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Unitarity of Little Higgs models signals new physics of UV completion

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Abstract

The "Little Higgs" opens up a new avenue for natural electroweak symmetry breaking in which the standard model Higgs particle is realized as a pseudo-Goldstone boson and thus is generically light. The symmetry breaking structure of the Little Higgs models predicts a large multiplet of (pseudo-)Goldstone bosons and their low energy interactions below the ultraviolet (UV) completion scale $\Lambda \sim 4\pi f \sim O(10)$ TeV, where f is the Goldstone decay constant. We study unitarity of the Little Higgs models by systematically analyzing the high energy scatterings of these (pseudo-)Goldstone bosons. We reveal that the collective effect of the Goldstone scatterings via coupled channel analysis tends to push the unitarity violation scale Λ_U significantly below the conventional UV scale $\Lambda \sim 4\pi f$ as estimated by naive dimensional analysis (NDA). Specifically, $\Lambda_U \sim (3\text{–}4) f$, lying in the multi-TeV range for $f \sim 1$ TeV. We interpret this as an encouraging sign that the upcoming LHC may explore aspects of Little Higgs UV completions, and we discuss some potential signatures. The meanings of the two estimated UV scales Λ_U (from unitarity violation) and Λ (from NDA) together with their implications for an effective field theory analysis of the Little Higgs models are also discussed.

1. Introduction

The Standard Model (SM) with an elementary Higgs scalar is a remarkably simple theory, but despite the simplicity, it still successfully accommodates all known experimental data (aside from neutrino oscillations). However, the hierarchy problem [1] puts the naturalness and completeness of this theory in doubt. Already at one-loop level, quadratic radiative corrections to the Higgs mass parameter destabilize the weak scale, pulling it up to the intrinsic ultraviolet (UV) cutoff. At best, the SM is an effective field theory behaving naturally only up to an UV cutoff Λ_{SM} that could be higher than the weak scale by merely a loop factor, $\Lambda_{SM} \sim 4\pi v \simeq 3$ TeV.

This hierarchy problem (or naturalness problem) has motivated most of the major extensions of the SM since the seventies. The two earliest and best known directions are dynamical symmetry breaking [2] and the addition of supersymmetry [3]. More recently, theories with large or small extra dimensions [4] have been used to eliminate the hierarchy problem. These avenues are quite rich and have been explored in depth.

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The newest addition to this list of candidates is an attractive idea called the "Little Higgs" [5–10]. Little Higgs theories seek to solve a *Little Hierarchy*, by only requiring the Higgs mass be safe from one-loop quadratic divergences. In this mechanism, the extended global symmetries enable each interaction to treat the Higgs particle as a Goldstone boson. However, once all interactions are turned on, the Higgs becomes a pseudo-Goldstone boson [11]. Thus quadratic divergences in the mass parameter can only appear at two-loops and higher. This allows the theory to be natural with an UV cutoff up to two-loop factors above the weak scale, roughly $\Lambda \sim (4\pi)^2 v \sim 10$–30 TeV. The required particle content and interactions are usually quite economical; there may be new heavy gauge bosons (W', Z' and B' for instance), new heavy quarks (t' and possible exotics), and new heavy scalars (electroweak singlets, triplets and/or extended Higgs doublet sector).

Many Little Higgs models have been constructed, most of which take just the minimal solution towards stabilizing the Little Hierarchy. This approach requires a very minimal addition of extra particles and interactions. At first glance, both experimentalists and theorists might find this approach depressing, since this just predicts a sparsely filled little desert at the LHC. However, as we will show in this Letter, the situation luckily seems much better. In fact, a new scale in the multi-TeV range is found to demand new physics beyond that required by the minimal Little Higgs mechanism.

To begin, we can take inspiration from our knowledge of the SM. After observing the W and Z gauge bosons, we could wonder whether their mere existence predicts any new physics to be discovered. The lesson here is well known. Since the scattering amplitudes for longitudinal weak bosons grow with energy, perturbative unitarity would be violated at a critical energy $E = \Lambda_{\rm U}$ in the absence of Higgs boson [12–17]. The classic unitarity analysis determines this energy scale as $\Lambda_{\rm U} \simeq 1.2$ TeV [14–19]. Note that this is noticeably lower than the cutoff scale for strong dynamics, $\Lambda \sim 4\pi v \simeq 3$ TeV, as estimated by naive dimensional analysis (NDA) [20,21].

The possible resolutions to this unitarity crisis are well known. If a Higgs scalar exists, the Higgs contributions to the scattering amplitude cut off the growth in energy. Alternatively, if strong dynamics breaks the electroweak symmetry, possible new vector particles (such as techni-ρ's) will save unitarity. Imposing perturbative unitarity, these new states must appear below or around the scale $\Lambda_{\rm U} \simeq 1.2$ TeV for the high energy theory to make sense. Independent of details in the UV completion, this bound ensures new physics to be seen at LHC energies.

Essentially the same lesson can be relearnt for the Little Higgs models. The low energy dynamics of the Little Higgs theories are described by the leading Lagrangian under the momentum expansion, which is the analog of the two-derivative operator in the usual chiral Lagrangian. Due to the two derivatives, the scattering amplitude of these scalars is expected to grow as E^2, and will eventually violate unitarity at an energy $E = \Lambda_{\rm U}$. So far, the only difference from the SM case is the symmetry breaking structure. The different effective chiral Lagrangians will predict different interaction strengths and relations which determine the unitarity bound. Most importantly, the bound $\Lambda_{\rm U}$ points to the UV completion scale of the Little Higgs mechanism, and in analogy with the SM, is expected to be at accessible energy scales, lower than the NDA cutoff $\Lambda \sim 4\pi f \sim 10$ TeV. Moreover, because the breaking of extended global symmetries of the Little Higgs models results in a large number of additional (pseudo-)Goldstones in the TeV range, we expect the collective effects of the Goldstone boson scatterings in a coupled channel analysis to further push down the unitarity bound $\Lambda_{\rm U}$.

