PERGAMON
Vision Research 41 (2001) 1489–1499
Vision Research
www.elsevier.com/locate/visres

Photographic reproduction of out-of-focus and distorted ocular imagery

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Received 1 February 1999; received in revised form 23 October 2000

Abstract

The necessary settings and parameters were determined for ordinary camera and lens systems to faithfully reproduce out-of-focus and distorted imagery as it falls upon the retina of the human eye. Theoretic considerations of both geometric and physical optics were used to calculate the 'relative blur' and distortion produced by refractive error added to ordinary camera lenses as opposed to refractive error in an arbitrary thick-lens optical system bounded by air and fluid (i.e. the eye). In both the camera and the eye, 'relative blur' was determined to be directly proportional to dioptric defocus and to aperture size, and effectively independent of the focal length. Distortion of imagery was also found to be independent of the focal length. Photographs corroborate the theoretic findings. A given amount of relative blur, however, appeared somewhat greater when recorded on photographic film than when appreciated by the human eye. The Stiles–Crawford effect, the chromatic aberration of the eye, and neural processing probably each contribute to this difference. Previous investigators have grossly exaggerated blur and distortion in photographs intended to simulate ocular imagery and have drawn misleading conclusions from their results.

Keywords: Blur circle; Relative blur; Distortion; Diffraction; Ocular imagery; Photography

1. Introduction

For over a century, ophthalmologists have tried to use cameras to simulate what the human eye sees. Photographs showing blurred and distorted images have appeared periodically in the ophthalmic literature with the purpose of simulating the blur and distortion seen by the uncorrected or corrected ametropic eye, with many demonstrations dealing with the Jackson cross cylinder test (Bull, 1896; Crisp, 1923, 1931, 1932; Williamson-Noble, 1943; Guyton, 1977; Sims & Durham, 1986; Sims, 1988; Zisser & Guyton, 1989).

The most common technique used to simulate ocular imagery has been to add an 'error lens' in front of a focused camera, with the seemingly straightforward assumption that the camera will see what the eye sees. This assumption, however, is not true. The placement of the error lens and the size of the camera aperture have significant effects on the imagery obtained.

Attempts at photographic simulation by Bull in 1896 led him to state, "I was inclined to suppose that my photographs were wholly incorrect . . . an observer looking through a corresponding lens was able to make out with . . . ease . . . a line which seemed in the photograph to be very bad." More recently, in 1986, Sims and Durham purported to show the distortion and blur effects experienced by an eye being tested with a ± eighth diopter cross cylinder (Fig. 1). It should be immediately clear to anyone who has been refracted with a low-power cross cylinder, however, that the optical effects in this photograph are grossly exaggerated. Though some authors have acknowledged that the camera is not an equivalent optical system to the eye (Bull, 1896; Crisp, 1943; Puntenney, 1946; Sims & Durham, 1986) no systematic attempt has been made to explain the differences in images produced, and previ-

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0042-6989/01/$ - see front matter
PII: S0042-6989(01)00014-1

ous authors have drawn misleading conclusions from their photographic demonstrations.

Another approach to photographic simulation has been to build an optical replica of the eye, with total power, pupil size, and focal length identical to that of the eye. This was recently accomplished by Holladay and colleagues to photograph the optical effects of bifocal intraocular lenses and has provided some useful results (Holladay, Van Dijk, Lang, Portney, Willis, Sun, & Oksman, 1990). Such a custom optical replica of the eye, however, is difficult to design and build and particularly difficult to incorporate into a camera. Moreover, the photographs do not take physiologic factors into account and may still produce more blur than is perceived by the human eye.

The purpose of this study has been to determine the appropriate settings for an ordinary camera which will simulate the relative blur of out-of-focus ocular images and the distortion produced by correction of astigmatic error. Relative blur may be defined as the diameter of the blur circle from each object point relative to the object detail. For example, if the eye one diopter out of focus creates blur circles subtending twice the angle of the strokes of a 20/40 optotype, one should like the camera one diopter out of focus to do so likewise. For reasons to be explained, photographs taken by placing an 'error' trial lens in front of a camera cannot be equated to images produced by the eye with an equivalent amount of refractive error unless specific parameters of the photographic system are set appropriately.

A geometric analysis, using Gaussian optics, will demonstrate that the relative blur produced by an imaging system, for a given object at a fixed distance, is directly proportional to the entrance pupil size and to

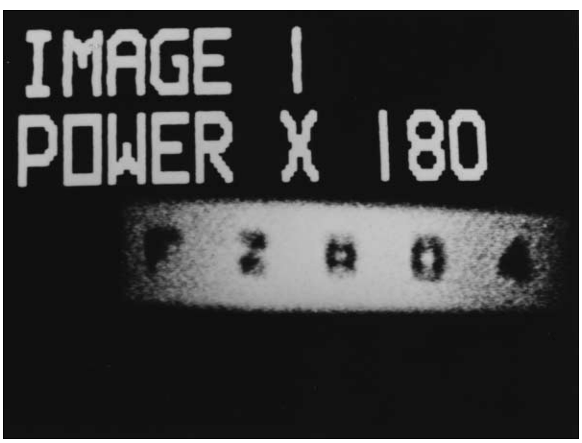

Fig. 1. Illustration from the literature showing blur produced by a $\pm$ eighth diopter cross cylinder placed in front of a camera. It is clear that the optical effects are grossly exaggerated and do not simulate the blur of out-of-focus ocular imagery (From Sims CN & Durham DG, 1986. The Jackson cross-cylinder disproved. *Transactions of the American Ophthalmological Society*, *86*, 355–386. Photograph reproduced with permission of the American Ophthalmological Society).

the degree of dioptric defocus. Moreover, in a system where the indices of refraction in the object and image spaces are equal, the relative blur is independent of the focal length of the system. In systems where the indices of refraction are different, the focal length does appear in the equation for relative blur. In the case of the eye, however, where the focal length is a very small number, its inclusion in the equation makes little difference, changing the calculation of relative blur by only 1% or so. Given the limits of photographic film resolution, the error in excluding this factor is found to be negligible, and one can therefore easily compare ocular imagery with the imagery produced by cameras with lenses of various focal lengths. The contributions made by spherical and chromatic aberrations shall also be ignored.

