Prediction of forming limit curve (FLC) for Al–Li alloy 2198-T3 sheet using different yield functions

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Abstract The Forming Limit Curve (FLC) of the third generation aluminum–lithium (Al–Li) alloy 2198-T3 is measured by conducting a hemispherical dome test with specimens of different widths. The theoretical prediction of the FLC of 2198-T3 is based on the M–K theory utilizing respectively the von Mises, Hill’48, Hosford and Barlat 89 yield functions, and the different predicted curves due to different yield functions are compared with the experimentally measured FLC of 2198-T3. The results show that though there are differences among the four predicted curves, yet they all agree well with the experimentally measured curve. In the area near the planar strain state, the predicted curves and experimentally measured curve are very close. The predicted curve based on the Hosford yield function is more accurate under the tension–compression strain states described in the left part of the FLC, while the accuracy is better for the predicted curve based on Hill’48 yield function under the tension–tension strain states shown in the right part.

1. Introduction

As a new type of aluminum alloy, Al–Li alloy is widely used in the aerospace field because of its low-density, low fatigue crack growth rate, high elastic modulus, high specific strength, high specific stiffness, weldability and other excellent comprehensive performance. Each 1% weight of Li alloyed with Al reduces the density by 3% and increases the Young’s modulus by 6% as compared with the pure Al. Using the new Al–Li alloy to replace the conventional high strength aluminum alloy makes it possible for the structure’s stiffness to increase by 15%–20% and the structural weight to decrease by 10%–20%.

The course of research and development of the Al–Li alloy can be generally divided into three stages, and corresponding Al–Li alloy products are divided into three generations. The chemical composition of the third generation Al–Li alloy has changed, which enables it to demonstrate significant advantages over the second generation Al–Li alloy and traditional aluminum alloy, such as low-density, high corrosion resistance, high fatigue strength, high tensile strength and high fracture toughness. As a representative of the third generation Al–Li alloy, 2198 Al–Li alloy has been used both in the manufacture of
first and second overall fuel tank barrels and circular end covers on rocket “Falcon 9” and in the manufacture of aircraft fuselage skin. Therefore, the study of basic material properties of 2198 and other third-generation Al–Li alloys is of great significance.

The forming limit is an important performance indicator and process parameter in the field of sheet metal forming which reflects the largest deformation the sheet can reach before plastic instability occurs in the process. Among a variety of methods for evaluating sheet metal formability, the FLC is of the greatest practical significance and is most widely used. The FLC is a very effective tool to evaluate sheet metal formability and solve sheet metal stamping problems. Usually there are two methods to determine the FLC: theoretical calculation and experiments. The theoretical calculation of the FLC is based on the specific plastic instability theories, including Swift’s diffuse instability theory, Hill’s localized instability theory, and Jones–Gilliss (JG) theory, and it uses different yield functions and plastic constitutive equations for theoretical calculation on the forming limit strain. Of these theories, the Swift’s diffuse instability theory (valid only when biaxial stress state exists) and Hill’s localized instability theory (no strain rate sensitivity is accounted for) have some limitations. The JG theory was originally applied to the tension test of a round bar and then extended to the right-hand side (RHS) and left-hand side (LHS) of the FLD using different yield functions and constitutive laws. In 1967 Marciniai and Kuczynski presented a groove hypothesis from the perspective of material damage, which is the most widely used damage instability theory today, known as the M–K theory.

The FLC of Al–Li alloy 5A90 was extensively studied in literature, including theoretical prediction and parameter influence of FLC based on an M–K model and the constitutive relationship of 5A90 Aluminum–lithium alloy at hot forming temperature. The FLC of 2090, 2091 and 8090 Al–Li alloys were studied in a study on the stamping limit of Al–Li alloy sheets. But the forming limit of 2198-T3 plate has not been reported. In order to characterize the measured FLC, a hemispherical dome test was performed in the present study, and the theoretical FLC of 2198-T3 based on the M–K theory was performed and different yield functions were compared with experimental data. The analysis can be used to prove the validity and accuracy of the theoretical predictions and to establish the theoretical prediction model of FLC for 2198-T3.

