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Application of stochastic control system in structural control

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Abstract

Stochastic control of a building frame subjected to earthquake excitation and fixed with an Active Tuned Mass Damper (ATMD) is presented in this paper. The control forces are computed using stochastic control algorithm. It is assumed that the system is partially observed and measurement error is present as a Gaussian white noise. The excitation is also assumed as a Gaussian white noise and the expected value of the state of the system is controlled using the Astrom's algorithm. The algorithm is obtained using dynamic programming technique leading to a recursive formulation. The equations used for calculating the control force are similar to the Algebraic Riccati's Equation (ARE). An eight storey building frame is taken as an example in which Gaussian white noise with zero mean is considered as excitation. The analysis is simulated using MATLAB with synthetically generated time history of acceleration as an excitation. Results obtained for various response quantities are presented. The results show comparison between the controls achieved by the stochastic algorithm and Linear Quadratic Gaussian (LQG) regulator.

Keywords: Seismic control; Stochastic control; ATMD; Semi-active control; Kalman Filter; LQG control;

1. Introduction

Seismic resistant design of structures has been a great challenge for the design engineers since many decades. A building should be designed in such a way that it controls or absorbs the vibration acting on it due to earthquake excitation. The control techniques adopted for earthquake resistant buildings can be broadly classified into passive, active, semi active and hybrid control. The traditional way of designing an earthquake resistant structure is done by increasing its mass, strength and ductility. Later on, passive control systems like base isolator, visco-elastic damper, tuned mass damper, etc. has been used to control

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structural vibrations. The inability of the passive control systems to adapt to the applied forces, led to the development of active control systems in which the control system adapts according to the response of the structure. Active tendon system, active mass damper, electro-magneto-rheological damper, etc comes under active control system. The semi-active control system combines both the active and passive control systems, like ATMD, variable orifice damper, controllable liquid column damper, etc.

Several algorithms like Proportional Integral Derivative (PID), Linear Quadratic Regulator (LQR), Linear Quadratic Gaussian (LQG) regulator, fuzzy controller, stochastic control, etc. have been developed for calculating the control force applied by the actuator. Stochastic control is one of the efficient control algorithms, where uncertainties are considered. In this study, stochastic control has been applied to an eight storey building frame and the results are compared with the performance of LQG regulator.

Nomenclature

A & A_d	State Matrix for continuous and discrete state space representation respectively
B & B_d	Control Input Matrix for continuous and discrete state space representation respectively
H_d	Input Matrix for Earthquake Acceleration
C_d & D_d	Output Matrix and feed through matrix respectively
E, r	Location matrix for ground acceleration and control force respectively
k	Step number
K_{LQR}	LQR gain
L	Control Gain
M, K, C	Mass, stiffness and damping matrix respectively
P & R	Riccati's Matices
Q_0 & Q_1	Weightage matrices for response quantity
Q_2	Weightage matrices for control force
$x, \dot{x}$ & $\ddot{x}$	Displacement, velocity and acceleration response vectors respectively
$\ddot{x}_g$	Ground acceleration
u & u_{LQR}	Control force by stochastic control algorithm and LQR algorithm respectively
v	Measurement Noise
V	Minimum expected value of loss function
z & y	State and observation vectors of the system respectively
y_v	Observation contaminated with measurement error
$\hat{z}, \hat{y}$	Estimated state and estimated observation vectors of the system respectively
Δt	Discrete time interval
ϕ	Performance index

2. Mathematical Modelling of the structure

The shear frame model of the structure is given in fig. 1. The mathematical equation governing the dynamics of the structure is expressed below

$$[M]\{\ddot{x}(t)\} + [C]\{\dot{x}(t)\} + [K]\{x(t)\} = -[M][r]\{\ddot{x}_g(t)\} + [E]\{u(t)\} \qquad (1)$$

The damping is calculated based on the classical damping, where it is considered to be proportional to mass and the stiffness matrix.

A damping ratio of 3% for the structure is considered for calculating the Rayleigh damping coefficients by considering the first two natural frequencies of the structure.

