Study on the Dynamic Input-Output Model with Coal Mine Safety

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Abstract

The dynamic input-output model with coal mine safety was established on the basis of the table of coal mine input-output with safety in this paper. And this paper gives definitions and proposes methods to make sure the investment coefficient of coal mine productive, investment coefficient of coal mine security, and consumption coefficient of accident. The dynamic input-output model with coal mine safety can be solved with reverse recursive solution, and be applied in analysis of input-output for coal mine safety and forecast of safety input.

Keywords: coal mine safety, dynamic input-output model, security investment coefficient, accident consumption coefficient.

1. Introduction

The input-output technology was established by Harvard Professor Wassily. Leontief (W. Leontief\textsuperscript{(1966)}\textsuperscript{[1]}), has been the continuous development from the 1930s. People used this technical analysis to study the pollution, population, energy balance and other social problems, achieving remarkable economic and social benefits. In the safety area, the application of the theory is still in the initial stage. Huang Shengren (2001) applied input-output theory to the safety production field, establishing a national safety production input-output table\textsuperscript{[2]}. Li Feng and Hu Zongyi (2001) using the input-output method constructed input-output correlative mode of safety incidents\textsuperscript{[3]}. Tian Shuicheng, Yang Bo (2007) advocated the use of input-output analysis into decision analysis for coal mine safety\textsuperscript{[4]}.

Static input-output model can well describe a static economic operation situation. But effect of coal mine input for safety is lag and long-term, therefore, dynamic model analysis should be established considering the time-delay. Through the dynamic model, we can predict the amount of input for safety, develop more rational investment plans, providing a scientific basis for investment decisions.
2. Establishing the table of input-output with coal mine safety

The table of input-output with coal mine safety is the base of dynamic input-output model with coal mine safety. The table of input-output with coal mine safety should be set up before we establish the dynamic input-output model with coal mine safety.

The table of input-output with coal mine safety sets up on the basis of general business input-output table, according to the special nature of coal mining enterprises, to which security has more important meaning. Input for safety is an essential and indispensable one in coal mine production, and also is in the input-output table of coal mine.

The specific form is shown in table 1.

3. Establishing Basic Model

For the coal mining enterprises, investment are mainly two aspects, one is for the expansion of production scale, the effect is mainly reflected the addition to product; the other is used to secure investment, the effect is mainly reflected the reduction in coal mine accidents. Since the formation of investment that is fixed capital that would have an effect in subsequent years, in dynamic input-output analysis, fixed capital formation basically determines and affects the future production and safety in coal mine (Liu, 2006)\(^5\).

Suppose the coal enterprises have set up n produce sectors.

The productive investment coefficient is:

\[
p_{ij} = \frac{c_{ij}}{\Delta X_j} (i, j = 1,2,\cdots,n)
\]  

(1)

which \(c_{ij}\) represent the product of i sector needed in the expand production scale of j sector; \(\Delta X_j\) represent the increasing production in j sector over the previous year, that is \(\Delta X_j = X_j(t+1) - X_j(t)\); \(p_{ij}\) is the product of i sector requiring in j sector, per increasing unit of product in j sector.

Security investment coefficient:

\[
q_{ij} = \frac{s_{ij}}{\Delta W_j} (i, j=1,2,\cdots,n)
\]  

(2)

Which \(s_{ij}\) represent the product of i sector needed for reducing accidents loss in j sector; \(\Delta W_j = W_j(t+1) - W_j(t)\), \(\max W\) represent the maximum incident loss of j sector in recent years, \(W_j\) is the absolute value of difference between incidents loss and \(\max W\), that is security benefit value in j sector, \(q_{ij}\) is per unit of security benefit requiring the product of i sector.
Table 1: The Input-output Table with Coal Mine Safety

<table>
<thead>
<tr>
<th>Output</th>
<th>Product consumption</th>
<th>Final product</th>
<th>Total output</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Production sector</td>
<td>Accidental loss</td>
<td>Investment</td>
</tr>
<tr>
<td>Coal mining</td>
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<td></td>
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<tr>
<td>Driving</td>
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<tr>
<td>Transport</td>
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<td>...</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Others</td>
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<td></td>
</tr>
<tr>
<td>Others</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Electric power</td>
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<td></td>
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<tr>
<td>Wage &amp; welfare</td>
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<tr>
<td>Management fee</td>
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<td></td>
<td></td>
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<tr>
<td>Others</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Security costs</td>
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<td></td>
<td></td>
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<tr>
<td>Costs of security facilities</td>
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<td></td>
<td></td>
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<tr>
<td>Monitor &amp; control of hazard</td>
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<td></td>
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<tr>
<td>Evaluation &amp; inspection of safety</td>
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<td></td>
</tr>
<tr>
<td>Safety education and training costs</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Occupational health protection costs</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Others</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Depreciation of fixed assets</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>taxes</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>profits</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total input</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Assume $\bar{Y}_i$ is final net product of coal mine, $Y_i$ is final product of coal mine.

$$Y_i = \bar{Y}_i + \sum_{j=1}^{n} c_{ij} + \sum_{j=1}^{n} s_{ij}, (i = 1, 2, \cdots, n)$$  \hspace{1cm} (3)

Leontief input-output model $X - AX = Y$ does not consider the loss of coal mine accidents. Mix the coal mine accident loss matrix in it, we obtain improved static input-output model $X - (A + B)X = Y$. Substitute Eq.(3) to improved static input-output model, we obtain

$$X_i - \sum_{j=1}^{n} a_{ij}X_j - \sum_{j=1}^{n} b_{ij}X_j - \sum_{j=1}^{n} c_{ij} - \sum_{j=1}^{n} s_{ij} = \bar{Y}_i \hspace{1cm} (i, j=1, 2, \cdots, n)$$  \hspace{1cm} (4)
$X_i$ is the total product of i sector; $a_{ij}$ is direct consumption coefficient (input coefficient), means that the product of i sector being consumed in per product in j sector. $a_{ij} = \frac{x_{ij}}{X_j}$, $x_{ij}$ is the product in i sector being consumed in the total product in j sector.