In the case of a cylindrical lens placed before the eye or camera, our analysis demonstrates that the distortion produced in both cases is independent of aperture size and focal length, but simply depends on the power of the cylindrical lens and on the distance of the lens from the entrance pupil of the system.

Following these derivations, are listed and explained conditions which must be met in the optical system such that effects of diffraction do not override the geometrical analysis for out-of-focus imagery. Graphical, mathematical and photographic support are presented for these relationships.

2. Geometric considerations

2.1. Relative blur of a defocused image point in an ideal thin lens camera

Fig. 2 illustrates an ideal thin lens camera in which the center of the aperture, the nodal point, and the refractive plane are coincident. Object of height O subtends the angle θ at the nodal point of the lens of power $D + \Delta D$. Image I is formed in front of the film plane at distance v_0 from the lens. A blurred image I' is formed on the film at distance v_f from the lens. n is the index of refraction. Simple ray tracing and algebra lead to the following expression for the relative blur of a defocused image point in an ideal thin lens system.

$$\text{Relative blur} = \frac{b}{I'} = \frac{2a(\Delta D)}{n \tan \theta} \quad (1)$$

Therefore, in an ideal thin lens camera, with the aperture coincident with the lens, and with the object a fixed distance from the camera, the relative blur (b/I') of out-of-focus images is directly proportional to the aperture size $2a$ and to the dioptric amount of defocus ΔD. Specifically, the relative blur is independent of focal distance, because $\tan\theta$ is constant.

Thus, if actual cameras and the eye behaved as ideal thin lens cameras, a camera could be used to simulate

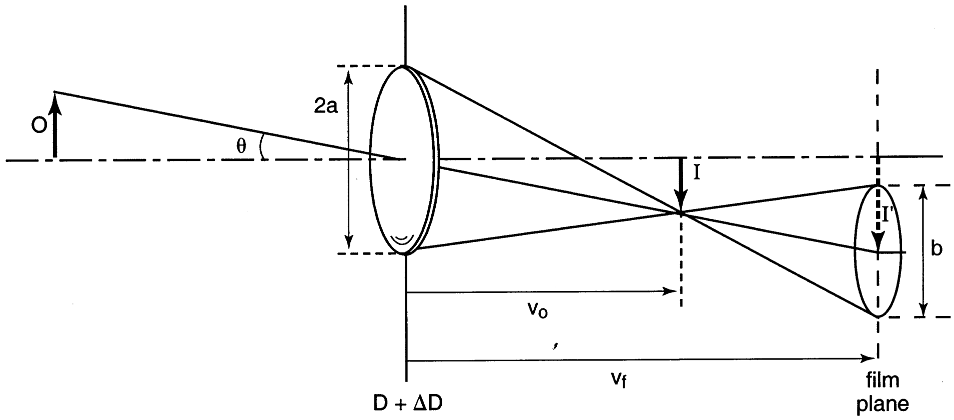

Fig. 2. An ideal thin lens camera with a lens of dioptric power D increased by ΔD. The object O subtends an angle θ at the nodal point. An image I is formed anterior to the film plane a distance v_0 from the lens. The diameter of the blur circles b that form the blurred image I', on the film at distance v_f from the lens, is related to the aperture size and dioptric amount of defocus.

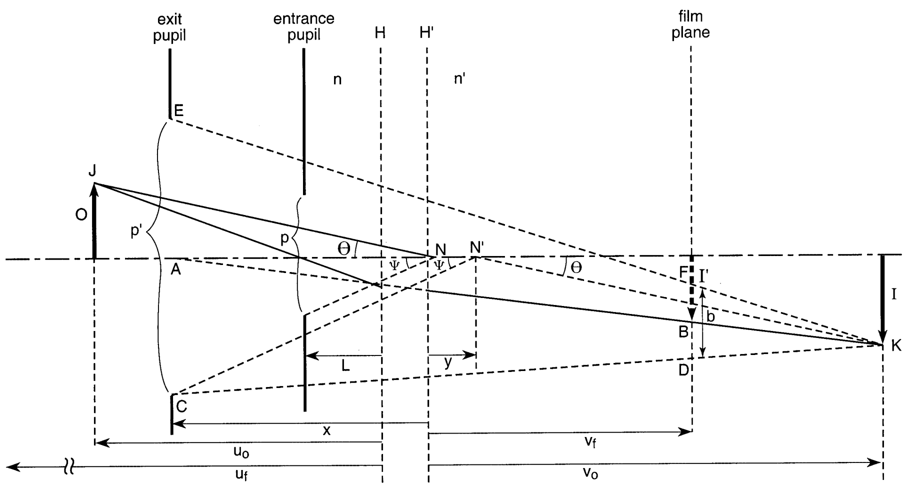

Fig. 3. Image formation in a general optical system. The object O subtends an angle θ at the nodal point N, as does the image I at N′, formed at distance v_0 from the second principal plane H′. The size of the image I is determined by the chief ray which leaves the tip of the object and passes through the center of the entrance pupil, exiting through the center of the exit pupil. ψ is the half angle the entrance pupil subtends at the first nodal point N and is equal to the half angle which the exit pupil subtends at the second nodal point N′. The separation of nodal points N and N′, equal to the distance separating the first and second principal planes H and H′, is a consequence of the difference in indices of refraction n and n' in the object and image space respectively. The diameter of the blur circles b that form the blurred image I' on the film plane, at distance v_f from the second principal plane, is related to the amount of dioptric defocus and to the exit pupil size p' which, in turn, is related to the size of the entrance pupil p.

ocular imagery simply by making the aperture of the camera equal in size to the pupil of the eye. For example, for a fixed dioptric amount of defocus ΔD, a camera aperture 5 mm in diameter would produce relative blur equal to that in an eye with a 5 mm pupil, independent of the focal length of the camera lens.

But actual cameras and the eye are not ideal thin lens systems. The apertures are not coincident with the refractive (principal) planes, and in the eye there is fluid on one side of the optical system. For these reasons, consideration of relative blur must be more detailed.

2.2. Relative blur of a defocused image point in a general optical system

Fig. 3 illustrates a general optical imaging system with different indices of refraction in the object and image spaces, thus separating nodal points N and N′ from principal planes H and H′. The entrance and exit pupils are shown, but the actual pupil has been omitted as it is not used in the analysis. The entrance pupil is the image of the actual pupil which is formed by the refractive elements anterior to the actual pupil. It can be seen from Fig. 3 that paraxial rays of light that enter the system are limited by the entrance pupil. The exit pupil may lie either anterior or posterior to the entrance pupil, depending on the particular system, and consists of the image of the actual pupil formed by the refractive elements of the system posterior to the actual pupil. The bundle of rays of light that fills the entrance pupil is the same bundle that fills the actual and exit pupils.