The Discrete state space representation of the equation of motion at any step '*k*' is given below

$$z(k+1) = A_d z(k) + B_d \ddot{x}_g(k) + H_d u(k) \quad (2)$$

The equation of the observer is expressed as

$$y(k) = C_d z(k) + D_d u(k) \quad (3)$$

The equation of the observer with signal noise becomes

$$y_v(k) = C_d z(k) + D_d u(k) + v(k) \quad (4)$$

The discrete state space matrices can be calculated from the continuous state space matrices as shown below

$$A_d = e^{A\Delta t} \quad (5)$$

$$B_d = \int_o^T B e^{A\lambda} d\lambda \quad (6)$$

$$H_d = \int_o^T H e^{A\lambda} d\lambda \quad (7)$$

The continuous state space matrices, in turn, can be obtained by the following relations

$$[A] = \begin{bmatrix} 0 & I \\ -[M]^{-1}[K] & -[M]^{-1}[C] \end{bmatrix} \quad (8)$$

$$[B] = \begin{bmatrix} 0 \\ -[r] \end{bmatrix} \quad (9)$$

$$[H] = \begin{bmatrix} 0 \\ [M]^{-1}[E] \end{bmatrix} \quad (10)$$

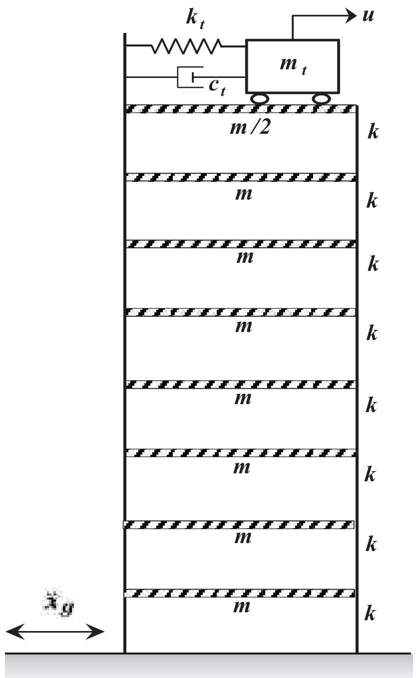


Fig. 1. Model of the structure

The ATMD is placed at the top floor as shown in the fig. 1 ATMD comprises a tuned mass damper and an actuator. The actuator is used to change the position of the mass to effectively control the vibration. The tuned mass damper consists of a mass attached to the structure through passive components which includes an elastic spring and a dashpot. The mass of the damper is taken as 5% of the total mass of the structure. The damping ratio of the dashpot is considered as 2%. The damper is tuned to the fundamental natural frequency of the structure and the stiffness of the spring is calculated accordingly.

3. LQG Controller

The Linear Quadratic Gaussian regulator essentially consists of a Linear Quadratic Regulator (LQR) and a Kalman filter. Here, the process noise and measurement noise is considered explicitly. The measurement noise is filtered by Kalman filter and the control gain is calculated using LQR algorithm. The LQR calculates the control gain by solving the ARE, which is given by

$$K_{LQR} = \frac{1}{2} R^{-1} H^T P \quad (11)$$

where R and P are the Riccati matrices obtained by optimizing the objective function. The control force is calculated by considering negative feedback

$$u_{LQR} = -K_{LQR} z(t) \quad (12)$$

The block diagram of the complete system controlled by an LQG regulator is given in fig. 2.

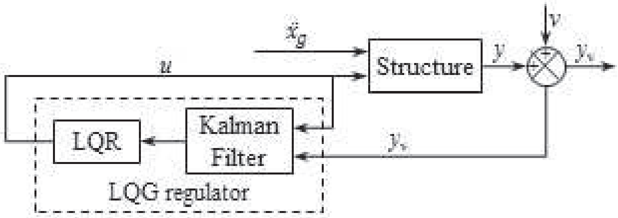


Fig. 2. Block diagram of system with LQG regulator

4. Stochastic control algorithm

The following assumptions are made while formulating the stochastic control algorithm

i. The structure is linearly elastic.
ii. The control algorithm is quadratic.
iii. The disturbance, i.e., earthquake acceleration is a Gaussian stochastic noise with zero mean and known covariance.
iv. The measurement noise is also a Gaussian stochastic noise with zero mean and known covariance.
v. The state of the system is not completely observed.

The performance index of the loss function of the dynamic system indicated in (2) is given below

$$\phi = z^{\mathrm{T}}(N)Q_0 z(N) + \sum_{\mathrm{k=k_0}}^{\mathrm{N-1}} \left\{ z^{\mathrm{T}}(k)Q_1 z(k) + u^{\mathrm{T}}(k)Q_2 u(k) \right\} \tag{13}$$

where Q_0 and Q_1 are the weightage matrices for structural response which are considered to be symmetric and non-negative. The matrix Q_2 is the weightage matrix for control force and is assumed to be positive definite. The weightage matrices are carefully selected to achieve an optimal control.