The rest of this Letter is organized as follows. We first perform a generic unitarity analysis for a class of Little Higgs models in Section 2, and then carry out an explicit unitarity study for the Littlest Higgs model of $SU(5)/SO(5)$ in Section 3. We discuss the potential new physics signals in Section 4, which is not intended to be exhaustive, but just gives a flavor of the possible phenomenology at the LHC. This section ends with a discussion of the interpretation and implications for the unitarity violation scale versus the NDA cutoff scale. Finally, we conclude in Section 5.

2. Unitarity of Little Higgs models: a generic analysis

As described in the introduction, Little Higgs models predict new physics in the TeV range, such as new

gauge bosons and new fermions. However, there can be substantial variation in these extra ingredients and thus their analysis is usually model dependent. On the other hand, the symmetry breaking structure of a given Little Higgs theory is completely determined. For instance, the scalars in the Littlest Higgs model [6] arise from the global symmetry breaking $SU(5) \to SO(5)$. This guarantees the existence of 14 "light" (pseudo-) Goldstone bosons, most of which are expected in the TeV range. At leading order in the momentum expansion, the interactions of these Goldstones are completely fixed by the global symmetry breaking pattern. This allows us to perform a generic analysis of the Goldstone boson scatterings and the corresponding unitarity bounds. Note that the local symmetries (as well as the fermion sector) in the Little Higgs theories can vary, but according to the power counting [22,23] they do not affect our analysis of the leading Goldstone scattering amplitudes. So we can apply our generic unitarity formula to each given theory and derive the predictions.

The setup is rather simple. As mentioned above, a Little Higgs model is defined by breaking its global symmetry $\mathcal{G}$ down to a subgroup $\mathcal{H}$. This guarantees the existence of $|\mathcal{G}| - |\mathcal{H}| = \mathcal{N}$ Goldstone bosons, denoted by π^a ($a = 1, \ldots, \mathcal{N}$). At the lowest order of the derivative expansion [22], the Goldstone interactions are fully fixed by the symmetry breaking structure,

$$\mathcal{L}_{\mathrm{KE}} = \frac{f^2}{8} \operatorname{Tr} |\partial_\mu \Sigma|^2. \tag{1}$$

In this expression, we define the non-linear field $\Sigma \equiv \exp[2i\pi^a T^a/f]$, where $\operatorname{Tr}(T^a T^b) = \delta^{ab}$ ensures the canonical normalization for the π^a's. The specific form of the broken generators T^a depends on the particular model under consideration. The scale f is the Goldstone decay constant and is usually taken to be order 0.7–1 TeV for naturalness. Note that the factor of $1/8$ is a consequence of the normalization $\operatorname{Tr}(T^a T^b) = \delta^{ab}$ and the definition for Σ. Changing the factor $1/8$ will correspond to a simple rescaling of f. We note that in general the ∂_μ's should be raised to covariant derivatives by gauge invariance. However, since we will be concerned only with the leading Goldstone scatterings (instead of the more involved gauge boson scatterings), it is enough to include the partial derivatives. This restriction also does not weaken the analysis because power counting [23] shows that the leading energy growth of the Goldstone scattering amplitudes completely arises from the derivative terms and is independent of the gauge couplings. Finally, we note that the only Little Higgs models which cannot be described by this Lagrangian are the simple group Little Higgses [8]. This is due to the fact that in those models, the vacuum expectation value $\langle \Sigma \rangle$ is not unitary and leads to a different structure.

Expanding Eq. (1) up to quartic Goldstone interactions, we arrive at

$$\mathcal{L}_{\mathrm{KE}} = \frac{1}{2}\partial_\mu \pi^a \partial^\mu \pi^a + \frac{\Gamma^{abcd}}{3f^2}\big(\partial^\mu \pi^a\big)\pi^b\big(\partial_\mu \pi^c\big)\pi^d + O\big(\pi^5\big), \tag{2}$$

where we have defined

$$\Gamma^{abcd} \equiv \operatorname{Tr}\big[T^a T^b T^c T^d - T^a T^c T^b T^d\big]. \tag{3}$$

To proceed with a coupled channel analysis, we will consider a canonically normalized singlet state under $\mathcal{H}$, consisting of $\mathcal{N}$ pairs of Goldstone bosons,

$$|S\rangle = \sum_{a=1}^{\mathcal{N}} \frac{1}{\sqrt{2\mathcal{N}}} \big|\pi^a \pi^a\big\rangle, \tag{4}$$

where the factor $1/\sqrt{2}$ is conventionally used to account for the identical particle states. The state $|S\rangle$ is a singlet since the π^a's form a real representation of the $\mathcal{H}$ symmetry in non-Simple Group models. Since the π^a's also form an irreducible representation of $\mathcal{H}$, this is the only singlet formed from two π^a's. The scattering amplitude $\mathcal{T}[S \to S]$ will contain $\mathcal{N}^2$ number of individual $\pi\pi \to \pi\pi$ channels, and is expected to be the largest amplitude for deriving the optimal unitarity bound. For instance, experience with the QCD $SU(2)$ chiral Lagrangian or the SM Higgs sector shows that the isospin singlet channel of $\pi\pi$ scattering results in the strongest unitarity bound [15,17–19]. We also note that among the π^a's there are would-be Goldstone bosons whose scattering describes the corresponding scattering of the longitudinal gauge bosons (such as (W_L, Z_L) and (W'_L, Z'_L, B'_L)) in the high energy range ($s \gg m_W^2, m_{W'}^2$) via the equivalence theorem [13,15,17,24]. So, at high energies our analysis is equivalent to a unitary gauge analysis.

