Evaluating the Thickness of Broken Rock Zone for Deep Roadways using Nonlinear SVMs and Multiple Linear Regression Model

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Abstract

Since the traditional methods to estimation of the thickness of broken rock zone (BRZ) are usually difficult, expensive and not feasible in many cases, the development of some predictive models for the thickness of broken rock zone (BRZ) for deep roadways will be useful. To describe the complex relationship between geological factors and BRZ, a nonlinear model-based support vector machines (SVMs) analysis was applied on the data pertaining to China mine to develop some predictive models for the thickness of BRZ for deep roadways from the indirect methods in this study. The type of kernel function was Radial basis function (RBF). 132 samples were trained by proposed models; the other 10 samples that were not used for training were used to validate the trained models. For the same two similarity groups, the developed SVMs model was also compared with the multiple linear regression analysis (MLRA) models and measured data. As a result of SVMs analysis, very good model was derived for BRZ estimation. It was shown that SVMs models were more reliable and precise than the regression models. Concluding remark is that the thickness of BRZ values of deep roadways can reliably be estimated from the indirect methods using SVMs analysis.

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Keywords: broken rock zone (BRZ); surrounding rock; support vector machines (SVMs); multiple linear regression analysis (MLRA)

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1. Introduction

In the development of a deep mine tunnel, the original stress balance is destroyed. When stress redistribution occurs, the radial stress decreases with distance while the tangential stress increases along with stress concentration. Furthermore, the three-dimensional stress state of the in situ rock mass can be approximated to a two-dimensional one, and the rock strength is greatly reduced. Consequently, the surrounding rocks will be unavoidably broken, and the area in which the rocks are broken is called broken rock zone around drifts (Dong 2001[1]; Kruschwitz and Yaramanci, 2004[2]; Cai and Kaiser, 2005[3]). According to the formation mechanism of the rock broken zone, the deformation of surrounding rock are mainly from the volume expansion of rock broken in the loose circle, and confining pressure of roadway is also caused mainly by loose circle (Jing 2004[4]). Therefore, knowledge of the degree and extent of the excavation disturbed zone (EDZ) or identification of the broken rock zone (BRZ) thickness is important for the design and construction of deep underground engineering.

In order to study the stress distribution features of the surrounding rocks along the mine roadway after the roadway excavated and the radius size of the released circle, many methods is used to determine the scope of broken rock zone, such as acoustic method, seismic wave method, multipoint extensometer method, complex resistivity method, infraction method and geologic radar method etc., among which acoustic method was used commonly (Kruschwitz and Yaramanci, 2004[2]; Jing 2004[4]; Chen et al. 2008[5]). However, the acoustic method is expensive, and is not feasible in many cases. Therefore, it is imperative to explore a more reasonable way to study the thickness of the loose ring. The extent and degree of the EDZ has been quantified by Cai and Kaiser, 2005[3] using microseismic monitoring data and the anisotropic softening model for the rock mass has been confirmed by field velocity measurement. Zou and Xiao 2010[6] put forward the mathematical model for determining EDZ of underground caverns. Jing et al. 2001[7] present a concept of “key part” of roadways and its stability criterion using the program of discontinuous deformation analysis (DDA) while Chen et al. 2008 [5] proposed a new arrangement mode of acoustic measuring boreholes for broken rock zone in gently inclined thin layer weakness structure. The broken zones of rectangle cavity under different conditions were calculated by Xia et al. 2010 [8] using FLAC3D. However, the thickness can not be gotten in advance. To solve this problem, the artificial neural network (ANN) was introduced by Zhu 1999[9] to predict the thickness of BRZ. Gao and Zheng, 2002[10] presented an evolutionary neural network (ENN) model on prediction of the thickness of the loosen zone around roadway. Jing 2004[4] and Xu et al. 2005 [11] introduced an emerging intelligent prediction method with adaptive neuro fuzzy inference system (ANFIS) into the thickness of BRZ prediction. Research shows that, the developed ANN model has some limitations, such as black box approach, arriving at local minima, less generalization capability, slow convergence speed, overfitting problem and absence of probabilistic output (Zhu 1999[9]; Gao and Zheng, 2002[10]). Furthermore, there is no proper method to determine the number of hidden layers in the ANN model. The developed FIS model determines the fuzzy rules with difficulty (Jing 2004[4]).