The principle of stochastic control is to minimize the expected value of the loss function as given below

$$V(x,\ k) \ = \ \min_{u_1,\ldots u_N} \ E\left\{ z^{\mathrm{T}}(N)Q_0 z(N) \ + \ \sum_{\mathrm{k=k_0}}^{\mathrm{N-1}} \left\{ z^{\mathrm{T}}(k)Q_1 z(k) \ + \ u^{\mathrm{T}}(k)Q_2 u(k) \right\} \right\} \tag{14}$$

The derivation of the above equation yields the Bellman equation, similar to that of the ARE. The Bellman equation is further solved using dynamic programming technique which yields the following equations.

$$u_{\mathrm{k}} = -L_{\mathrm{k}} z_{\mathrm{k}} \tag{15}$$

$$L_{\mathrm{k}} = \left[Q_2 + B_{\mathrm{d}}^{\mathrm{T}} S_{\mathrm{k+1}} B_{\mathrm{d}} \right]^{-1} B_{\mathrm{d}}^{T} S_{\mathrm{k+1}} A_{\mathrm{d}} \tag{16}$$

where $S_{\mathrm{k}} = Q_1 + A_{\mathrm{d}}^{\mathrm{T}} S_{\mathrm{k+1}} \left(A_{\mathrm{d}} - B_{\mathrm{d}} L_{\mathrm{k}} \right)$ (17)

with initial condition $S_{\mathrm{N}} = Q_0$ (18)

The above equations are solved recursively to obtain the control gain.

The equation (15) is substituted in (2) to compute the controlled response of the structure as follows

$$z(k+1) = \left[A_{\mathrm{d}} - H_{\mathrm{d}} L(k) \right] z(k) + B_{\mathrm{d}} \ddot{x}_{\mathrm{g}}(k) \tag{19}$$

5. The Kalman filter

Kalman filter is an optimal estimator which is used to estimate the required parameters from the incomplete, inaccurate and noisy observations. This technique is a recursive approach for minimizing the mean square error of the estimate. Linear Kalman filter has been used in this study for removing the signal noise present in the observation. Linear Kalman filters are efficient only when the noise and disturbance are Gaussian stochastic process with known mean and variance.

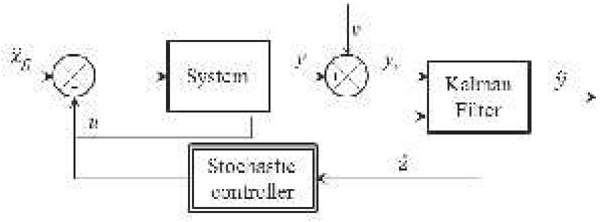


Fig. 3. Block diagram of the control system with Kalman filter

Displacement sensors used in this study are assumed to be contaminated with errors. These errors are filtered with the help of Kalman filter and a better estimate is made. It is also assumed that the structure is partially observed i.e., only four displacement sensors are available for measuring the responses. Kalman filter is also used to estimate the unobserved states. The block diagram of the control system with Kalman filter is shown in the fig. 3.

6. Results and discussion

The ground acceleration used in this study is a Gaussian white noise with a frequency range of 0 to 30 rad/s, which is synthetically generated. The PGA of the ground motion is taken as 0.35g. The ground acceleration is synthetically generated using MATLAB with zero mean.

The time history plot of the ground acceleration and signal noise is shown in the fig. 4.

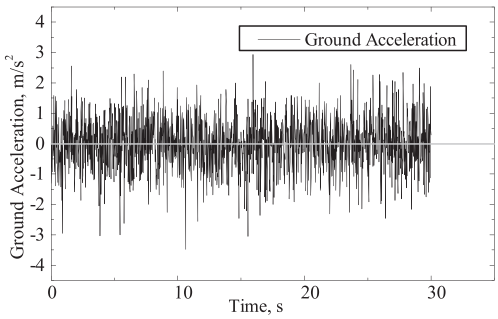
The displacement response of the first and top storey of the structure for both uncontrolled and controlled case using LQG regulator is given in fig. 5.

The displacement response of the first and top storey of the structure for both uncontrolled and controlled case by stochastic controller is given in fig. 6. The velocity responses are also plotted for the first and top story and are given in fig. 7.

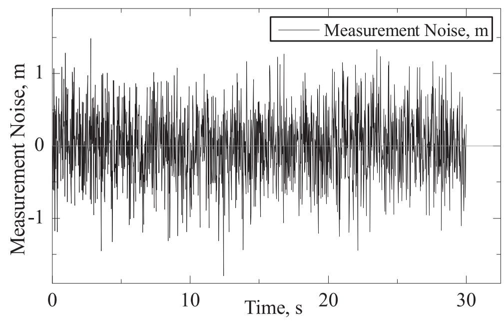
The time history plot of control force obtained by both LQG regulator and stochastic control algorithm is presented in fig. 8. The displacement response of TMD using LQG regulator and stochastic controller is given in fig. 9.