Rays EF and CD are the limiting rays of the bundle of rays passing through the exit pupil from the tip of the object and are the rays which determine the diameter of the blur circle b. Thus, by similar triangles in Fig. 3, the diameter of the blur circle for an object point can be seen to be directly proportional to the diameter of the exit pupil p'.

v_f is the distance from the second principal plane H′ to the plane where the out-of-focus image I' of object O is observed (film plane).

v_0 is the distance from the second principal plane H′ to the plane where the in-focus image I of object O would be in focus.

x is the distance from the second principal plane to the exit pupil (a negative number in Fig. 3).

Simple ray tracing and algebra lead to the following expressions in this lens system.

$$b = \frac{-p'(\Delta D)v_0 v_f}{n'(v_0 - x)} \quad (2)$$

This yields the diameter of the blur circle of an image point ΔD diopters out of focus.

The size of the blurred image in the plane of observation is given by:

$$I' = \frac{(v_f - x)}{(v_0 - x)}(v_0 - y)\tan\theta \quad (3)$$

and thus,

$$\frac{b}{I'} = \frac{-(p')\Delta D}{\tan\theta}\frac{1}{\acute{n}}\frac{v_0 v_f}{(v_f - x)(v_0 - y)} \quad (4)$$

Yielding the relative blur in the general optical system ΔD diopters out of focus.

With straightforward ray tracing and algebra this formula can be transformed into Eq. (5) containing the more usual parameters of an optical system. Details of these derivations as well as alternative approaches (Smith, 1982), are available as adjunctive material deposited with the National Auxiliary Publications Service (NAPS, 2001).

$$\frac{b}{I'} = \underset{A}{\frac{p\Delta D}{n\tan\theta}}\ \underset{B}{\frac{1}{(1 - L/u_f)}}\ \underset{C}{\frac{1}{1 + (1 - n'/n)f/u_0}} \quad (5)$$

By thus grouping the relationship as a product of three factors, A, B and C, it is seen immediately that the relative blur in a general optical system differs from that in the ideal thin lens system by the appearance of two additional factors, here labeled B and C. Factor B may be said to be related to the non-coincidence of the entrance pupil with the first principal plane. Since L, the distance from the first principal plane H to the entrance pupil, is practically always a much smaller number than u_f, which is the distance from the first principal plane to the plane in object space for which the camera is focused, factor B $(1/(1 - L/u_f))$ can usually be equated to 1. The third factor C appears as a result of differences in the indices of refraction in the object and image spaces. In most camera systems, $n = n'$ and Factor $C = 1$. It can be seen then that the relation for the ideal thin lens system applies to almost any camera.

In the case of the human eye, it is instructive to determine the range of values of the product of factors B and C.

For the human eye (Gullstrand model) (Bennett & Francis, 1962)

$$n = 1,\ n' = 4/3,\ L = -1.61 \text{ mm},\ f = 17 \text{ mm}$$

Factor B has as its minimum value 1 when $u_f = -\infty$, that is, when the eye is focused at infinity.

If u_f is taken to be as short as -10 cm (the eye focused at 10 cm from its first principal plane), Factor B then has a value of 1.0163. Factor C has a maximum value of 1 when $u_0 = -\infty$, that is, when the object being viewed is at infinity. Its value decreases with closer objects. If we take $u_0 = -20$ cm, Factor C = 0.9724.

Multiplying Factors B and C for these rather extreme conditions where $u_0 = -20$ cm and $u_f = -\infty$ (the object is out-of-focus 5 diopters; $\Delta D = -5$), one has;

$$B \times C = 0.9724$$

The error in equating the product of Factors B and C to 1.0 under such extreme conditions is thus less than 3%. For more typical distances; i.e. $u_0 = -2$ m and $u_f = -25$ cm, the error is nearly an order of magnitude less, only 0.36%. It is concluded, therefore, that for practical purposes for an object at a given distance, it is accurate to equate the relative blur produced by the eye with that produced by a camera of arbitrary focal length when there is equal dioptric defocus and equal entrance pupil size in both systems.

For the special situation where the optical system is focused at infinity and the index of refraction is the same in the object and image space, the relation (5) for the relative blur in a general optical system simplifies to:

$$\frac{b}{I'} = \frac{-p}{O} \quad (6)$$

The minus sign indicates negative dioptric defocus.

For a camera focused at infinity, therefore, the relative blur of out-of-focus images is independent of the focal length *and* dioptric defocus. It depends, in this case, only on the entrance pupil diameter and the object height. In other words, though a decrease in the distance between the object and first principal plane would increase the negative dioptric defocus, image size also increases proportionally such that the quotient of the two remains fixed.

In a non-accommodating eye focussed at infinity (for example, an emmetrope with total presyopia), factors B and C in general relation (5) can still nearly always be approximated by 1, but now $\tan\theta = I/(v_0 - y)$. However, since $v_0 \gg y$ ($v_{0\,\min} = 14.41$ mm and $y = 0.33$ mm), one may again approximate and set $\tan\theta = I/v_0$. Using the relation $\Delta D = n/u_0$, Eq. (6) is obtained again with an error still on the order of 3%.

2.3. Distortion of blurred imagery

Distortion of optical imagery, whether in-focus or blurred imagery, can be analyzed in terms of meridional magnification (Ogle & Madigan, 1945; Ogle, 1972; Guyton, 1977). A plus cylinder, for example, added in front of a camera or an eye, will produce meridional magnification in the meridian perpendicular to the axis of the plus cylinder. The amount of meridional magnification can be found by determining the angle that the chief ray from the tip of the object in that meridian (the ray that passes through the center of the entrance pupil of the original system) makes with the optical axis (Fig. 4) (Ogle, 1968).

The amount that the added cylindrical lens will change this angle will be dependent on the dioptric power of the added cylindrical lens and its distance in front of the entrance pupil. This amount will be the same whether the optical system subsequently receiving the ray is the eye or an arbitrary imaging system. Distortion of ocular images from cylindrical lenses, therefore, can be simulated by an arbitrary camera if the same power cylindrical lens is mounted the same distance anterior to the camera's entrance pupil as the distance of the cylindrical lens from the entrance pupil of the eye. The amount of distortion is not a function of entrance pupil diameter, amount of defocus, or the focal distance of the original optical system.