Among artificial intelligence (AI) tools, Support vector machines(SVM) is an efficient machine learning (ML) technique derived from statistical learning theory by Vapnik 1995 [12]. It is a machine learning tools being based on statistical theory and following the structural risk minimization principle. As a representative nonlinear technique, SVM will be used since it has been shown to be an effective technique for regression nonlinear dataset (Gun 1998[13]; Gopalakrishnan and Kim, 2011[14]; Chang and Lin 2000[15]). It is therefore motivating to investigate the capability of SVM in broken zone thickness prediction.

In the current study, the SVM is applied to predict the broken zone thickness and the predicted values accord well with the in situ measured ones. Thereby the SVM provides a new approach to obtaining the broken zone thickness.
2. Multiple linear regression analysis

Multiple linear regression attempts to model the relationship between two or more explanatory variables and a response variable by fitting a linear equation to observed data (Kutner et al. 2004[16]; Yilmaz et al. 2007[17]). When there are i independent variables $X_1, X_2, \ldots, X_i$, the linear multiple regression equation is in the general form

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_i X_i$$

where $Y$ is the dependent variable; $X_1, X_2, \ldots, X_i$ are the independent variables (explanatory variables); $\beta_0$ is the constant, where the regression line intercepts the $Y$ axis, representing the amount the dependent $Y$ will be when all the explanatory variables are 0; $\beta_1, \beta_2, \ldots, \beta_i$ are the regression coefficients, representing the amount the response variable $Y$ changes when the explanatory variable changes 1 unit.

3. Support Vector Machines

Support vector machines (SVM) (Vapnik, 1995[12]) have been introduced as an effective model in both machine learning and data mining communities for solving both classification and regression problems. This section focuses on some highlights representing crucial elements in using this method. Further detailed mathematical description over SVM can be referred from Ref. (Vapnik, 1995[12]; Gunn, 1998[13]; Gopalakrishnan and Kim, 2011[14]; Chang and Lin, 2001[15])

SVMs are linear learning machines which means that a linear function ($f(x) = wx + b$) is always used to solve the regression problem. The best line is defined to be that line which minimises the following cost function ($Q$):

$$Q = C \sum_{i=1}^{N} L^\varepsilon (x_i, y_i, f) + \frac{1}{2} \|w\|^2$$

subject to

$$y_i - (wx_i + b) \leq \varepsilon + \xi_i$$

$$\varepsilon - (wx_i + b) - y_i \leq \varepsilon - \xi_i^*$$

$$\xi_i, \xi_i^* \geq 0$$

The first part of this cost function is a weight decay which is used to regularize weight sizes and penalizes large weights. Due to this regularization, the weights converge to smaller values. Large weights deteriorate the generalization ability of the SVM because they can cause excessive variance. The second part is a penalty function which penalizes errors larger than $\varepsilon$ using a so-called $\varepsilon$-insensitive loss function $L^\varepsilon$ for each of the $N$ training points. The positive constant $C$ determines the amount up to which deviations from $c$ are tolerated. Errors larger than $\varepsilon$ are denoted with the so-called slack variables $\xi$ (above $\varepsilon$) and $\xi^*$ (below $\varepsilon$), respectively. The third parts of the equation are constraints that are set to the errors between regression predictions ($wx_i + b$) and true values ($y_i$). The values of both $\varepsilon$ and $C$ have to be chosen by the user and the optimal values are usually data and problem dependent.

The minimisation of Eq. (2) is a standard problem in optimisation theory: minimisation with constraints. This can be solved by applying Lagrangian theory and from this theory it can be derived that the weight vector, $w$, equals the linear combination of the training data

$$w = \sum_{i=1}^{N} (\alpha_i - \alpha_i^*) x_i$$

In this formula, $\alpha_i$ and $\alpha_i^*$ are Lagrange multipliers that are associated with a specific training point. The asterisk again denotes difference above and below the regression line. From Eqs. (2) and (3), the following solution is obtained for an unknown data point $x$:

$$f(x) = \sum_{i=1}^{N} (\alpha_i - \alpha_i^*) \cdot (x_i, x) + b$$
By using a mapping function, the regression function Eq. (4) can be changed into:

\[ f(x) = \sum_{i=1}^{N} (\alpha_i - \alpha_i^*) \cdot K(x_i, x) + b \]  

(5)

In Eq. (5), \( K \) is the so-called kernel function which is proven to simplify the use of a mapping. The most used kernel functions are the Gaussian RBF with a width of \( g \): \( K(xi, x) = \exp(-0.5 ||x-x_i||^2/g^2) \), \( xi, x \) are the input feature vectors. so the type of kernel function is RBF in this paper.