2. Formability test

2.1. Test material

The test pieces investigated in this work are made of 2198-T3 Al–Li alloy with a 1.5 mm thickness. The sheet was solution treated, quenched and naturally aged to a substantially stable condition (T3 heat treatment). The chemical compositions are shown in Table 1.

2.2. Uniaxial tension test

All tests were carried out at room temperature. The specimens were cut along three different directions (rolling direction, diagonal and transverse direction) from a 2198-T3 sheet. They were selected in an uniaxial tension test according to the standard of GB/T 228-2002 (Metallic materials-Tensile testing at ambient temperature). Three specimens at least were tested for each condition. Scatter is negligible so that only one curve was plotted. The true stress–strain curves of the specimens in different directions are shown in Fig. 1.

The basic formability parameters were calculated according to the standards of GB/T 5027-1999 (Metallic materials-Sheet and strip-Determination of plastic strain ratio (r-values) and GB/T 5028-1999 (Metallic materials-Sheet and strip-Determination of strain hardening exponent (n-values)). The K-value is the hardening coefficient. The r-values were thick anisotropy coefficients for a plastic deformation of 10%. See Table 2.

| Table 1 Chemical composition of 2198 alloy. |
|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| Element           | Cu                | Li                | Zn                | Mn                | Mg                | Zr                | Si                | Ag                | Fe                |
| wt.%              | 2.9–3.5           | 0.8–1.1           | <0.35             | <0.5              | 0.25–0.8          | 0.04–0.18         | <0.08             | 0.1–0.5           | <0.01             |

<p>| Table 2 Basic formability parameters of 2198-T3. |
|-------------------|-------------------|-------------------|-------------------|</p>
<table>
<thead>
<tr>
<th>Orientation (°)</th>
<th>Yield stress (MPa)</th>
<th>Ultimate tensile strength (MPa)</th>
<th>Uniform elongation (%)</th>
<th>K (MPa)</th>
<th>n-value</th>
<th>r-value</th>
</tr>
</thead>
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<tr>
<td>0</td>
<td>385.0</td>
<td>475.0</td>
<td>14.5</td>
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<td>0.168</td>
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<td>45</td>
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<td>455.0</td>
<td>15.9</td>
<td>714</td>
<td>0.172</td>
<td>0.779</td>
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<tr>
<td>90</td>
<td>322.5</td>
<td>432.5</td>
<td>17.2</td>
<td>757</td>
<td>0.180</td>
<td>2.073</td>
</tr>
</tbody>
</table>

Fig. 1 True stress-true strain curves of 2198-T3.
2.3. Forming limit test

The forming limit diagram was obtained by conducting a hemispherical dome test based on the GB/T 15825.8-2008 standard (sheet metal formability and test methods-Guidelines for the determination of forming-limit diagrams). Before testing, all the specimens were electro-etched using a grid of circles of 2 mm in diameter. Then the specimens were placed between the die and blank holder and pressed by a blank holding force at room temperature. The middle part of the test piece showed bulging deformation and formed a convex hull under the action of the punch force (shown in Fig. 2). At the same time, the circular grid on the surface of the test piece was distorted. The principal strains at fracture were estimated by measuring the distortion of the grid as near as possible to the fracture zone, which were used to define the limit principal strains of the local surface that the sheet metal could withstand. The width of the test piece and the lubrication conditions were changed to get a different strain state.  

The test specimens were prepared by varying the width of the blanks from 20 mm to 220 mm (respectively 20, 50, 80, 100, 120, 140, 160 and 220 mm), while the horizontal direction (aligned in the transverse direction) was fixed in length. The thickness of the sample is 1.5 mm. In order to obtain enough test data under different strain states to describe accurately the forming limit curve of the sheet at room temperature, the plates were processed into the sample forms shown in Fig. 3.
The tested specimens were shown in Fig. 4. The strain of each test piece was measured and selected according to the standards: (1) discard the scattered data point from the three points in one group if it is far away from the other two close points; (2) keep these three points if they are gathered together or relatively dispersed in the same coordinate system.19

3. Theoretical prediction of FLC

3.1. M–K theoretical model

The core of the M–K theory is the famous assumption of the initial inhomogeneity factor: due to geometric or physical causes, there is an initial inhomogeneity factor on the direction perpendicular to the direction of the maximum principal stress when a biaxial tension exists on a sheet metal surface. That means there is a linear groove before deformation occurs on the sheet surface. The strain concentration will appear and grow in the groove with the degree of deformation increasing. Under this assumption, the localized instability of the sheet is actually caused by the existence of initial surface defects.20

The theoretical model diagram is shown in Fig. 5, in which part \( B \) is the uneven deformation zone which is called the groove part and part \( A \) is the uniform deformation area. \( t_{A4} \) and \( t_{B4} \) represent the thicknesses of the part \( A \) and part \( B \). \( \sigma_1 \) and \( \sigma_2 \) are the major stresses.