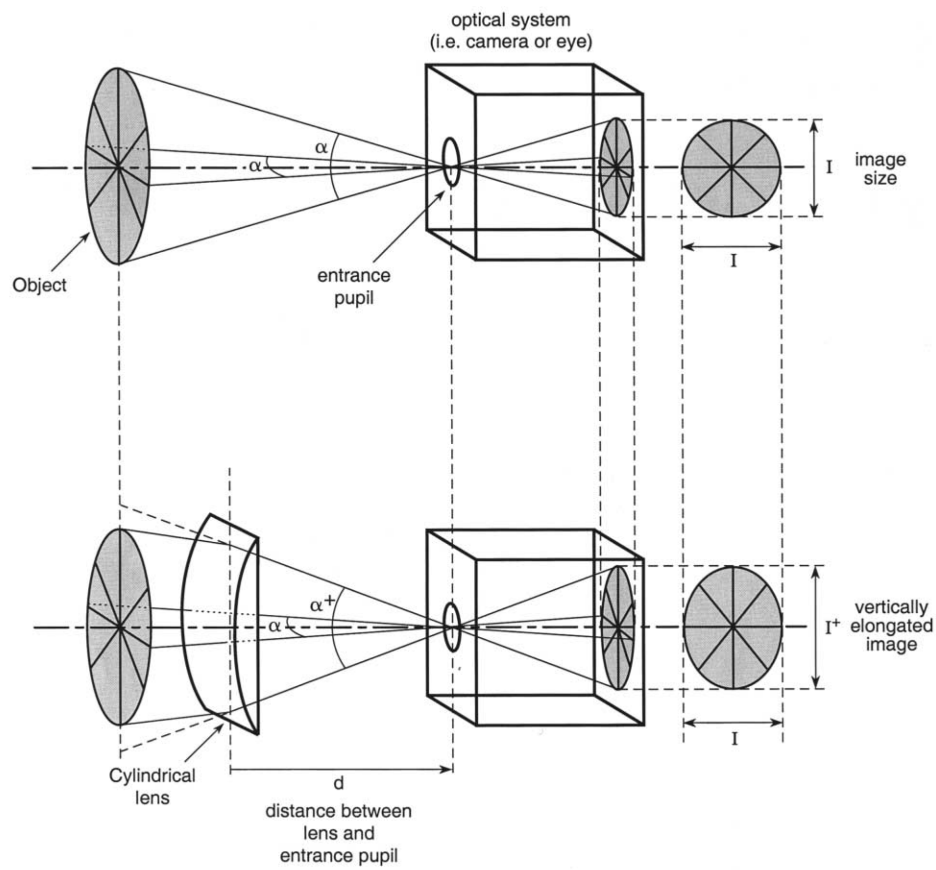

Fig. 4. Meridional magnification (distortion) produced by an added cylindrical lens is equal to α^+/α, whether the optical system is an eye or a camera, using the chief ray to the center of the entrance pupil to define these angles. The degree of magnification is dependent upon the power of the cylindrical lens and its distance *d* from the entrance pupil of the system.

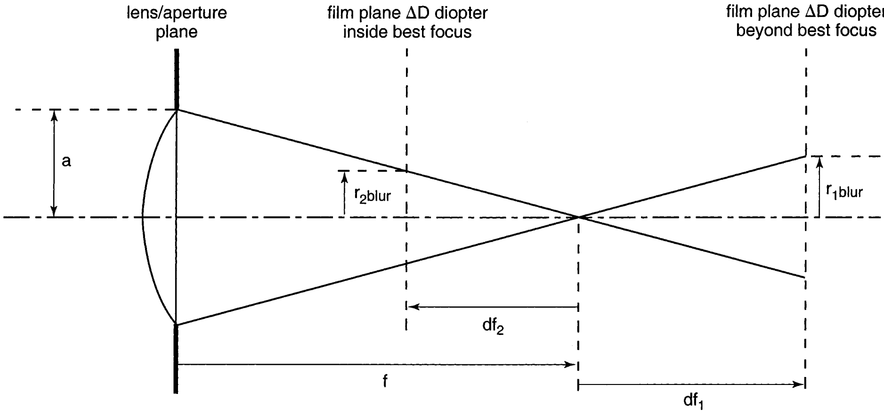

Fig. 5. The geometric blur circle radius, r_{blur}, is directly related to the distance *df* from the point of focus and to the aperture radius *a*.

3. Heuristic considerations for the applicability of geometrically-derived equations

Thus far, this discussion of relative blur has been based on a conventional geometric analysis which does not take into account the effects of diffraction.

While geometric optics predicts infinite light concentration at a point of focus, the true limit is known to be given by the Airy formula for Fraunhofer (in-focus) diffraction. This formula shows that for light converging toward a point focus and diffracted by a circular aperture of radius a, 84% of the total light intensity is concentrated at the center of the resulting image within an area called the Airy disc whose radius r_{Airy} is given by:

$$r_{\mathrm{Airy}} = 1.22\lambda f/n2a \quad (7)$$

where f is the focal distance to the plane where the best in-focus image is formed, λ is the light wavelength in object space, and n is the index of refraction in the image space (Hecht, 1987). The remaining 16% of the diffracted light is spread out over a large area and may usually be neglected.

Diffraction, therefore, prevents us from obtaining the sharp images predicted by geometric optics. Instead, it introduces a blur circle the size of the Airy disc in the plane of best focus, directly proportional in size to f, the focal distance. Since image size I is also directly proportional to the focal distance, it was noted that the quotient r_{Airy}/I, or relative blur, for in-focus diffracted imagery is independent of the focal distance of the system.

The Airy disc formula pertains to focused images, and the equations describing in-focus, or Fraunhofer, diffraction are valid only in the vicinity of the focus. If df represents the distance away from the focus, Fraunhofer diffraction analysis is valid only for $df \ll f$.

Knowing the minimum attainable blur circle size from physical optics allows one to calculate the minimum amount of dioptric defocus, ΔD_{min}, that must be present before a geometric analysis for values of relative blur becomes comparatively valid.