Using the interaction Lagrangian in Eq. (2), we can readily determine the singlet scattering amplitude at

Table 1
Summary of unitarity bounds in various Little Higgs theories

Little Higgs model	G	H	$\mathcal{N}$	$\lvert\mathcal{C}\rvert$	Λ_U/f	$m_{W'}/f$	$m_{t'}/f$
Minimal Moose [5]	$SU(3)^2$	$SU(3)$	8	24	2.89	2.37	1
Littlest Higgs [6]	$SU(5)$	$SO(5)$	14	35	3.17	1.67	2
Antisymmetric condensate [7]	$SU(6)$	$Sp(6)$	14	26	3.68	1.67	2
$SO(5)$ Moose [9]	$SO(5)^2$	$SO(5)$	10	15	4.09	3.35	$\sqrt{2}$
$SO(9)$ Littlest Higgs [10]	$SO(9)$	$SO(5)\otimes SO(4)$	20	35	3.79	2.37	2

tree level,

$$\mathcal{T}[S\to S]=\frac{\mathcal{C}}{\mathcal{N}f^2}s, \tag{5}$$

where we have defined the group-dependent coefficient

$$\mathcal{C}=\sum_{a,b=1}^{\mathcal{N}}\Gamma^{aabb}. \tag{6}$$

To derive this result, we have used the relation for Mandelstam variables $s+t+u\approx 0$ after ignoring the small pion masses relative to the large energy scale $\sqrt{s}$. Here we note that because $\Gamma^{aaaa}=0$, only the $\mathcal{N}(\mathcal{N}-1)$ inelastic channels, $\pi^a\pi^a\to\pi^b\pi^b$ $(a\neq b)$, contribute.

It is now straightforward to compute the 0th partial wave amplitude from Eq. (5),

$$a_0[S\to S]=\frac{1}{32\pi}\int_{-1}^{1}dz\,P_0(z)\mathcal{T}(s,z)=\frac{\mathcal{C}}{16\pi\mathcal{N}f^2}s, \tag{7}$$

which, as expected, grows quadratically with the energy and is subject to the unitarity constraint,

$$|\Re\mathrm{e}\,a_0|<\frac{1}{2}. \tag{8}$$

Hence, we find that perturbative unitarity holds for energy scales

$$\sqrt{s}<\sqrt{\frac{8\pi\mathcal{N}}{|\mathcal{C}|}}f\equiv\Lambda_U. \tag{9}$$

Since $\mathcal{C}$ tends to scale as $\mathcal{N}^{3/2}$ for large $\mathcal{N}$, the unitarity bound should scale as $\mathcal{N}^{-1/4}$ [25]. Hence, we expect the unitarity bound to be quite low since $\mathcal{N}$ is reasonably large in the Little Higgs models.

Using this general formula, we can readily compute the coefficient $\mathcal{C}$ and determine the unitarity bounds on the various Little Higgs theories. We compile our results in Table 1. Note that for Moose models, there is a four times replicated non-linear sigma model structure. But, we have chosen to analyze only one of the non-linear sigma model fields. Any interaction between the different non-linear sigma model fields is model-dependent, so this restriction is consistent with our approach.

Table 1 shows that indeed the Little Higgs models generically contain a large number of Goldstone bosons, $\mathcal{N}=O(10\text{–}20)$, and our unitarity bound Λ_U is significantly lower than the conventional cutoff of the theory, $\Lambda\sim 4\pi f\simeq 12.6f$, as estimated by NDA. The observation that the unitarity violation scale turns out much lower than Λ is an encouraging sign, indicating that aspects of the Little Higgs UV completions may be possibly explored at the LHC. We will discuss more about the interpretations of our results and highlight the possible collider signatures in Section 4.

To add a reference frame for the unitarity bounds in Table 1, we also give the masses of the W' gauge boson and the t' quark (using our current normalization of f). For the gauge boson, the mixing angle between the two $SU(2)$ gauge couplings has been set to $\theta=1/5$. To scale to a different angle θ_{new}, just multiply by $\sin(2/5)/\sin 2\theta_{\mathrm{new}}$. A relatively small mixing angle is required since electroweak precision analysis restricts $m_{W'}\gtrsim 1.8$ TeV [9,30]. For the t' quark, we have minimized its mass, corresponding to maximizing the naturalness; in the particular case of two Higgs doublet models we have set $\sin\beta=1$ (for other β values, just divide by $\sin\beta$).

A striking feature of Table 1 is that $2m_{W'}>\Lambda_U$ holds for almost all Little Higgs models except the Antisymmetric Condensate model [7] where Λ_U is only slightly higher than the corresponding value of $2m_{W'}$. Such a low Λ_U means that for the center of mass energy $\sqrt{s}<\Lambda_U$, the $W'W'$ scattering processes will not be kinematically allowed. From the physical viewpoint, this strongly suggests that additional new parti-

cles (having similar mass range) have to co-exist with W''s in the same effective theory so that their presence can properly restore the unitarity. But these new states should enter the Little Higgs theory in such a way as to ensure the cancellation of one-loop quadratic divergences [26]. From the technical viewpoint, this obviously implies the equivalence theorem no longer holds for predicting the $W'_L W'_L$ scattering amplitude by that of the corresponding Goldstone scattering. But the exact $W'_L W'_L$ scattering amplitude could only differ from the Goldstone amplitude by $m^2_{W'}/s = O(1)$ terms at most, and thus are not expected to significantly affect our conclusion.

3. Unitarity of the Littlest Higgs model: an explicit analysis

In this section we will explicitly analyze the Littlest Higgs model of $SU(5)/SO(5)$ [6] by writing all Goldstone fields in the familiar electroweak eigenbasis of the SM gauge group. Then we will extract the leading Goldstone scattering amplitudes and derive the unitarity bounds, in comparison with our generic analysis of Section 2.