The core equations of the M–K theory include11,19:

1. The volume of the sheet remains the same along with the sheet deformation:

\[
d\epsilon_1 + d\epsilon_2 + d\epsilon_3 = 0
\]  
(1)

where \( d\epsilon_1 \) is the strain increment, the number \( i (=1,2,3) \) represent the rolling, transverse and thickness direction, respectively.

2. The principal stresses in the three directions of part \( A \) increase in proportion:

\[
d\epsilon_{1A} = \frac{d\epsilon_{2A}}{\epsilon_{2A}} = \frac{d\epsilon_{3A}}{\epsilon_{3A}}
\]  
(2)

where \( \epsilon_{1Y} \) is strain of part \( A \) or \( B \) (the letter \( Y \) represent the part \( A \) or \( B \)); the number \( i \) represents the three directions. The \( d\epsilon_{1Y} \) is strain increment of part \( A \) or \( B \).

The ratio of strains is unchanged through the loading process:

\[
d\epsilon_{3A} = \frac{\epsilon_{3A}}{\epsilon_{2A}}
\]  
(3)

3. The increments of transverse strain (minor strain) are the same both in part \( A \) and part \( B \):

\[
de_{2A} = de_{xB} = de_2
\]  
(4)

(4) The force equilibrium condition should be satisfied at each moment during deformation:

\[
\sigma_{1A}t_{A4} = \sigma_{1B}t_{B4}
\]  
(5)

where \( \sigma_{1Y} \) is stress, the number \( i \) represents the three directions, the letter \( Y \) represents the part \( A \) or \( B \).

5. The initial inhomogeneity factor:

\[
f_0 = \frac{t_{B0}}{t_{A0}}
\]  
(6)

where \( t_{A0} \) and \( t_{B0} \) is the initial thicknesses of part \( A \) and \( B \).

3.2. Theoretical prediction of FLC

When plastic deformation occurs, the strain increases faster in the groove than outside the groove. Therefore the stress states inside and outside the grooves are different. If we assume that the stress state remains constant in part \( A \) because of the stationary linear load, the load route of part \( B \) changes nonlinearly along different levels of the yield surface. The stress state and stress intensity have to be changed to meet the geometric coordinate conditions and static equilibrium conditions in order to reach the planar strain state, in which the groove deepens (\( \Delta\epsilon_{1B} > \Delta\epsilon_{1A} \)) and the material is thought to lose its ability to bear the deformation, and then localized necking occurs.

The strain increment \( \Delta\epsilon_{1A} \) of part \( A \) is given, with which we can calculate \( \Delta\epsilon_{2A} \) and get the value of \( \Delta\epsilon_{3A} \) with the volume conditions:

\[
de_\epsilon = -(\Delta\epsilon_1 + \Delta\epsilon_2)
\]  
(7)

All the strain values change with the increase of the strain increment as follows:

\[
e = e_0 + de
\]  
(8)

where \( e_0 \) is the initial train.

The compatibility condition is used to link the two regions \( A \) and \( B \) with the algorithm:

\[
(\epsilon_{1A} + de_{1A})/\phi_A = f_0(e_{1A} + de_{1B})/\phi_B
\]  
(9)

where \( \epsilon_{1Y} \) is equivalent strain of part \( A \) or \( B \) (the letter \( Y \) represents the part \( A \) or \( B \)). \( \Delta\epsilon_1 \) is increment of equivalent strain. \( \sigma_Y \) is equivalent stress. The process parameter \( \phi_Y \) is the ratio of the equivalent stress and major stress (\( \phi_Y = \sigma_Y/\sigma_1 \))