$b_{ij}$ is the consumption coefficient of accident loss, means accident loss of i sector in the proportion of per unit product in j sector. $b_{ij} = \frac{w_{ij}}{X_j}$, $w_{ij}$ is accident loss of i sector in the proportion of total product in j sector. Put Eq. (1) and Eq. (2) in Eq. (4).

$$X(t) = \sum_{j=1}^{n} a_{ij} X_j(t) - \sum_{j=1}^{n} b_{ij} X_j(t) - \sum_{j=1}^{n} p_{ij} X_{ij}^{t+1} + \sum_{j=1}^{n} q_{ij} W_j(t) = \bar{Y}(t)$$

\[(i=1, 2, \ldots, n)\] (5)

That is

$$\sum_{j=1}^{n} p_{ij} X_j(t) + \sum_{j=1}^{n} q_{ij} W_j(t) = \bar{Y}(t)$$

\[(i=1, 2, \ldots, n)\] (6)

Written in matrix form:

$$PX(t+1) + QW(t+1) = (I - A - B)X(t) + PX(t) + QW(t) - \bar{Y}(t)$$

(7)

P, Q, A, B respectively being n-order matrix of $p_{ij}, q_{ij}, a_{ij}, b_{ij}$, I is n-order unit matrix. $X(t+1), W(t+1), X(t), W(t), \bar{Y}(t)$ respectively are the n-dimensional column vector of $X_j(t+1), W_j(t+1), X_j(t), W_j(t), \bar{Y}_j(t)$.

Eq. (7) establishes the relationship between the coal mine input for production and security and output of product and security benefit of the next year.

Consider the more general case, assuming a time-delay of s years. The model is as follows.

$$X(t) - AX(t) - BX(t) - P[X(t+s) - X(t+s-1)] - Q[W(t+s) - W(t+s-1)] = \bar{Y}(t)$$

(8)

4. The solution and application of model

4.1. The solution

The coal security dynamic input-output model can be solved by the reverse recursive method to reckon the variable value of the previous year by the variable value of one year. Such as by the production scale and structure in $(t+1)$ year we could reckon the production scale and structure in $t$ year. Starting from a target year, the variable value can be calculated year by year. So production arrangements and safety input in the planning year can be given considering the situation in future fully. Let

$$G(t) = (A + B - I - P)X(t) + PX(t+1) + \bar{Y}(t)$$

(9)
Eq. (7) deforms as follow.

\[ W(t) = Q^{-1}G(t) + W(t + 1) \]  \hspace{1cm} (10)

4.2. Fix the parameters and coefficient

4.2.1 Fix consumption coefficient

Consumption coefficient involves the direct consumption coefficient matrix A and accident consumption coefficient matrix B. As direct consumption coefficient represents the production technical structure, it is relatively stable in a period of time (e.g., 5 years). So according to data in the input-output table for consecutive 5 years, the direct consumption coefficient can be calculated by averaging the amount in 5 years. Accident consumption coefficient matrix B is established on the basis of the matrix table of accident loss, is determined by weighted average accident consumption investment coefficient in consecutive 5 years, and is revised according to the actual situation.

4.2.2 Fix investment coefficient

Productive investment coefficient \( p \) is based on the productive investment matrix, is determined by weighted average productive investment coefficient in consecutive 5 years, and is revised according to the actual situation.

Security investment coefficient \( q \) is based on the Security investment matrix, is determined by weighted average security investment coefficient in consecutive 5 years, and is revised according to the actual situation.

4.2.3 Fix exogenous variables

For the exogenous variables final net product, it can be measured and calculated through econometric method or the artificial neural network method.

4.3. Application of the model

Dynamic input-output model can be used in forecasting and planning of security input. Supposing that the safety output vector \( W(t + 1) \) is known, we can determine matrix \( Q \) \( G(t) \) according to the methods to fix parameters and coefficient described above. So we can obtain \( W(t) \) by Eq. (10), and we can plan and arrange the security inputs for each sector according to Eq. (2).

5. Conclusions

In connection with the characteristics of coal mine, the table of coal mine safety input-output is established. On the basis of the definitions of productive investment coefficient, security investment coefficient, and accident consumption coefficient, we establish dynamic input-output model with coal mine safety which the time lag is 1 year and s years respectively, import the dynamic relationship between safety input and security benefits. By discussing methods to solve the model, we obtain the ways to fix parameters and coefficient of model. Applying the model, it can arrange and forecast the security inputs in each sector, provide scientific basis for the coal mining enterprises to develop a more rational plan.
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References