4. Applications

4.1. Determination of broken rock zone (BRZ) factors

There were many factors affecting the form and thickness of the surrounding rock of broken rock zone, such as rock mass physical and mechanical parameters, rock mass structure, section shape, initial stress field and explosion effect etc (Dong 2001[1]; Jing 1999 [18]; Gao and Zheng, 2002[10]; Jing 2004[4]). Interaction mechanism between various factors forming broken rock zone was complicated, often showing a strong non-linear, at present there is no mathematical model of a generic. According to available information, the following factors were selected as influencing the he broken zone thickness factors: (1) the embedding depth of drifts (ED, m), (2) the drift span (DS, m), (3) the strength of surrounding rock blocks (RBS, MPa) and (4) the joint index (J) of surrounding rock masses. Corresponding to the classification of rock mass structures, the joint index is taken as 1 to 5 representing intact rock, blocky rock, stratified rock, disintegrated rock, and unconsolidated rock, respectively.

4.2. Dataset

In this study, a dataset generated by Jing 2004[4] were used for constructing nonlinear models-based SVMs and multiple linear regression analysis (MLRA) to estimate the thickness of BRZ. 132 test results were selected as training samples of model in this paper. Table 1 indicates the relevant parameters as well as their respective symbols used to develop BRZ prediction models range with their max, min, mean, standard deviation and skew, respectively. The scatter plot matrix of the original data set is given in Fig. 1 while the boxplot of the original data set is given in Fig. 2. For the most of the data groups, the median is not in the centre of the box, which indicates that the distribution of the most of the data groups is not symmetric (Fig. 2). In addition, dependent variable of ED does not have any outliers whereas DS, RBS, J and BRZ have at least one outlier (Fig. 2). Another 10 test results (Xu et al. 2005[11]) were used as the testing samples for accuracy of the model, which are shown in Tab.2. In the present study, training and testing analysis of MLRA and SVM have been carried out using MATLAB.

Table 1  Descriptive statistics of the input and output parameters with their max, min, mean, standard deviation and skew for SVM modeling (the number of 132 samples)

<table>
<thead>
<tr>
<th>Type of data</th>
<th>Parameter</th>
<th>Max</th>
<th>Min</th>
<th>Mean</th>
<th>Standard deviation</th>
<th>Skew</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inputs</td>
<td>Embedding depth (ED, m)</td>
<td>97.0</td>
<td>1159.0</td>
<td>565.318</td>
<td>261.770</td>
<td>0.410</td>
</tr>
<tr>
<td></td>
<td>Drift span (DS, m)</td>
<td>2.4</td>
<td>10.0</td>
<td>3.947</td>
<td>1.339</td>
<td>2.755</td>
</tr>
<tr>
<td></td>
<td>Rock block strength(RBS, MPa)</td>
<td>7.8</td>
<td>110.2</td>
<td>30.237</td>
<td>23.488</td>
<td>1.411</td>
</tr>
<tr>
<td></td>
<td>Joint index (J)</td>
<td>1.0</td>
<td>5.0</td>
<td>3.216</td>
<td>1.016</td>
<td>-0.039</td>
</tr>
<tr>
<td>Output</td>
<td>Broken zone thickness (BRZ, m)</td>
<td>0.3</td>
<td>3.5</td>
<td>1.522</td>
<td>0.585</td>
<td>0.514</td>
</tr>
</tbody>
</table>
Fig. 1 Scatter plot matrix of the original data  

Fig. 2 Boxplot of the original data set of BRZ

Table 2  Testing data of the thickness of BRZ

<table>
<thead>
<tr>
<th>No.</th>
<th>Embedding depth (m)</th>
<th>Drift span (m)</th>
<th>Rock block strength (MPa)</th>
<th>Joint index</th>
<th>Broken zone thickness (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>700.00</td>
<td>4.20</td>
<td>52.00</td>
<td>3.00</td>
<td>1.70</td>
</tr>
<tr>
<td>2</td>
<td>750.00</td>
<td>4.20</td>
<td>52.00</td>
<td>3.00</td>
<td>1.70</td>
</tr>
<tr>
<td>3</td>
<td>690.00</td>
<td>4.20</td>
<td>52.00</td>
<td>3.00</td>
<td>1.40</td>
</tr>
<tr>
<td>4</td>
<td>690.00</td>
<td>4.60</td>
<td>52.00</td>
<td>3.00</td>
<td>1.50</td>
</tr>
<tr>
<td>5</td>
<td>690.00</td>
<td>4.60</td>
<td>40.00</td>
<td>3.00</td>
<td>1.60</td>
</tr>
<tr>
<td>6</td>
<td>690.00</td>
<td>4.20</td>
<td>40.00</td>
<td>3.00</td>
<td>1.60</td>
</tr>
<tr>
<td>7</td>
<td>660.00</td>
<td>3.60</td>
<td>2.00</td>
<td>5.00</td>
<td>2.50</td>
</tr>
<tr>
<td>8</td>
<td>615.00</td>
<td>3.60</td>
<td>25.64</td>
<td>3.00</td>
<td>1.50</td>
</tr>
<tr>
<td>9</td>
<td>670.00</td>
<td>3.60</td>
<td>16.80</td>
<td>4.50</td>
<td>2.35</td>
</tr>
<tr>
<td>10</td>
<td>685.00</td>
<td>3.60</td>
<td>25.64</td>
<td>3.00</td>
<td>1.70</td>
</tr>
</tbody>
</table>