3.3. Hardening law formulation

The selection of a material hardening law is essential to the accuracy of stress calculation from measured strains. Uniaxial tension tests were performed for getting the stress–strain curves of Al–Li alloy 2198-T3 sheet. The true stress–true strain data measured in the test were fitted to the equation as follows:21

\[
\sigma = Ke^n
\]  
(10)

The hardening model parameters \( K \) and \( n \), respectively represent the strength coefficient and strain hardening exponent. The values of \( K \) and \( n \) can be obtained from the fitting calculation of uniaxial tension test based on the constitutive model equation above. Fig. 6 is a comparison diagram between the experimen-
3.4. Yield functions

Four different yield functions were selected to calculate the FLC of 2198-T3 based on the M–K theoretical model in this paper: the Mises yield function, Hill’48 yield function, Hosford yield function and Barlat–Lian’89 yield function.

(1) Mises yield function

In 1913, Mises revised the Tresca yield function and established Mises yield function for the convenience of calculation. He proposed that a material begins plastic deformation when the RMS value of the three principal shearing stresses reaches a critical value in any stress state. This criterion applies only to the isotropic materials. \( \sigma_s \) is the yield stress. The mathematical expression is:

\[
\frac{1}{\sqrt{2}} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} = \sigma_s
\]  

(11)

(2) Hill’48 yield function

In 1948, Hill introduced the concept of anisotropy into the yield equation for the first time. He proposed a yield function for orthotropic materials following the Mises yield function as a mode and established a reasonable mathematical model to describe the anisotropic plastic flow of sheet metal which laid the foundation for the establishment of the theory of anisotropic plastic deformation.

\[
F(\sigma_{xy} - \sigma_{zx})^2 + G(\sigma_{xz} - \sigma_{xy})^2 + H(\sigma_{xy} - \sigma_{yz})^2 + 2L\sigma_{xy}^2 + 2M\sigma_{yx}^2 + 2N\sigma_{xy}^2 = 1
\]  

(12)

In the formula, \( x, y, z \) are the orthotropic axes, respectively. \( F, G, H, L, M, N \) are the independent anisotropic characteristic parameters determined by experiments according to different materials. A simplified quadratic yield equation facing to a planar isotropic and thick anisotropy material is used in the calculation:

\[
\sigma_1^2 - \frac{2r}{1 + r} \sigma_1 \sigma_2 + \sigma_2^2 = \sigma_0^2
\]  

(13)

where \( \sigma_0 \) represents the yield stress. The \( r \)-value is thick anisotropy coefficient.

(3) Hosford yield function

Logan and Hosford proposed the following yield function for the planar stress state of anisotropic materials in 1979:

\[
|\sigma_1|^{m} + |\sigma_2|^{m} + r|\sigma_1 - \sigma_2|^{m} = (1 + r)^{m}
\]  

(14)

where \( m \) in Hosford yield function is not adjustable; for body-centered cubic metals, \( m = 6 \), while for face-centered cubic metals, \( m = 8 \).

(4) Barlat–Lian’89 yield function

In 1989, Barlat pointed out that the Hosford yield function could not handle the situation that the main axes of anisotropy were not aligned with the main stress axes, because it did not contain a shear stress component. So Barlat proposed a yield function considering anisotropism in the planar stress conditions:

\[
\frac{1}{2} [ a |K_1 + K_2|^M + a |K_1 - K_2|^M + c |2K_2|^M ] = \sigma_0^M
\]  

(15)

\[
K_1 = \frac{\sigma_{11} + h\sigma_{22}}{2}
\]  

(16)

\[
K_2 = \sqrt{\left(\frac{\sigma_{11} - h\sigma_{22}}{2}\right)^2 + \rho \sigma_{22}}
\]  

(17)

where \( \sigma_0 \) represents the yield stress of uniaxial tension test. \( a, c, h, p \) are the anisotropy parameters. \( K_1 \) and \( K_2 \) are process parameters. For body-centered cubic metals, \( M = 6 \), for face-centered cubic metals, \( M = 8 \).