The blur circle radius for a defocused image in geometric optics is found using similar triangles and is expressed in terms of the aperture radius a, the focal distance f, and the distance $df_{1,2}$ between the imaging plane from the point of focus (Fig. 5).

$$r_{1,2\,blur}/a = df_{1,2}/f \quad \text{or} \quad r_{1,2\,\mathrm{lur}} = (a)df_{1,2}/f \quad (8)$$

If the blur circle radii corresponding to Fraunhofer diffraction is equated to geometric defocus at a small distance df_1 or df_2 away from the focus, one has

$$r_{1,2\,\mathrm{Airy}} = 1.22\lambda(f \pm df_{1,2})/n2a = a(df_{1,2})/f = r_{1,2\,\mathrm{blur}} \quad (9)$$

Rearranging, and using $\Delta D = n(\mp df_{1,2})/f(f \pm df_{1,2})$, this gives:

$$\pm\Delta D_{\mathrm{min}} = 1.22\lambda/2a^2 \quad (10)$$

Thus, if the geometric analysis is constrained to exclude regions where the defocus is less than $1.22\lambda/2a^2$ diopters, one may expect it to provide a good approximation to the actual, physical optics image (Fig. 6). Calculating the minimum defocus necessary to be in the realm of good geometrical analysis for $\lambda = 550$ nm:

$\pm\Delta D_{\mathrm{min}} = 1.22\lambda/2a^2$

$\pm\Delta D_{\mathrm{min}} = 1.342$ Diopters for a 0.5 mm radius pupil.

$\pm\Delta D_{\mathrm{min}} = 0.335$ Diopter for a 1 mm radius pupil.

Noting that these small boundary limits of the Fraunhofer diffraction image ($\pm\Delta D_{\mathrm{min}}$ just calculated) are independent of the focal length f of the system, one may conclude from a physical optics analysis that relative blur in the Fraunhofer region produced by systems of different focal distances are equal, and that the defocus limits of this region are independent of the focal length f. Outside this region, geometrical optics provides a good approximation to the actual image and it has already been shown geometrically that the relative blur of this image is independent of focal distance.

In certain extreme situations of small apertures and long focal distances, however, the diffracted light distribution is asymmetric about the plane of focus. This phenomenon has been described as the 'focal shift' and has no counterpart in geometric theory (Li & Wolf, 1981, 1984). It is never encountered in the human eye and rarely in most camera systems. However, if blurred imagery is produced by defocusing an image projected on a screen by a projector with a focal length on the order of meters with a small aperture, a significant focal shift may occur. In these instances, focal shift must be included and considered as a source of error, much as is currently the case for errors induced by spherical or chromatic aberrations. Further considerations on the applicability of geometric analysis are discussed in adjunctive material deposited with the NAPS (NAPS, 2001).

4. Materials and methods

Sets of photographs of a retroilluminated 20/200-equivalent line from a Snellen acuity chart and a pinhole light source were taken with a Nikon F-3 camera using two different lenses. The first set corresponds to out-of-focus and distorted photographs taken with a 50 mm Nikkor f1.4 lens. The second set was taken using a 105 mm Micro-Nikkor f2.8 lens. To determine the location of the first principal plane within the camera lens assembly, a traveling microscope, a camera lens, and a target object were serially mounted along an optical bench. Peering through the microscope and through the lens (both focused at infinity) at the target, the lens was displaced until the target came sharply into focus. A distance equal to the stated lens focal length was then measured back from the target object to determine the location of the principal plane for that lens. The camera was then placed with the lens in focus at the line from the Snellen acuity chart 40 cm anterior to the first principal plane such that the height of the 20/200 line letters would subtend 50 min of arc at the first nodal point of the camera lens. This assured that the relative blur created in the camera would be equal to that in the eye for same-size letters held 40 cm anterior to the viewing eye.

Entrance pupils were formed from brass plates and secured with a standard filter mount screwed onto the lens frame. This configuration permitted exact knowledge of entrance pupil size and location. The amount of dioptric defocus was determined by placing refracting lenses of the appropriate power immediately adjacent to the entrance pupil. This assured that the angular size of the image, as determined by the chief ray passing through the center of the entrance pupil, remained constant for different degrees of dioptric defocus. This configuration did have the disadvantage of displacing the added change in refracting power from the first principal plane; but its effect was calculated to be small. An alternative, though cumbersome, set-up to provide exact degrees of defocus consists of setting the lens

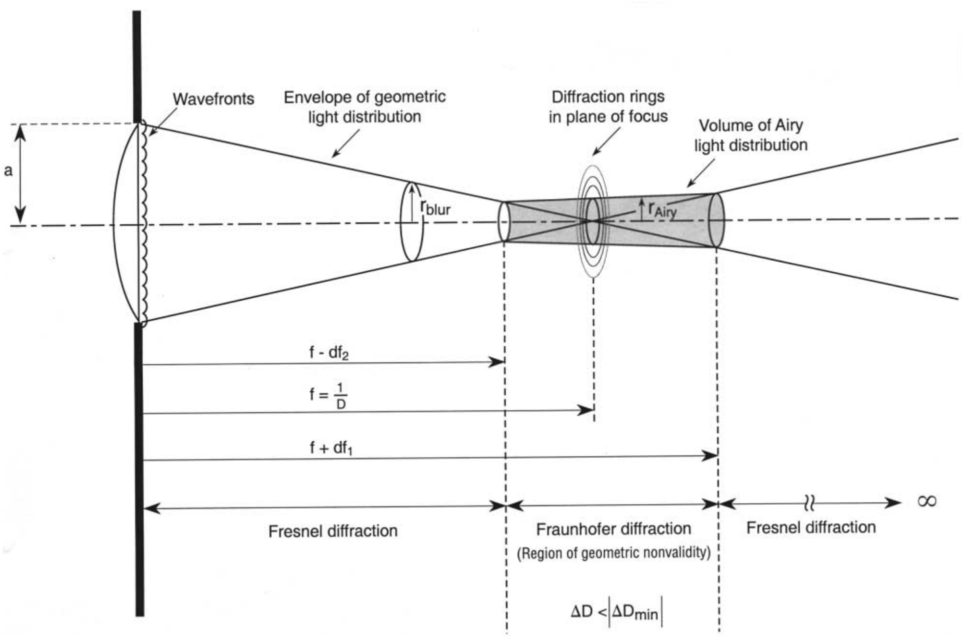

Fig. 6. Comparison of blur circles from geometric theory with the physically diffracted light pattern, where df is much smaller than f, the focal length of the system. In physical reality, the blur circle radius cannot be less than r_{Airy}, the radius of the Airy disc in Fraunhofer, or in-focus, diffraction. When the blur circle radius calculated via geometric optics, r_{blur}, is equal to or greater than r_{Airy}, we are then in the region of Fresnel, or out-of-focus diffraction, and geometric optics can be used as an approximation to the physical reality. About 16% of the diffracted light energy falls outside the Airy disc within dim surrounding rings. These are spread over a large area, and can be neglected.