A three dimensional plot has been made between time, displacement response and velocity response of the structure and is given in fig. 10.

The summary of the maximum responses and reduction achieved by both LQG and stochastic controller is given in table 1.

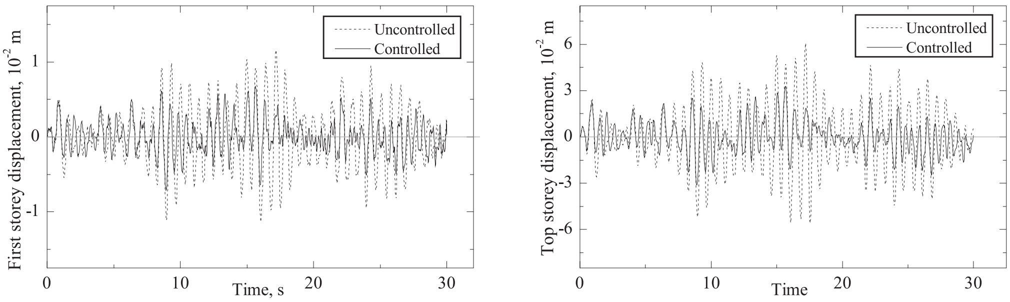


(a)



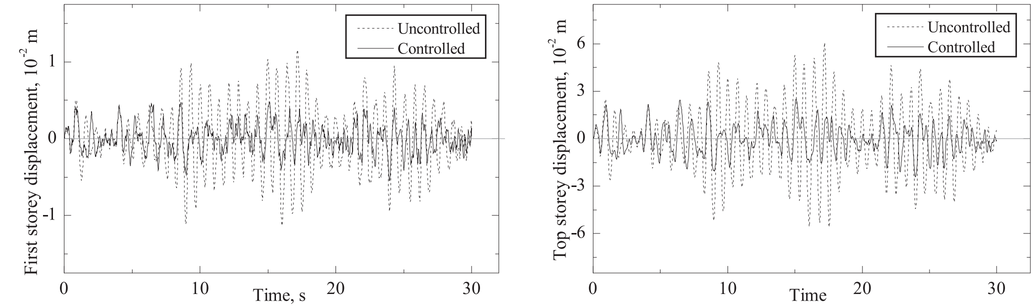
(b)

Fig. 4. Time history plots of (a) Ground acceleration (b) Signal noise



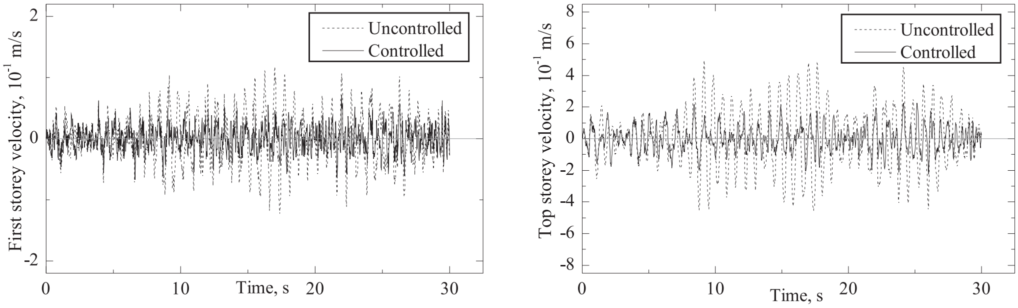
(a) (b)

Fig. 5. Displacement response, controlled by LQG regulator, of (a) First storey (b) Top storey



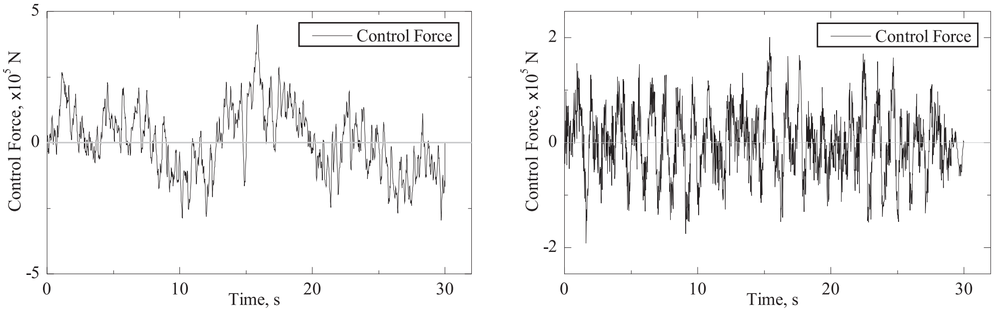
(a) (b)

