Salient Features of Cluster based Routing for Mobile Networks
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Abstract
Clustering maps a physical network to its corresponding virtual network of interconnected clusters of nodes. A clustering based approach reduces traffic and communication overhead during the routing process. In this context, control messages propagate between different clusters. Various cluster based routing protocols in mobile networks are available till now. The extended features of such routing protocol have been introduced in this paper. The concept of conflict graph on that routing problem includes assigned disjunctive constraint. The routing method is expressed as a Multi objective optimization problem and then, the possible solution for that problem has been discussed. In addition, an approximation based approach is introduced on the routing method to obtain near optimal solution for the performance metric. The usefulness of such approximation is stated in terms of application perspective.

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1. Introduction

The architecture of mobile cellular networks is stationary. There is a base station (BS) in every cell. The BSs (nodes) communicate with each other using multi-hop links. A node (BS) in a network can broadcast the calls to its neighboring nodes. The sharing of Quality of Service (QoS) parameter from this forwarded call may lead to increase the network traffic. One approach to reduce traffic during the routing process is to divide the network into clusters. Clustering basically transforms a physical network into a virtual network of interconnected clusters or groups of nodes [3, 4, 5]. Control messages may have to propagate within a cluster. Thus, multilevel hierarchy reduces the storage requirement and the communication overhead of large networks. A new base station can add or remove to/from the cluster in the cellular network. As a result, a new link comes up or breaks into the network.

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Thus clustering is necessary for providing scalability, adaptability and autonomy to achieve strong connectivity in the resulting network. One of the clustering based routing techniques for mobile networks is addressed in [1]. This procedure is resolved with the concept on minimum spanning tree (MST) [2]. In short, the procedure excludes the disjunctive constraints assigned on that system. Simultaneously, the different cost metric is required to assign on the edges of the tree. In such scenario, some salient features related to this routing method are introduced to meet user satisfaction in practice.

The salient features of the cluster based routing problem (Π'), in [1], for mobile networks are introduced in this paper. The disjunctive constraint for pairs of edges in the routing tree is represented here with the introduction of conflict graph approach and therefore, it is formulated as Integer Linear Programming (ILP) expression. Consequently, the routing problem is transformation into a Multiobjective optimization problem based on different cost metric assigned on the edges in the tree. The possible solution for that Multiobjective optimization has been discussed. In addition, an approximation based scheme is introduced to obtain near optimal solution considering the degree of routing tree. All of these features are reflected on user satisfaction perspective.

The rest of the paper is organized in the following sections. We begin with a brief presentation of the cluster based routing approach for completeness of the work in section 2. The proposed model is described in section 3. Finally in section 4, we conclude with a short discussion on advantages of the model proposed here.

2. Cluster based Call Routing Approach

A constraint based clustering algorithm designed for call routing in mobile networks is described in [1]. The clusters are formed on the basis of dynamic threshold value (τ). Here, the cluster head (CH) is elected with respect to different weight metrics for improving call scheduling. The leader (CH) election procedure is described for link breakage and link emergence respectively. Once the clusters are formed and maintained, these can be used to handle incoming call requests. So, the routing path of the call request is represented as follows:

\[ C_S \rightarrow B_S \rightarrow \text{Leader}_S \rightarrow \text{Leader}_R \rightarrow B_R \rightarrow C_R \]

Where, \( C_S, B_S \) and \( \text{Leader}_S \) denote caller, Base station and Cluster head at the sender side respectively. Similarly, \( C_R, B_R \) and \( \text{Leader}_R \) denote the analogous representation for the receiver side. If \( \text{Leader}_S \) and \( \text{Leader}_R \) (BSS) are predetermined, the next step is the call routing between \( \text{Leader}_S \) and \( \text{Leader}_R \). Here, the shortest path between \( \text{Leader}_S \) and \( \text{Leader}_R \) (BSS) is found with the concept of minimum spanning tree \( T \) for a connected weighted graph \( G' = (v, e) \) where \( v \) and \( e \) denote the set of vertices and edges respectively. This is done by using Kruskal’s algorithm. So, to map the problem of call routing to the Kruskal’s algorithm, a connected weighted graph \( G \) is constructed. The vertices of \( G \) represent cluster heads (\( \forall CH_k \in v \)) and the weight (\( \omega \)) on the edges represents the Euclidean distances \( \omega_{d(e_k)}: \{CH_i, CH_j\} \forall CH_k \in v \land \forall e_k \in e \) between different cluster heads (CHs) as shown in Fig. 1.
So, the routing problem can be formulized as follows.

\[
\text{objective} \quad \text{minimize} \quad \sum_{e_k \in e} \omega_d^{(e_k)} \tag{1}
\]

subject to

\[
\sum_{e_k \in e} e = v - 1
\]

\[
\bigcup_{k=1} T_k = T \quad \forall (v_k, e_k) \in T \tag{2}
\]

### 3. Proposed Model

The proposed model describes the important features of the routing method introduced in previous by following sequences.