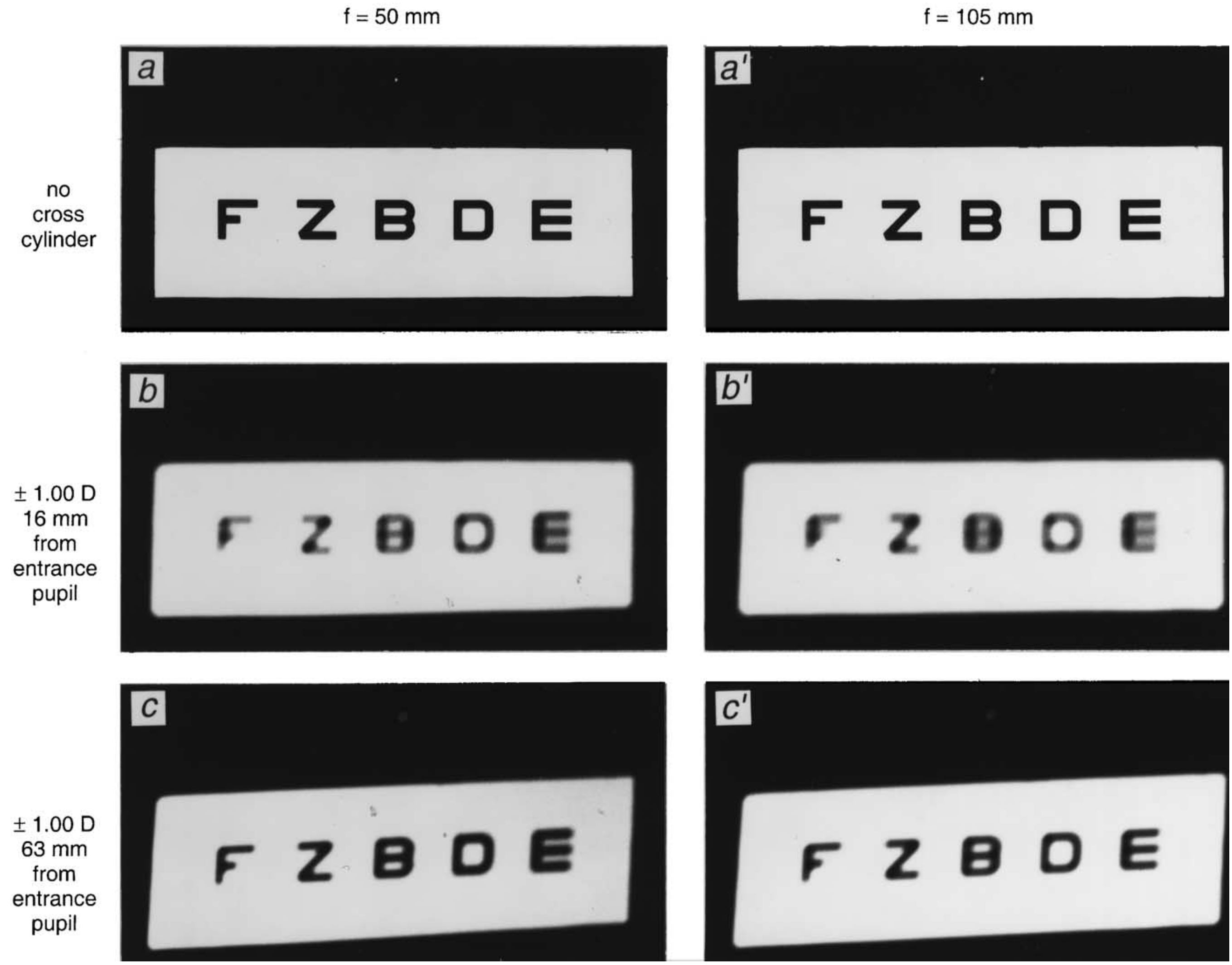

Fig. 7. Photographs taken with lenses of two different focal lengths, scaled for equal magnification to demonstrate the direct proportionality of relative blur to dioptric defocus and to aperture size, and the independence of relative blur to focal distance. Photographs are of the 20/200 line of a Snellen visual acuity chart and should be viewed at about 35 cm (14 in.) distance for the letters to subtend 50 min of arc at the viewer's eye. The small pinhole image just above the line is used to compare blur circle sizes. Photographs *a–d* in the first column were taken using a 50 mm focal length camera lens, while *a′–d′* in the second column were taken using a 105 mm focal length lens. No difference in the degree of blur or pinhole imagery can be distinguished between corresponding photos in the two columns, demonstrating that relative blur is independent of the focal length. Photographs *b* and *b′* in row two, along with photographs *c* and *c′* in row three, demonstrate that doubling of the defocus doubles the blur. Comparison of photos *b* and *b′* in the second row with photographs *d* and *d′* in the fourth row, demonstrates that alternatively, doubling of the aperture size doubles the blur to the same degree.

focus at infinity and taking the reciprocal of the object distance to the first principal plane as the dioptric defocus. Different target object sizes would then be necessary to preserve a constant angular size as measured at the entrance pupil while changing the degree of dioptric defocus when changing object placement with respect to the first principal plane.

To obtain best possible tonal reproduction and resolution, Kodak Technical Pan Film was used. The negatives were developed with Technidol developer for 9 min at 68° Fahrenheit (20°C) with agitation for 5 s every 30 s. Prints were then made on grade 2 paper, using a diffuser enlarger. This assured the highest quality of resolution and tonal reproduction (Stroebel, Compton, Current, & Zakia, 1986).