4.3. Multiple linear regression analysis

In this study, a stepwise multiple linear regression analysis was carried out to determine the relations between dependent variable the thickness of BRZ and the independent variables ED, DS, RBS and J. To achieve this goal, regression analysis was carried out using Matlab. Stepwise regression procedures select the most correlated independent variable first, and then select the second independent variable which most correlates with the remaining variance in the dependent variable. This procedure continues until selection of an additional independent variable does not increase the R-squared ($R^2$) by a significant amount, usually a significance of at least 95%. Accordingly, the variables ED, DS, RBS and J were included in the regression model, and independent variable RBS was omitted from the model due to lack of statistical significance. The most reliable and meaningful regression equation that could be obtained by the statistical analysis as follows Eq. (6)–(7), and the trained models were applied to predicting the thickness of BRZ, as shown in Fig. 3.

$$BRZ=-0.6681+0.0008ED+0.1134DS-0.0012RBS+0.4074J \ (R^2=0.7321) \ (6)$$

$$BRZ=-0.7291+0.0008ED+0.1083DS+0.4219J \ (R^2=0.7307) \ (7)$$

Where 4 re-fit linear regression, 3 re-fit linear regression was calculated by Eq. (6), Eq. (7), respectively.
4.4. Nonlinear Models-Based SVMs and Its Applications

Then, broken rock zone (BRZ) prediction with nonlinear model-based support vector machines can be carried out as follows: firstly, the factors influenced behavior of rock mechanics should be determined; secondly, training and predicting samples were collected; thirdly, the model were trained, and reasonable parameters of SVM structures were obtained; finally, the trained models were applied to predicting the thickness of BRZ, as shown in Fig. 4. ED, DS, RBS and J, were selected as the input variables. the thickness of BRZ was selected as outputs of the SVM model. So the mapping $R^n \rightarrow$ BRZ was established. $R^n$ is input variables of the proposed model, $n$ is the variable dimension.

When applying SVR, the goodness of fit is determined by the penalty factor $C$ and insensitive parameter $g$. LIBSVM (Chang and Lin, 2001[15]) provides a parameter selection tool using the RBF kernel: cross validation via parallel grid search method (GSM) (Lin and Huang [19]). As shown in Fig.4, the framework of optimizing the SVM’s parameters with GSM is presented; While cross validation is available for both support vector classification (SVC) and support vector regression (SVR), for the grid
search, currently we support only C-SVC with two parameters \( C \) and \( g \). In this study, the free parameters of SVR were selected followed a 5-fold cross-validation experiment to control generalization capability of SVM, and the RBF kernel is used as the kernel function of the SVR because it tends to give better performance. Gaussian kernel function is adopted as the kernel function of the samples training, obtaining best parameters by GSM. Fig. 5 shows an example of the GSM result, where the \( x \)-axis and the \( y \)-axis are \( \log_2 C \) and \( \log_2 g \), respectively. The \( z \)-axis is the 5-fold average performance. The findings of this experiment were that SVR is quite robust against parameter selections.

The result of the SVR parameter selection by GSM is shown in Figure 5(3D view), when the penalty factor \( C \) is 8, \( g=1 \) and the average value of MSE is \( \text{CVmse} = 0.023088 \). 132 sets of training sample data were back evaluated one by one using the SVM model of rock fragmentation and compared with the actual situation. The compared predicted and Measured of BRZ test results of training data are shown in Figure 6 and Figure 7. The regression mean-square error of the study sample is 0.0098502, and the square correlation coefficient is 0.9314. From figure 6, SVR have good performance for regression forecast, which prove that the model has stable and reliable prediction ability. Therefore, the SVM model is feasible and effective for BRZ forecasting and can be put into use. Shown in Fig 6 and Fig 7, the prediction curve obtained by SVR training sample fits good.