As mentioned earlier, the Littlest Higgs model has the global symmetry breaking structure $SU(5) \to SO(5)$, resulting in 14 Goldstone bosons which decompose under the SM gauge group $SU(2)_W \otimes U(1)_Y$ as

$$\mathbf{1}_0 \oplus \mathbf{3}_0 \oplus \mathbf{2}_{\pm 1/2} \oplus \mathbf{3}_{\pm 1}. \tag{10}$$

Here the $\mathbf{1}_0 \oplus \mathbf{3}_0$ denotes a real singlet χ^0_y and a real triplet $\chi^{\pm,0}$. They will become the longitudinal components of gauge bosons (B', W', Z') when the gauged subgroups $[SU(2) \otimes U(1)]^2$ are Higgsed down to the diagonal subgroup $G_{\rm SM}$. The $\mathbf{2}_{\pm 1/2}$ includes a Higgs doublet H and $\mathbf{3}_{\pm 1}$ a complex Higgs triplet Φ, defined as

$$H^T = \begin{pmatrix} \pi^+ \\ \frac{v+h^0+i\pi^0}{\sqrt{2}} \end{pmatrix}, \qquad \Phi = \begin{pmatrix} \phi^{++} & \frac{\phi^+}{\sqrt{2}} \\ \frac{\phi^+}{\sqrt{2}} & \phi^0 - i v' \end{pmatrix}, \tag{11}$$

where the would-be Goldstones $\pi^{\pm,0}$ will be absorbed by the light gauge bosons $(W^\pm, Z^0)$ when electroweak symmetry breaking is triggered by the Yukawa and gauge interactions via the Coleman–Weinberg mechanism [27]. There will be some small mixings between the scalars in H and Φ due to the non-zero triplet VEV v', but the condition $M_\Phi > 0$ requires [28]

$$v' < \frac{v^2}{4f} \ll v, \tag{12}$$

so that for the current purpose it is enough to expand the tiny ratio v'/v and keep only its zeroth order at which the two sets of Goldstone bosons do not mix. This greatly simplifies our explicit analysis.

Collecting all the 14 Goldstone bosons we can write the non-linear field $\Sigma = \exp[i2\Pi/f]\Sigma_0$ for the $SU(5)/SO(5)$ model where the 5×5 Goldstone matrix is given by

$$\Pi = \begin{pmatrix} \frac{1}{2}X & \frac{1}{\sqrt{2}}H^\dagger & \Phi^\dagger \\ \frac{1}{\sqrt{2}}H & \frac{2}{\sqrt{5}}\chi^0_y & \frac{1}{\sqrt{2}}H^* \\ \Phi & \frac{1}{\sqrt{2}}H^T & \frac{1}{2}X^* \end{pmatrix}, \tag{13}$$

and

$$X = \begin{pmatrix} \chi^0 - \frac{\chi^0_y}{\sqrt{5}} & \sqrt{2}\chi^+ \\ \sqrt{2}\chi^- & -\chi^0 - \frac{\chi^0_y}{\sqrt{5}} \end{pmatrix},$$

$$\Sigma_0 = \begin{pmatrix} & & \mathbf{1}_{2\times 2} \\ & 1 & \\ \mathbf{1}_{2\times 2} & & \end{pmatrix}. \tag{14}$$

Similar to Eq. (1), we derive the leading order Goldstone boson Lagrangian

$$\begin{aligned} \mathcal{L}_{\rm KE} &= \frac{f^2}{8}\,\mathrm{Tr}\,|\partial_\mu \Sigma|^2 \\ &= \frac{1}{2}\,\mathrm{Tr}\big(\partial^\mu \Pi\big)^2 \\ &\quad + \frac{1}{3f^2}\,\mathrm{Tr}\big[\big(\Pi\partial^\mu\Pi\big)^2 - \big(\partial^\mu\Pi\big)^2\Pi^2\big] \\ &\quad + O\big(\Pi^5\big), \end{aligned} \tag{15}$$

where the first dimension-4 operator gives the canonically normalized kinetic terms for all Goldstone fields in Π, and the second term gives the quartic Goldstone interactions.

To derive the optimal unitarity limit from the Goldstone scatterings, we will consider a canonically normalized $SO(5)$ singlet state consisting of 14 pairs of Goldstone bosons,

$$|S\rangle = \frac{1}{\sqrt{28}}\big[2|\pi^+\pi^-\rangle + |\pi^0\pi^0\rangle + |h^0h^0\rangle + 2|\chi^+\chi^-\rangle + |\chi^0\chi^0\rangle + |\chi_y^0\chi_y^0\rangle + 2|\phi^{++}\phi^{--}\rangle + 2|\phi^+\phi^-\rangle + |\phi_1^0\phi_1^0\rangle + |\phi_2^0\phi_2^0\rangle\big], \tag{16}$$

where we have defined $\phi^0 \equiv \phi_1^0 + i\phi_2^0$. This is essentially a re-expression of our general formula (4) with all $\mathcal{N} = 14$ Goldstone fields in the electroweak eigenbasis. But the expanded form of the quartic interactions in (15) is extremely lengthy in the electroweak eigenbasis, making the explicit calculation of the whole amplitude $\mathcal{T}[S \to S]$ tedious. Before giving a full calculation of $\mathcal{T}[S \to S]$, we will *explicitly* expand Eq. (15) and illustrate the unitarity limits for the two sub-systems (χ^a, χ_y^0) and $(\pi^{\pm,0}, h^0)$. From Eq. (15), we derive the corresponding interaction Lagrangians

$$\begin{aligned}\mathcal{L}_{\rm int}^{\pi h} = \frac{1}{12f^2}\{&\big[-(2vh+h^2)\big(\partial_\mu\pi^a\partial^\mu\pi^a\big) - (\partial_\mu h)^2\big(\pi^a\big)^2 + 2(v+h)(\partial_\mu h)\big(\pi^a\partial^\mu\pi^a\big)\big] \\ &+ \big[\big(\partial_\mu\pi^+\big)^2\big(\pi^-\big)^2 - \big[\big(\partial_\mu\pi^0\big)^2 + \partial_\mu\pi^+\partial^\mu\pi^-\big]\pi^+\pi^- \\ &+ 2\big(\pi^0\partial_\mu\pi^0\big)\big(\pi^+\partial^\mu\pi^-\big) - \big(\pi^0\big)^2\big(\partial_\mu\pi^+\partial^\mu\pi^-\big) + \text{h.c.}\big]\}, \\ \mathcal{L}_{\rm int}^{\chi} = \frac{1}{6f^2}\{&\big(\partial_\mu\chi^+\big)^2\big(\chi^-\big)^2 - \big[\big(\partial_\mu\chi^0\big)^2 + \partial_\mu\chi^+\partial^\mu\chi^-\big]\chi^+\chi^- \\ &+ 2\big(\chi^0\partial_\mu\chi^0\big)\big(\chi^+\partial^\mu\chi^-\big) - \big(\chi^0\big)^2\big(\partial_\mu\chi^+\partial^\mu\chi^-\big) + \text{h.c.}\},\end{aligned} \tag{17}$$