5. Results

To properly view these photographs, the viewer should place the letters at about 35 cm (14 in.) from the eyes such that the letters subtend 50 min of arc at the eye. Fig. 7 consists of an arrangement of eight photographs of the 20/200 line of a Snellen visual acuity chart with a small pinhole aperture placed just above the line to allow for comparison of blur circle sizes. Photographs in the first column were taken using the 50 mm lens whereas those in the second column were taken using the 105 mm lens. The photographic print sizes were scaled to offset the effects of lens focal length on magnification and allow for direct comparison of blur between the two sets. Photographs are arranged by row according to aperture (entrance pupil) size and power of the defocussing trial lens placed immediately before the entrance pupil. Photographs in the first row were taken with the camera sharply in focus, while for those in the second row, a + 0.50 diopter trial lens was placed at the 4 mm diameter aperture for both the 50 and 105 mm camera lens assemblies. A comparison of photographs in the second row reveals letters of the 20/200 line of the Snellen acuity chart to be equally blurred in both columns and that the size of the blur circle, evidenced by light exiting through the pinhole, is identical in both photographs. This confirms the theoretical finding that relative blur is independent of focal length.

In the third row, the power of the defocusing trial lens placed before the entrance pupil was doubled, to + 1.00 diopter. Again, the degree of relative blur, as measured by pinhole blur circle sizes and blurring of the letters in the two photographs, is identical. Moreover, a comparison of blur circle sizes with those obtained in the second row confirms the direct proportionality of blur circle size to dioptric defocus. Photographs in the fourth row, where the dioptric defocus is kept at + 0.50 diopter, but the aperture size is doubled, confirm the direct proportionality of relative blur to aperture (entrance pupil) size. As demonstrated in the previous rows of the two sets of photographs, these relations remain true whatever the focal length of the assembly. As was determined crucial for the validity of our findings, all photographs were taken under conditions well within the domain of geometric applicability for our equations. The lowest degree of dioptric defocus tested, 0.50 diopter, was well above the $\pm \Delta D_{\min}$ for a 4 mm aperture (± 0.08 diopter). The photographs in Fig. 7 provide direct experimental evidence for asserting the independence of focal length in determining relative blur, and the direct proportionality of both dioptric defocus and entrance pupil size to relative blur as revealed by Eq. (5) for a system where the indices of refraction are the same in both the object and image space, the distance L between the entrance pupil and first refractive plane is small compared to the distance u_f of the object in-focus.

To demonstrate the relevant variables producing distortion, we took a series of photographs shown in Fig. 8. This consists of an arrangement of six photographs of the 20/200 line of the Snellen acuity chart along with the pinhole. As before, photographs in the first column were taken with the 50 mm lens, and those in the second column with the 105 mm lens. Again, photographs in the first row were taken with the camera sharply in-focus and the photographic print sizes in both columns were scaled to offset image magnification. In the second row, however, a ± 1.00 diopter cross cylinder was placed 16 mm in front of the entrance aperture. Sixteen millimeters is the approximate distance between the cross cylinder placed in a trial lens frame and the entrance pupil of the eye. Distortion is now distinctly noted, but no difference is perceived between the two columns. Meridional magnification, thus, is independent of focal length. In the third row, the distance of the cross cylinder to the entrance pupil is increased from 16 to 63 mm, the approximate distance between the cross cylinder in a phoropter to the entrance pupil of the eye viewing through the phoropter. Distortion, or meridional magnification, is increased, but to the same extent in both focal length systems.

6. Discussion

By pursuing a geometric analysis of out-of-focus imagery and taking consideration of physical optics to determine the conditions for its validity, the parameters have been determined which are of importance in reproducing out-of-focus imagery in different optical systems. Defining relative blur as the ratio of blur circle size to image height, we have derived a general equation to describe relative blur in any optical system (Eq. (5)). This equation shows that relative blur of out-of-focus images is directly proportional to the entrance pupil size and to the dioptric amount of defocus and is practically independent of the focal length. A review of the formula for relative blur clarifies several known observations. For a given dioptric amount of defocus and fixed pupil size, the defocused image of an object at a distance has a much greater relative blur than the defocused image of the same size object at near, where it subtends a larger angle. However, the blur circle diameters in both images are equal. Thus, the distant object is not really 'more blurred', but simply appears so because its image is smaller.

Using a schematic approach, we also found distortion produced by cylindrical lenses to be independent of the focal length of the system, and related only to the power of the cylindrical lens and its distance from the entrance pupil of the system.

A heuristic approach to considerations of physical optics demonstrated the geometrically-derived conclusions to be valid in most situations, that is whenever we exceed a certain very minimal amount of dioptric defocus ($\pm \Delta D_{\min} = 1.22\lambda/2a^2$). Further conditions on the applicability of geometric analysis, rarely encountered in most camera systems, are discussed in the NAPS adjunctive material (NAPS, 2001).

The end result, that relative blur and distortion are independent of focal length, allows us to infer that out-of-focus and distorted imagery can easily and reliably be reproduced in various optical assemblies. Scaling down of instruments to physiological size or immersion of miniature cameras into fluid to emulate the aqueous and vitreous humors, has previously been painstakingly accomplished by some investigators to simulate ocular imagery, partly out of a belief that only such measures would provide a fail-safe way of obtaining accurate results (Holladay et al., 1990; Keates, Pearce, & Schneider, 1987). For many purposes, this is no longer necessary. Blurred and distorted ocular imagery simulations, for instance, can be used for objective comparison of bifocal intraocular lenses prior to the implementation of subjective and costly human trials. Also, illustrations of what uncorrected ametropes see can be made with greater confidence.

Accurate photographic simulation and verification of ophthalmic refractive techniques also become possible.