![Fig. 5 The fitness curve of selecting best parameters by GSM](image-url)

![Fig. 5 The fitness curve of selecting best parameters by GSM](image-url)
4.5. Evaluation and Discussion

The trained models are applied to predicting the BRZ of the other 10 samples. Results of SVM were compared to that of MLRA, and measured data, which were presented in Table 3 and Fig. 8. From Table 3 and Fig. 2, we know that the results using SVM are more feasible and precise than that using MLRA.

In estimating the SVR Model prediction performance, The results of SVR models are compared with MRVR method, computing indexes such as correlation coefficient \( R^2 \) and Root Mean Square Error (RMSE) can be used to evaluate the prediction accuracy of SVR and MRVR model. These indexes can be calculated by the following equation (8) and (9):

\[
R^2 = 1 - \frac{\sum (O_i - T_i)^2}{\sum (O_i)^2}
\]

(8)

\[
\text{RMSE} = \sqrt{(O_i - T_i)^2 / n}
\]

(9)

Where, \( T_i \), \( O_i \) and \( n \) represent the measured output, the predicted output and the number of input-output data pairs, respectively.

To compare the accuracy of SVM to MLRA, the \( R^2 \) and RMSE of two methods were listed in Table 3. From the prediction results of training and testing samples, the RMSE and \( R^2 \) between the observed and predicted values of SVR model are found to be 0.0099 and 0.9314 respectively for training data, and 0.0059 and 0.9418 respectively for testing data. From Table 3, the same for the predicted values by using Eq. (6) are found to be 0.3074 and 0.7321 respectively for training data, while 0.0703 and 0.9697 respectively for testing data, the corresponding values for Eq. (7) are 0.3070 and 0.7321 for training data, while 0.0703 and 0.9697 respectively for testing data. So for SVR method it can be seen that though the \( R^2 \) value is very high (0.9314 and 0.9418) showing good correlation. In Fig. 8 comparison of the predicted BRZ using MVRA and SVR method for test samples and their deviation from the observed one has been made. It is observed that the SVR predicted values are less scattered and are close to observed values signified by its closeness to the line of equality. Furthermore, various factors affected the thickness of BRZ prediction, as long as the corresponding data can be input to the SVM as variables, and the number of factors is not limited. Therefore, SVM can be more comprehensive consideration of rock mass physical and mechanical parameters, rock mass structure, section shape, initial stress field, explosion effect etc and the relationship between factors. Therefore, in the case of limited training samples, SVM based on small samples has more feasible and precise accuracy than MLRA. In conclusion, nonlinear model-based SVM
have good generalization ability and nonlinear dynamic data processing capabilities. It better makes up for the shortcomings of traditional method (MLRA). It has a very good state of adaptability to the thickness of BRZ prediction.

Table 3  Compared results of the $R^2$ and RMSE of two methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Train set MSE</th>
<th>Train set $R^2$</th>
<th>Test set MSE</th>
<th>Test set $R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>MLRA</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Eq.(6)</td>
<td>0.3074</td>
<td>0.7321</td>
<td>0.0703</td>
<td>0.9697</td>
</tr>
<tr>
<td>Eq.(7)</td>
<td>0.3070</td>
<td>0.7307</td>
<td>0.0703</td>
<td>0.9697</td>
</tr>
<tr>
<td>SVM</td>
<td>0.0099</td>
<td>0.9314</td>
<td>0.0059</td>
<td>0.9418</td>
</tr>
</tbody>
</table>

Fig.8. Comparison of forecasting results of test samples

5. Conclusions

A nonlinear model-based support vector machines (SVMs) analysis was applied on the data pertaining to China mine to develop some predictive models for the thickness of BRZ for deep roadways from the indirect methods in this study. The 132 samples were trained by proposed models; the other 10 samples were tested by trained models. The correlation coefficients of SVM model for predicting the thickness of BRZ is more than 0.90, which show the models are highly correlated and have good fitting performance. The accuracy of SVM was compared to that of MLRA; the relative errors of two methods were obtained. Results show that prediction accuracy of SVM has improved more greatly than that of the MLRA. Nonlinear Model-based SVM have good generalization ability and nonlinear dynamic data processing capabilities, which has a very good state of adaptability to the thickness of BRZ prediction.

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