#### 3.1. Cluster based Routing with Conflict Graph Approach (CRCGA)

There exist incompatibilities for certain pair of edges in \( G \) consists of \( n \) vertices and \( m \) edges. It means that such conflictness related to \( \omega_d^{(e)} \) at most one edge can occur in \( T \) due to equal Euclidean distances between different pairs of cluster heads. Now, these symmetric conflict relations are represented by means of an undirected conflict graph \( G' = (v', e') \), where every vertex \((v_k \in v)\) of \( G' \) corresponds uniquely to an edge in \( G \) and an edge \((e_k \in e)\) in \( G' \) implies that the two adjacent vertices, i.e. edges in \( G \), cannot occur together in an MST solution. In contrast to \( G \), \( G' \) is not necessarily connected and may contain isolated vertices (i.e. edges of \( G \) which can be combined with every other edges in MST solution).

Now consider for a set of vertices \( F \subseteq v \) in \( G \), \( e(F) \) be the set of edges in \( G \) that have both of its endpoints in \( F \). Then \( CRCGA \) is stated by the following ILP formulation:

\[
\text{(CRCGA)} \quad \text{min} \quad \sum_{e_k \in e} \omega_d^{(e_k)} \times x_{e_k} \tag{3}
\]

Subject to:

\[
\sum_{e_k \in e} x_{e_k} = n - 1 \tag{4}
\]

\[
\sum_{e_k \in e(F)} x_{e_k} \leq |F| - 1 \quad \forall \phi \neq F \subseteq v \tag{5}
\]

\[
x_{e_k} + x_{f_k} \leq 1 \quad \forall (e_k, f_k) \in e' \tag{6}
\]

\[
x_{e_k} \in \{0, 1\} \quad \forall e_k \in E \tag{7}
\]
The conflict constraint on pairs of edges is solved with (6). The detailed discussion on such formulation is described by the following example.

**Example:** In Fig. 2, let us assume a graph $G$ with the set of vertices and the set of edges are denoted by the following.

$$V = \{A, B, C, D, E, F\} \text{ and } E = \{AD, AB, BE, DE, EF, CF, EC, AE, DB\}$$

and subsequently, the names for the edges in $G$ are as follows.

$$AD = a, AB = b, BE = c, DE = d, AE = e, BD = f, EC = g, CF = h \text{ and } EF = i.$$
tree, so a set of vertices is taken). Therefore, it is given that $F = \{A, B, C, D, E, F\}$ and $E(F) = \{AD, AB, BE, DE, EF, CF, EC, AE, DB\}$. So, it implies that $|F| = 5$ and $\sum_{e \in E(F)} X_e \leq |F| - 1$. In this example, it can be written as $X_A + X_B + X_C + X_D + X_E + X_F + X_G + X_H + X_I \leq |F| - 1$ and furthermore, the minimum cost for $G$ is 19 by following $\sum_{e \in E} w(e) \times X_e$. The following Fig. 4 shows the minimum spanning tree for $G$.

Fig. 4: MST for $G$

3.2. Routing Applications towards Multi objective Optimization

In real life, the other cost factors like transmission cost, hop count, energy of the nodes, delay etc. [6] are simultaneously responsible for determining the effectiveness of the system rather than only Euclidean distance as proposed in [1]. Therefore, the Multi objective optimization for the problem $(\Pi'')$ is represented as follows.

3.2.1. Multi objective Optimization for $\Pi''$

Let us consider the case for $d$ weightings $\omega_1, \omega_2, \ldots, \omega_d \in R^n$ are assigned on $n$ edges of $G$. That is, a real value is assigned for every $\omega_i$ to each element of $[n]$. We assume $W \in R^{d \times n}$ be a matrix with rows $\omega_1, \omega_2, \ldots, \omega_d$. Now we define the incidence vector of $S$ as $e_i := \sum_{e \in E} x_e \in R^n$ for each spanning tree $S$ of $G$. Hence the cost of the $S$ is determined by the inner product of any $\omega_i$ and its corresponding $e_i$ with respect to $\omega_i$. In order to clarify this idea, an example is given in the following with respect to Fig. 1.

Example: Let us consider here $d = 2$, and $n = 11$ from Fig. 1. Clearly the edges in Fig. 1 are ordered as $(\{CH_1, CH_2\}, \{CH_1, CH_4\}, \{CH_2, CH_3\}, \{CH_2, CH_4\}, \{CH_3, CH_5\}, \{CH_4, CH_5\}, \{CH_5, CH_6\}, \{CH_5, CH_7\}, \{CH_6, CH_7\})$. Then the first weight $\omega_1$ and the second one $\omega_2$ are considered as $(4, 7, 8, 9, 2, 8, 5, 3, 10, 5, 2)$ and $(4, 7, 4, 6, 6, 7, 13, 10, 2, 1, 2)$. So the matrix $W$ can be represented as follows.

\[
\begin{pmatrix}
4 & 7 & 8 & 9 & 2 & 8 & 5 & 3 & 10 & 5 & 2 \\
4 & 7 & 4 & 6 & 6 & 7 & 13 & 10 & 2 & 1 & 2
\end{pmatrix}
\]

The incidence vectors for minimum spanning trees $(S_1, S_2)$ are $(1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1)$ and $(1, 0, 1, 1, 1, 0, 0, 1, 1, 0)$ respectively. Thus the cost of $S_1$ is $e_{S_1} \cdot \omega_1 (= 24).$ Similarly, the cost of $S_2$ is evaluated as 23. So, the set of all spanning trees $S_M$ of $G$ can be found by applying this procedure.