where the $U(1)$ Goldstone χ_y^0 does not enter $\mathcal{L}_{\rm int}^{\chi}$ at this order. The Goldstones $(\pi^{\pm,0}, h^0)$ form the SM Higgs doublet H which also has a renormalizable Coleman–Weinberg potential. But unlike $\mathcal{L}_{\rm int}^{\pi h}$, this potential only contributes constant terms to the Goldstone amplitudes and thus do not threaten the unitarity, especially when the pseudo-Goldstone Higgs h^0 is relatively light as favored by the electroweak precision data.

The Lagrangian $\mathcal{L}_{\rm int}^{\pi h}$ describes the leading derivative interactions of the Higgs doublet H, characterized by the Goldstone decay constant f and originated from the global symmetry breaking $SU(5) \to SO(5)$. In analogy with the SM case [15], we find that $(\pi^{\pm,0}, h^0)$ form an electroweak singlet state $|S_H\rangle = \frac{1}{\sqrt{8}}[2|\pi^+\pi^-\rangle + |\pi^0\pi^0\rangle + |h^0h^0\rangle]$. The corresponding s-wave amplitude is $a_0[S_H \to S_H] = 3s/(64\pi f^2)$, where we have dropped small terms suppressed by the extra factor $(v/f)^2 \ll 1$. Imposing the condition (8), we deduce the unitarity limit

$$\sqrt{s} < \Lambda_{\rm U} = \sqrt{\frac{32\pi}{3}}\, f \simeq 5.79 f, \tag{18}$$

which is lower than the NDA cutoff $\Lambda \sim 4\pi f$ by a factor of 2.2. Note that contrary to the scatterings of Goldstone π^a's (or W_L/Z_L's) in the SM, the $\pi\pi$ scatterings in the Littlest Higgs model grow with energy due to the derivative interactions in $\mathcal{L}_{\rm int}^{\pi h}$. Next, we turn to the $(\chi^\pm, \chi^0)$ system. The Lagrangian $\mathcal{L}_{\rm int}^{\chi}$ for the Goldstone triplet is the same as the familiar $SU(2)$ chiral Lagrangian. So we define the normalized isospin singlet state $|S_{\chi^a}\rangle = \frac{1}{\sqrt{6}}[2|\chi^+\chi^-\rangle + |\chi^0\chi^0\rangle]$, and derive its s-partial wave amplitude $a_0[S_{\chi^a} \to S_{\chi^a}] = s/(16\pi f^2)$. Using the condition (8), we arrive at

$$\sqrt{s} < \Lambda_{\rm U} = \sqrt{8\pi}\, f \simeq 5.01 f, \tag{19}$$

which is lower than $\Lambda \sim 4\pi f$ by a factor of 2.5.

After the above explicit illustrations, we will proceed with a full analysis of this model in the electroweak eigenbasis. The key observation is that the $SO(5)$ singlet state $|S\rangle$ in Eq. (16) can be decomposed into 4 smaller orthonormal states formed from two π^a's,

$$|S\rangle = \sqrt{\frac{2}{7}}\,|S_H\rangle + \sqrt{\frac{3}{14}}\,|S_{\chi^a}\rangle + \frac{1}{\sqrt{14}}\,|S_{\chi_y^0}\rangle + \sqrt{\frac{3}{7}}\,|S_\Phi\rangle, \tag{20}$$

each of which is an *electroweak singlet* state, defined as

$$\begin{aligned}|S_H\rangle &\equiv \frac{1}{\sqrt{8}}\sum_{a=1}^{4}|\pi^a\pi^a\rangle \\ &= \frac{1}{\sqrt{8}}\big[2|\pi^+\pi^-\rangle + |\pi^0\pi^0\rangle + |h^0h^0\rangle\big], \\ |S_{\chi^a}\rangle &\equiv \frac{1}{\sqrt{6}}\sum_{a=5}^{7}|\pi^a\pi^a\rangle = \frac{1}{\sqrt{6}}\big[2|\chi^+\chi^-\rangle + |\chi^0\chi^0\rangle\big],\end{aligned}$$

$$|S_{\chi_y^0}\rangle \equiv \frac{1}{\sqrt{2}}|\pi^8\pi^8\rangle = \frac{1}{\sqrt{2}}|\chi_y^0\chi_y^0\rangle,$$

$$|S_\Phi\rangle \equiv \frac{1}{\sqrt{12}}\sum_{a=9}^{14}|\pi^a\pi^a\rangle = \frac{1}{\sqrt{12}}\big[2|\phi^{++}\phi^{--}\rangle + 2|\phi^+\phi^-\rangle + |\phi_1^0\phi_1^0\rangle + |\phi_2^0\phi_2^0\rangle\big]. \quad (21)$$