3.5. Determination of the initial inhomogeneity factor

The influence of the initial inhomogeneity factor on the prediction of forming limit based on the M–K model cannot be ignored. The determination of initial inhomogeneity factor \( f_0 \) is very complex and error-prone because it depends on many factors, including the thickness of the sheet metal, surface quality, grain size and other material properties. The theoretical forming limit curve is defined by adjusting the value of \( f_0 \) in practical calculations to make the theoretical FLC (when minor strains vanish) close to the experimentally measured FLC in a planar strain state. Therefore, the initial inhomogeneity factor is an adjustable parameter in the calculation.

The influence of the initial inhomogeneity factor on the FLC of 2198-T3 at room temperature is shown in Fig. 7. Generally, the forming limit curve is high when the \( f_0 \) value is big, while the curve is low when the \( f_0 \) value is small. The forming
limit curve drops down with the decrease of value $f_0$ up to a particular location which corresponds to a constant value of $f_0$.

With reference to the forming limit curve shown in Fig. 7, the theoretical forming limit curve exhibits good agreement with the experimentally measured forming limit curve by adjusting the value of $f_0$ when predicting the FLC at room temperature. 0.99 is selected to be the value of the initial inhomogeneity factor.

3.6. Influence of $n$-value (hardening exponent)

The hardening exponent is another important parameter with a significant impact on the FLC. Fig. 8 is a comparison of different curves based on different $n$-values. We can draw a conclusion that the curve is higher with the increase of the $n$-value. Due to the fact that the differences of the tested $n$-values shown in Table 2 are not significant, the influence of the $n$-value is within an acceptable range. The minimum $n$-value is selected in the calculation for safety, because the corresponding curve is the lowest.

3.7. Influence of $r$-value (thick anisotropy coefficient)

The $r$-value (thick anisotropy coefficient) is the ratio of strains in the width direction to the thickness direction. Due to the significant variation of the $r$-value shown in Table 2, the influence of the $r$-value on the FLC should be taken into account. The forming limit curves based on the same parameters but three different $r$-values are calculated and compared in Fig. 9. We can conclude that the influence of the $r$-value on the forming limit curve can be ignored. The $r$-value used in the calculation is the average of the $r$-values obtained from uniaxial tension tests in three directions (0°, 45° and 90° directions). (See Eq. (18)). So the theoretical calculation based on the basic formability parameters is credible.

$$r = (r_0 + 2r_{45} + r_{90})/4 \quad (18)$$

4. Theoretical prediction of FLC

The theoretical predictions are compared with the forming limit data received from the punch bulging test based on the same parameters in order to verify their feasibility and validity.

Fig. 8 is the comparison diagram between the forming limit curves of experimental data and theoretical prediction at room temperature. We can draw a conclusion that the experimentally measured data points are intensive; at the same time all the four theoretical prediction curves agree well with the experimentally measured curve near the planar strain state but are different from the experimentally measured curve on both sides of the curve away from the planar strain state. The experimentally measured data points on the left side are distributed evenly around the theoretical prediction curves based on four different yield functions under the tension–compression strain states. The prediction curve derived from Hosford yield function is more accurate, and the anisotropic index ($r$) has little effect on the FLC. The prediction curves on the right side are slightly lower than the experimentally measured data under the tension–tension strain states, and the prediction curve based on Hill’48 yield function is closer to the experimentally measured curve. The prediction of FLC is more accurate when the value of $r$ is greater than 1 due to the applicability of the Hill’48 yield function. In general, the theoretical prediction of the FLC shows good agreement with the measured results, which means the theoretical FLC based on material parameters, different yield functions and M–K theory is valid.
5. Conclusions

(1) The basic formability of the third generation 2198-T3 was characterized by a uniaxial tension test at room temperature. And the Forming Limit Curve (FLC) of 2198-T3 was measured by conducting a hemispherical dome test with specimens of different widths.

(2) The comparison result proves the feasibility and validity of the theoretical prediction of the FLC which is based on the M–K theoretical model and four different yield functions.

(3) The four theoretical prediction curves all agree well with the experimentally measured curve near the planar strain state. The theoretical prediction curve based on Hosford yield function is more accurate under the tension–compression strain states described in the left part of the FLC. The accuracy of the theoretical prediction curve based on Hill’48 yield function under the tension–tension strain states shown in the right part of FLC is better.

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References


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