{O}.$$

LP(5): Optimized air flow; $\{u^*(n),\ n \in [0,4]\} = \{0.8; 0.3; 0.3; 0.3; 0.3\}$;

$$y^{+}(5) = 13.78 \text{ cm H}_2\text{O}, \qquad y = 10.66 \text{ cm H}_2\text{O}.$$

Optimal solution: solution of LP(5).

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REGULAR MATRIX POLYNOMIALS WITH GIVEN SPECTRAL DATA

by EIVIND STENSHOLT[36]

1. *The algorithm*

The $n \times n$ matrix polynomials of degree $\leqslant p$ over a field K form a $(p = 1)n^2$-dimensional vector space $V_{n,p}$. The notation will be as follows:

$$a(\lambda) = \sum_{i=0}^{p} a_i \lambda^{p-i} \in V_{n,p}. \tag{1}$$

Instead of formal derivatives, it is easier to work with some closely related matrix polynomials:

$$a^{[k]}(\lambda) = \sum_{i=0}^{p-k} \binom{p-i}{k} \cdot a_i \lambda^{p-k-i}. \tag{2}$$

Multiplication by $k!$ in (2) yields the kth derivative of $a(\lambda)$.

We let $m > 0$ and $\omega \in K$ be arbitrary, and consider the block triangular $m \times m$ matrices

$$Kt(m, a(\lambda), \omega) = \begin{bmatrix} a(\omega) & 0 & 0 & \cdots & 0 \\ a^{[1]}(\omega) & a(\omega) & 0 & \cdots & 0 \\ a^{[2]}(\omega) & a^{[1]}(\omega) & a(\omega) & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ a^{[m-1]}(\omega) & a^{[m-2]}(\omega) & a^{[m-3]}(\omega) & \cdots & a(\omega) \end{bmatrix}. \tag{3}$$

These matrices are important because they combine two properties: Firstly,

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