Now we will perform a full coupled-channel analysis for the Goldstone scatterings among these 4 electroweak singlet states and prove that the maximal eigenchannel just corresponds to the amplitude $\mathcal{T}[S \to S]$ in Section 2 with $|S\rangle$ given by Eq. (20) (equivalently, Eq. (16) or (4)). There are 16 such individual scattering channels in total. Denoting each singlet state in Eq. (21) as

$$|S_j\rangle \equiv \frac{1}{\sqrt{2\mathcal{N}_j}}\sum_{a=a_j^{\min}}^{a_j^{\min}-1+\mathcal{N}_j}|\pi^a\pi^a\rangle$$

with $j = H, \chi^a, \chi_y^0, \Phi$. We can now readily derive any amplitude $\mathcal{T}[S_j \to S_{j'}]$ by using the general formulas (5), (6),

$$\mathcal{T}[S_j \to S_{j'}] = \frac{\mathcal{C}_{jj'}}{\sqrt{\mathcal{N}_j\mathcal{N}_{j'}}f^2}s, \quad (22)$$

where $\mathcal{C}_{jj'} = \sum_{a=a_j^{\min}}^{a_j^{\min}-1+\mathcal{N}_j}\sum_{c=c_{j'}^{\min}}^{c_{j'}^{\min}-1+\mathcal{N}_{j'}}\mathcal{C}^{aacc}$ will be explicitly evaluated for $SU(5)/SO(5)$. So, with all the singlet states $|S_j\rangle$, we deduce a 4×4 matrix of the leading s-wave amplitudes

$$\mathcal{A}_0 = \frac{s}{16\pi f^2}\begin{pmatrix} \frac{3}{4} & \frac{\sqrt{3}}{4} & \frac{5}{\sqrt{2}} & \sqrt{\frac{3}{8}} \\ \frac{\sqrt{3}}{4} & 1 & 0 & \frac{1}{\sqrt{2}} \\ \frac{5}{\sqrt{2}} & 0 & 0 & 0 \\ \sqrt{\frac{3}{8}} & \frac{1}{\sqrt{2}} & 0 & \frac{3}{2} \end{pmatrix}. \quad (23)$$

It has the eigenvalues $a_{0j} = \frac{s}{16\pi f^2}\big(-1, \frac{1}{2}, \frac{5}{4}, \frac{5}{2}\big)$, where the maximum channel $a_0^{\max} = 5s/(32\pi f^2)$ corresponds to a normalized eigenvector $(\sqrt{2/7}, \sqrt{3/14}, \sqrt{1/14}, \sqrt{3/7})$, which in this basis is precisely the singlet state in Eq. (20). Imposing the condition (8), we derive the best unitarity limit for the Littlest Higgs model,

$$\sqrt{s} < \Lambda_U = \sqrt{\frac{16\pi}{5}}f \simeq 3.17f, \quad (24)$$

in perfect agreement with the optimal bound in Table 1.

With the information in Eq. (23), we can also analyze the optimal unitarity limits for all *subsystems* via partial coupled-channel analysis, as summarized below.

Subsystem	Λ_U	Subsystem	Λ_U
$\{H\}$:	$5.79f$	$\{H, \chi^a\}$:	$4.35f$
$\{\chi^a\}$:	$5.01f$	$\{H, \Phi\}$:	$3.69f$
$\{\Phi\}$:	$4.09f$	$\{\chi^a, \Phi\}$:	$3.45f$
$\{H, \chi^a, \chi_y^0\}$:	$3.71f$	$\{H, \chi_y^0, \Phi\}$:	$3.45f$
$\{\chi^a, \chi_y^0, \Phi\}$:	$3.45f$	$\{H, \chi^a, \Phi\}$:	$3.27f$

(25)

It clearly shows that as more states are included into the coupled channel analysis, the unitarity limit Λ_U becomes increasingly stronger and approaches the best bound (24) in the full coupled-channel analysis. It also demonstrates the limit Λ_U to be fairly robust since omitting a few channels does not significantly alter the result. Finally, for the subsystems $\{H\} = \{\pi^{\pm,0}, h^0\}$ and $\{\chi^a\}$, we see that Eq. (25) non-trivially agrees with Eqs. (18), (19) derived from explicitly expanding (15).

In summary, taking the Littlest Higgs model as an example, we have explicitly analyzed the unitarity limits from the Goldstone scatterings via both partial and full coupled-channel analyses, with the Goldstone fields defined in the familiar electroweak eigenbasis. These limits are summarized in Eqs. (25) and (24). We find that the best constraint (24) indeed comes from the *full coupled-channel analysis* including all 14 Goldstone fields in the $SO(5)$ singlet channel (Eq. (4) or (20)), in complete agreement with Table 1 (Section 2). We have also systematically analyzed the smaller subsystems where some channels are absent. Most of the resulting unitarity limits in Eq. (25) are fairly close to the best limit, so Eq. (24) is relatively robust.

4. Implications for new physics signals

As shown in Sections 2, 3, the unitarity constraints already indicate that Little Higgs theories have an

important intermediate scale Λ_U, which is in the multi-TeV region and below the conventional NDA cutoff $\Lambda \sim 4\pi f$. Somewhere below Λ_U, new particles should appear in order to unitarize the Goldstone scattering of π^a's. In particular, the longitudinal $W_L W_L/Z_L Z_L$ scattering (or the corresponding Goldstone scattering $\pi\pi \to \pi\pi, hh$) will be measured by experiments. This process should start to exhibit resonance behavior at least by the scale Λ_U, although what actually unitarizes the amplitude depends upon the UV completion. For the case of the minimal Moose [5], we can rely on our intuition from the QCD-type dynamics. If it is dynamical symmetry breaking that generates the $SU(3)^2 \to SU(3)$ breaking, the new states should be the analogous vector meson multiplet, i.e., TeV scale $(\rho, K^*, \omega, \phi)$ particles. On the other hand, we could envision a linear sigma model completion (with/without supersymmetry). As an example, there could be a scalar Σ that transforms as a $(3, \bar{3})$ and gets a VEV proportional to the 3×3 unit matrix. In this case, we can expect new singlets and heavy octet scalars to appear in addition to the octet of Little Higgs bosons. If the Little Higgs theory respects T-parity (cf. second reference in [9]), these new states would have to be even under this parity. This means they can be singly produced and also have restricted decay channels, allowing only an even number of T-odd particles in the final state. So, selecting a specific UV completion can predict a very interesting phenomenology. This direction will be pursued further [26]. In order to investigate the phenomenology of these new states, realistic UV completions should be searched for. For instance, Ref. [29] provides an interesting dynamical UV completion, but more constructions should also be actively sought.