Now let $c_{S_1}$ is the cost vector for spanning trees $S_1$ where the first entry is the cost of spanning tree $S_1$ with respect to $\omega_1$ and the second entry is the cost of spanning tree $S_1$ with respect to $\omega_2$, then the following cost vectors are obtained for Fig. 1 as $c_{S_1} = (24, 39)$ and $c_{S_2} = (38, 23)$ respectively.
3.2.2. Pareto optimality for $\Pi'$

Generally, there are different techniques used to select the optimal spanning tree or set of optimal spanning trees using different tie-breaking criteria. The Pareto optimization is one of the possible techniques due to given different weightings, a different spanning tree might be best for each edge weighting. The following rule is given for finding Pareto optimality for Multi objective optimization.

Rule: The $\min_{\text{Pareto}}$ is to be found for vectors $a, b$ as $a \leq b$ iff $a_i \leq b_i \forall a, b \in R^d$. Furthermore, it is true that $a < b$ if $a \leq b$ and $a \neq b$.

Now each point $c_{S_{ij}}$ is marked with the spanning tree that it corresponds to. Hence, the Pareto optima are found with the help of predefined rule. Thus, we can find the feasible space for $c_{S_{i1}}$ and $c_{S_{i2}}$ in Fig. 5 corresponds to Fig. 1. The Pareto optima points have the smallest cost in regards to at least one of the weights related to real-life applications [7].

![Fig.5: Pareto optimal solutions for $\Pi'$](image)

3.2.2. Objective Function for Multi objective Optimization of $\Pi'$

With the discussion of Multi objective optimization and subsequent Pareto optimality for $\Pi'$, the objective function for $\Pi'$ is described by the following.

\[
\begin{align*}
\text{objective} & \quad \min_{\text{Pareto}} (W e_{S'}) \\ 
\text{subject to} & \quad (8) \\
& \begin{cases} 
  c_{S_{ia}} \leq c_{S_{ib}} & \text{if } f(c_{S_{ia}}), \leq (c_{S_{ib}}), \forall c_{S_{ia}}, c_{S_{ib}} \in R^d \\
  c_{S_{ia}} < c_{S_{ib}} & \text{if } f(c_{S_{ia}}) \leq c_{S_{ib}} \text{ and } c_{S_{ia}} \neq c_{S_{ib}} 
\end{cases} \\
\end{align*}
\]

3.3. Approximation for problem ($\Pi'$)

The difficulty of computing lies in creating a minimum set of solutions for a multi-objective optimization problem that represents approximately the whole Pareto curve within a desired accuracy. In our work, the problem can be reduced as a bi-objective optimization problem due to $d = 2$. The generation of Pareto optima solutions for a bi-objective problem ($\Pi'$) is also NP-hard [8].

Generally, the degree of a vertex for a spanning tree is denoted by the number edges incident to that vertex, and subsequently, the degree of a spanning tree is determined by the maximum of the degrees of its vertices. Related
to our Multi objective optimization formulation, the problem is to find a spanning tree whose degree is minimum. Thus minimizing the degree of a spanning tree finds with a smallest integer $l$ for which there exists a spanning tree with each vertex has at most $l$ incident edges [11].

In the routing method addressed in [1], the cluster head (as represented the vertex of the graph) is elected on the basis of the degree ($D$) of the vertex due to nearest neighbourhood principle [1, 9]. It denotes the number of edges connected to it. Here if we can calculate the maximum degree with respect to the degrees of all vertices in graph, and consequently the corresponding degree of that spanning tree can be found. Thus the objective of the problem is now to minimize the degree of the spanning tree with a smallest integer $k$. So, the approximation guarantees that there exists a spanning tree in which each vertex has at most $k$ incident edges. This approximation can be useful for such BS (considered as CH) in the network equips with $k$ interfaces. Therefore a connected $k$ - edge coloring subnetwork can be chosen to transport data. Since all the adjacent links in this sub network can use different channels to reduce wireless interference and increase network throughput accordingly [10].

4. Conclusions

The salient features for the cluster based call routing has been focused in this work. These features are correlated with the real life scenario as well as many applications. The solution for disjunctive constraint on the equal Euclidean distances is introduced with the concept of conflict graph. Furthermore the routing method is expressed as a Multi objective optimization problem. The possible solution for such optimization has been discussed. Hence the problem is classified as an NP-hard problem. In addition an approximation is suggested for the routing problem to obtain near optimal solution.

References


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