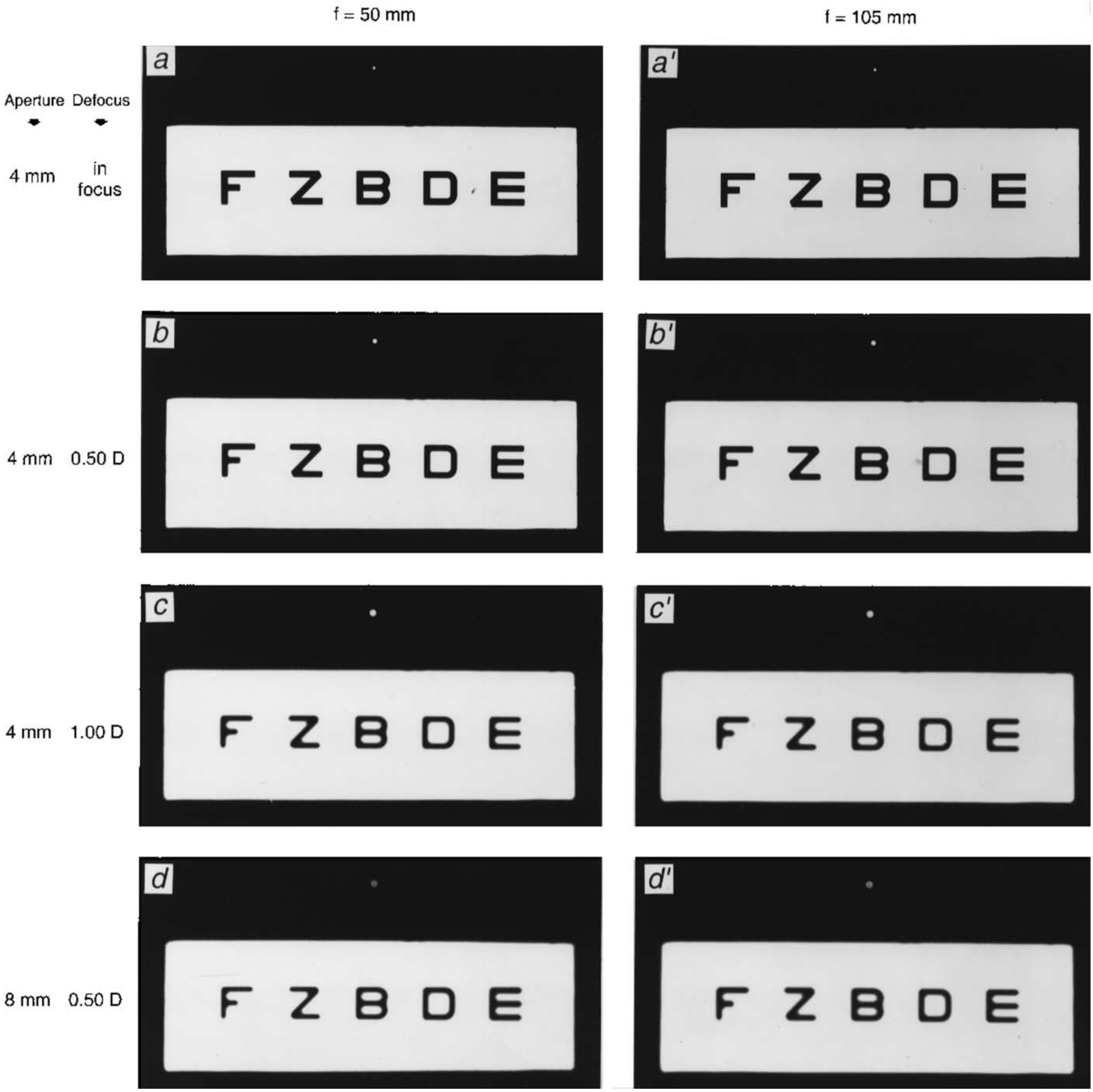

Fig. 8. Photographs taken with lenses of two different focal lengths, scaled for equal magnification to demonstrate the effect of cylinder lens power and distance from entrance pupil on distortion, and the independence of distortion to focal distance. Sixteen millimeters is the approximate distance of the cross-cylinder in a trial lens frame to the entrance pupil of the eye. Sixty-three millimeters is the approximate distance of the cross-cylinder in a phoropter to the entrance pupil of the eye peering through a phoropter. Photographs *a–c* in the first column were taken with a 50 mm lens, while photos *a′–c′* were taken with a 105 mm lens. No difference is noted between the two columns, demonstrating that distortion is independent of the focal length. The distortion produced by a $\pm$ one diopter cross cylinder placed 16 mm in front of the entrance pupil seen in photos *b* and *b′*, increases, however, when the same lens is placed 63 mm from the entrance pupil seen in photos *c* and *c′*.

The greatly exaggerated blur obtained by previous investigators misled some to believe, for example, that the end points of the Jackson cross cylinder technique for astigmatic correction are not equally blurred, as is predicted by theory, but that they are otherwise different (Sims & Durham, 1986; Sims, 1988). These experimental findings were likely due to small degrees of residual, uncorrected astigmatism present in the system but not perceived by the eye. Such remaining blur, even if unnoticeable to the eye, could easily be magnified sufficiently by a camera system to produce visibly unequal end points with the cross cylinder. Similarly, due to the unappreciated magnification of blur by their photographic apparatus, some investigators have mistakenly concluded that the axis test for astigmatic correction should be performed at much smaller intervals than is advocated in common practice (Sims & Durham, 1986).

Despite the derivations, and the supporting photographic evidence collected, it has been found that the image appreciated by the human eye is still somewhat better than the blurred image which has been described so far which falls upon the inner layer of the retina. This can be appreciated by the reader using the series of photographs in Fig. 7 and an appropriate trial lens. A + 3.75 diopter lens is used to provide approximately + 1.00 diopter of defocus to the eye, with the additional + 2.75 diopters needed to compensate for the 35 cm viewing distance when full distance refractive correction is worn. One may then compare the + 1.00 D out-of-focus photographs (middle row) with blur appreciated by the eye looking at the in-focus photographs (top row) through the hand-held lens (assuming, of course, an equal 4 mm diameter pupil). The blurred ocular image appears less blurred than the out-of-focus photograph despite identical dioptric defocus and pupil size. Probably retinal architecture effects contribute to this, such as the Stiles–Crawford effect, which is a consequence of the light-directional properties of the photoreceptors. Also, the added depth of field from spherical aberration and chromatic aberration in the eye in conjunction with neural processing is probably important. The next step will be to investigate and simulate optically the Stiles–Crawford effect to simulate ocular imagery at the photoreceptor level.

Acknowledgements

The authors thank David L. Kays and David G. Emmert for their technical advice and assistance in producing the photographs, Katherine Weaver and Nancy Held for drawing the diagrams and Babak Rezaian, Ph.D. for helping to provide insight into quantitative Fresnel analysis. We also thank Glen A. Fry, Ph.D., Yajun Li, Ph.D., and Emil Wolf, Ph.D. for their thoughtful comments.

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