One might also wonder if small mixing angles or coupling constants would render these new states hard to observe experimentally. We clarify this by noting that the approximate global symmetry $\mathcal{H}$ relates the scattering of the $\mathcal{H}$ singlet to the scattering of light longitudinal W/Z bosons in the following manner. Neglecting $\mathcal{H}$ breaking effects, the general amplitude of $\pi\pi$ scattering is given by

$$\mathcal{T}\left(\pi^a\pi^b \to \pi^c\pi^d\right) = \sum_j c_j^{abcd} A_j(s,t,u), \tag{26}$$

where j is a finite integer, c_j^{abcd} is a constant tensor invariant under $\mathcal{H}$, and $A_j(s,t,u)$ is a kinematic function depending on the Mandelstam variables. The $\mathcal{H}$ singlet amplitude is a specific linear combination of the kinematic functions. At the lowest order, we have seen that these functions grow with s and this specific combination needs to be altered at least by Λ_U. However, longitudinal W/Z scattering is just another linear combination of these kinematic functions. Thus, at the scale Λ_U, unitarizing only the $\mathcal{H}$ singlet scattering but keeping the SM-type scattering channels unaffected will require an accidental cancellation in the group theory space. So, generically any new resonance should be shared among all allowed individual scattering channels even though an amplitude for the SM-type channel alone violates unitarity at a relatively higher scale [25]. At worst, a possibly suppressed coefficient should only arise from the projection into the SM-type channel, rather than a small mixing or coupling (up to $\mathcal{H}$ breaking effects).

The scale Λ_U certainly opens up encouraging possibilities at the LHC, not only to test the minimal Little Higgs mechanism, but also to start probing possible new signs of its UV completion dynamics. We note that the unitarity bound $\Lambda_U \sim (3\text{–}4) f$ puts an *upper limit* on the scale of new states which are going to restore the unitarity of the Little Higgs effective theory up to the UV scale ~ 10 TeV or above. So the masses of these new states can be naturally at anywhere between $\sim f$ and Λ_U, but their precise values must depend on the detailed dynamics of a given UV completion. For instance, QCD-like UV dynamics would predict the lowest new resonance to be a ρ-like vector boson which is expected to be relatively heavy and close to our upper limit Λ_U. But when the UV dynamics invokes supersymmetry, the lowest new state that unitarizes the $W_L W_L$ scattering would be scalar-like and can be substantially below Λ_U, say $\sim 0.5 f$ according to the lesson of supersymmetric SM. (Note that the classic unitarity bound for the Higgsless SM only requires $\sqrt{s} < \Lambda_U = \sqrt{8\pi}\, v \simeq 5.0 v \simeq 1.2$ TeV [14–19], but the minimal supersymmetric SM unitarizes the $W_L W_L$ scattering by adding 2-Higgs-doublets with the lightest Higgs boson mass $M_h \lesssim 130$ GeV $\simeq 0.5 v$ [3], which is typically a factor ~ 10 below Λ_U.) So, it is legitimate to expect the lightest new state in the UV completion of Little Higgs models to lie anywhere in the range $0.5 f \lesssim M_{\text{new}}^{\min} \leqslant \Lambda_U$, though its precise mass value is highly model-dependent. The natural size for the scale f is ~ 1 TeV [5–10]. The updated precision

analyses [10,30,31] showed that the Little Higgs models are readily consistent with the current data which constrain $f \gtrsim 0.5$–1 TeV at 95% C.L. (depending on details of the parameter space in each given model),[1] so f is allowed to be around its natural size ~ 1 TeV. Taking $f \sim 1$ TeV for instance, we expect the lightest new state to be around 0.5 TeV $\lesssim M_{\rm new}^{\rm min} \lesssim$ 3–4 TeV. So, if lucky, the LHC may produce the lightest new resonance, or if it is too heavy, detect the effect of its resonance-tail (via higher order model-dependent contributions in the low energy derivative expansion) [32]. But a quantitative conclusion has to be highly model-dependent. To be conservative, we warn that the limited LHC center-of-mass energy does not guarantee the discovery for such state, especially when $M_{\rm new}^{\rm min}$ is close to the upper limit $\Lambda_{\rm U}$. Further precision probe may be done at future e^+e^- Linear Colliders, and the proposed CERN CLIC with $E_{\rm cm} = 3$–5 TeV and $\mathcal{L} = 10^{35}\ {\rm cm}^{-2}\,{\rm s}^{-1}$ [34] is particularly valuable. The definitive probe of the Little Higgs UV dynamics is expected at the future VLHC ($E_{\rm cm} = 50$–200 TeV and $\mathcal{L} \gtrsim 10^{35}\ {\rm cm}^{-2}\,{\rm s}^{-1}$) [33]. Incorporating the new signatures of UV completion into relevant collider analyses will expand upon the existing phenomenological studies [28,31,36].

Next, we discuss the meanings of the two estimated UV scales, $\Lambda_{\rm U}$ and Λ, and their implications for an effective field theory analysis in the Little Higgs models. We note that these UV scales are determined by two different measures of perturbativity breakdown. Our lowest unitarity limit $\Lambda_{\rm U}$ is derived from the Goldstone scatterings in the singlet channel via the s-partial wave. (Weaker bounds may be obtained for the non-singlet channels via the higher order partial waves.) On the other hand, the NDA estimate of the UV cutoff is based on the consistency of the chiral perturbation expansion, i.e., one estimates the coefficient of an operator (counter term) of dimension-D from its renormalization-group running induced by one-loop contributions of an operator of dimension-$(D-2)$ and so on [20,22], because the former's size should be at least of the same order as the latter's one-loop contribution (about $O(1)/16\pi^2$ multiplied by an $O(1)$ logarithm) barring an accidental cancellation. So one obtains the *original* NDA result [20],

$$\frac{f^2}{\Lambda^2} \gtrsim \frac{O(1)}{16\pi^2} \implies \Lambda \lesssim 4\pi f, \tag{27}$$