Fig. 6. Displacement response of (a) First storey (b) Top storey, controlled by stochastic controller



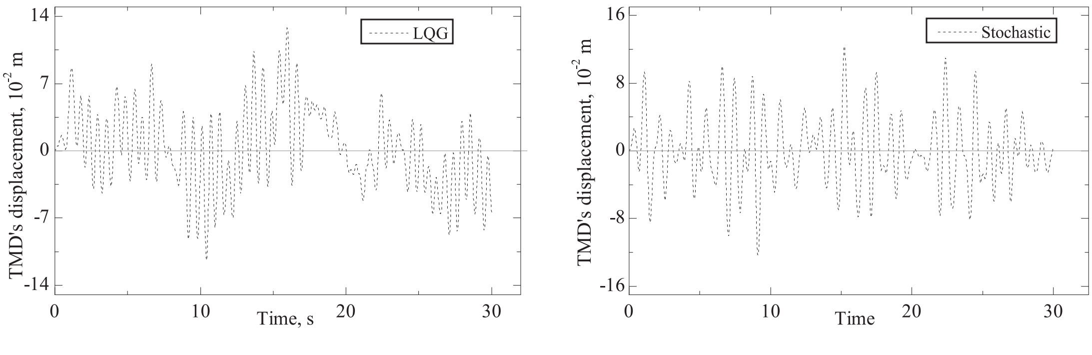
(a) (b)

Fig. 7. Velocity response of (a) First storey (b) Top storey, controlled by stochastic controller



(a) (b)

Fig. 8. Control force calculated by (a) LQG regulator (b) Stochastic controller



(a) (b)

Fig. 9. Time history plots of TMD displacement using (a) LQG control (b) Stochastic control

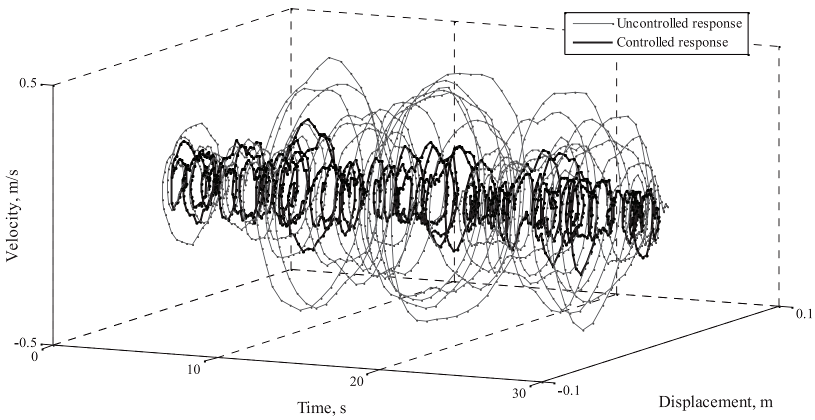


Fig. 10. Plot between time, displacement and velocity response

Table 1. Summary of responses and percentage reduction

Case		Displacement, m			Percentage Reduction				Maximum Control Force kN
		1st storey	4th storey	Top storey	1st storey	4th storey	Top storey	Average	
Uncontrolled		0.0116	0.0420	0.0606	-	-	-	-	-
Controlled	LQG	0.0071	0.0247	0.0329	38.7931 %	41.1905 %	45.7096 %	41.8977 %	449.3005
	Stochastic	0.0054	0.0187	0.0251	53.4483 %	55.4762 %	58.5806 %	55.8350 %	201.2905

The above graphs show that the response reduction achieved by stochastic controller is better than that of the LQG controller. Fig. 10 clearly shows that both the displacement response and velocity response is controlled efiiciently.

7. Conclusion

ATMD has good efficiency in controlling the structural vibrations. ATMD is suitable for practical applications as they have both passive and active components. Even if the actuator stops due to power failure, the passive components will help in considerable vibration control. The measurement errors are inherent in the practical sensor application which has been considered in the study. The simulation has been carried out using stochastic control algorithm for calculating the control forces and Kalman filter has been used for removing the signal noise. The results show that the average response reduction achieved by stochastic control is around 56 %, which is around 14 % more than control achieved by LQG regulator. The control force is less in stochastic control compared to that obtained by LQG control. The control force calculated by stochastic algorithm is almost 55 % lesser than the control force calculated using LQG regulator. Hence there is reduction in power consumption and the actuator can be operated through battery also.

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