which is a conservative *upper bound* on the UV cutoff. The true cutoff for the effective theory should be $\min(\Lambda_{\rm U}, \Lambda)$. From low energy QCD, the chiral perturbation theory breaks down as the energy reaches the ρ-resonance at $M_\rho = 0.77$ GeV which is below but still close to the upper limit $4\pi f \simeq 1.2$ GeV. So we know this original NDA upper bound $4\pi f$ describes the UV scale of the low energy QCD quite well.[2] But, the dynamics of Little Higgs UV completions can of course be very different from QCD dynamics (or even supersymmetric). In fact, for an underlying gauge interaction with large color N_c and flavor N_f, a generalized dimensional analysis (GDA) [21,25] gives

$$\Lambda \lesssim \min\left(\frac{a}{\sqrt{N_c}}, \frac{b}{\sqrt{N_f}}\right) 4\pi f, \tag{28}$$

where a and b are constants of order 1. So we see that as long as N_c or N_f is much larger than that of QCD, the GDA cutoff will indeed be lower than the original NDA estimate. Furthermore, the observation that the unitarity of Goldstone scatterings indicates a lower UV cutoff for the chiral perturbation was made in [25], where it was shown that for a symmetry breaking pattern $SU(N)_L \otimes SU(N)_R \to SU(N)_V$ $(N \geqslant 2)$, the $\pi\pi$ scattering in the $SU(N)_V$-singlet and spin-0 channel would impose a unitarity violation scale

$$\Lambda \lesssim \frac{4\pi f}{\sqrt{N}}, \tag{29}$$

signaling a significantly lower UV scale for new resonance formation in comparison with the original

[1] E.g., it was shown [31] that the early precision bound in the Littlest Higgs model is essentially relaxed by just gauging the subgroup $SU(2) \times SU(2) \times U(1)$.

[2] The best unitarity limit of the low energy QCD $\pi\pi$ scattering comes from the $I = 0$ isospin-singlet channel, $\Lambda_{\rm U} \simeq \sqrt{8\pi} f \simeq 0.47$ GeV. This lies significantly below the upper limit $4\pi f$ by a factor of 2.5. It is interesting to note that in the physical spectrum, besides the ρ meson, there are good evidences for a relatively light and broad σ meson in the ~ 0.5 GeV range [37] which unitarizes the $I = 0$ channel and agrees well with the unitarity limit $\Lambda_{\rm U} \simeq 0.47$ GeV. The fact that QCD chiral Lagrangian works quite well is largely because σ is a very broad $I = 0$ resonance and hard to detect [37].

NDA estimate. This is consistent with our current unitarity analysis for the Little Higgs models.

Finally, in an effective field theory analysis of the Little Higgs models, which UV cutoff is more relevant for suppressing the higher-dimensional operators? The precise answer has to be very model-dependent, relying on what type of heavy state(s) is integrated out when generating a given effective operator. Without knowing the true UV dynamics, the original NDA estimate $\Lambda \sim 4\pi f$ could be considered as a conservative analysis where the UV scale is the highest possible. So far all the electroweak precision analyses [30,31,35] adopted the NDA estimate of Λ. But we should keep in mind that the actual UV cutoff Λ could be significantly lower, as suggested by Λ_U, although Λ has to be fixed by the underlying dynamics (cf. GDA estimate in Eq. (28)). Hence it will be instructive to take the two UV scales Λ_U and $\Lambda \sim 4\pi f$ as guidelines and allow the predictions to vary in between. The ultimate determination of the UV scale can only come from future experiments.

5. Conclusions

In this Letter, we systematically studied the unitarity constraints in various Little Higgs models using a general formalism in Section 2. Our analysis of the Goldstone scatterings is rather generic and mainly independent of the choices of parameters, gauge groups and fermion interactions, etc. This is because the leading Goldstone interactions in the derivative expansion are completely governed by the structure of global symmetry breaking, allowing us to perform a coupled channel analysis for the *full* Goldstone sector in a universal way. We observed that because the global symmetry breaking in the Little Higgs theories generically predict a large number of (pseudo-)Goldstone bosons, their collective effects via coupled channel analysis of Goldstone scatterings tend to push the unitarity violation scale Λ_U significantly below the conventional NDA cutoff $\Lambda \sim 4\pi f \simeq 12.6 f$. Specifically, $\Lambda_U \sim (3\text{–}4) f$ (cf. Table 1), which puts an *upper limit* on the mass of the lightest new state, i.e., $M_{\text{new}}^{\min} \leqslant \Lambda_U \sim (3\text{–}4)$ TeV for $f \sim 1$ TeV.

As a comparison, in Section 3 we took the Littlest Higgs model of $SU(5)/SO(5)$ as an example and explicitly analyzed the Goldstone scatterings in their electroweak eigenbasis. We performed both partial and full coupled-channel analyses. We derived various unitarity violation limits for this minimal model and demonstrated that as more Goldstone states are included into the coupled channel analysis, the unitarity limit Λ_U becomes increasingly stronger, close to the best bound (cf. Eqs. (25) and (24)). This concrete analysis shows that the optimal unitarity limits in Section 2 are fairly robust.

We stress that these tight unitarity limits strongly suggest the encouraging possibility of testing the precursors of the Little Higgs UV completion at the upcoming LHC (although no guarantee is implied). A definitive test is expected at the future VLHC [33]. In Section 4 we discussed some implications for the UV completions and the related collider signatures. Finally, we concluded Section 4 by discussing the meanings of the two estimated UV cutoff scales Λ_U (from unitarity violation) and Λ (from NDA/GDA). Deciding which estimate to be more sensible in an effective field theory analysis of Little Higgs models is unclear before knowing the precise UV dynamics. Only future experiments can provide an ultimate, definitive answer.

Note added

As this work was being completed, a related preprint [38] appeared which did an explicit unitary-gauge calculation of only light W_L/Z_L scattering in the Littlest Higgs model. Unfortunately its result is incorrect due to, for instance, mistaking the upper bound on the Higgs triplet VEV which leads to erroneously large gauge-Higgs triplet couplings.

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