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Calculation of watershed flow concentration based on the *grid drop* concept

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Abstract: The *grid drop* concept is introduced and used to develop a micromechanism-based methodology for calculating watershed flow concentration. The flow path and distance traveled by a grid drop to the outlet of the watershed are obtained using a digital elevation model (DEM). Regarding the slope as an uneven carpet through which the grid drop passes, a formula for overland flow velocity differing from Manning's formula for stream flow as well as Darcy's formula for pore flow is proposed. Compared with the commonly used unit hydrograph and isochronal methods, this new methodology has outstanding advantages in that it considers the influences of the slope velocity field and the heterogeneity of spatial distribution of rainfall on the flow concentration process, and includes only one parameter that needs to be calibrated. This method can also be effectively applied to the prediction of hydrologic processes in un-gauged basins.

Key words: *micromechanisms of watershed flow concentration; grid drop; overland flow velocity formula; spatial velocity field; watershed runoff concentration time; digital elevation model*

1 Introduction

Watershed flow concentration theory has been researched and applied for a century and a half, ever since Mulvaney developed the rational formula for routing flood peak discharge in 1851. However, the most popular watershed flow concentration methodologies all over the world are still the unit hydrograph and the isochronal methods developed by Sherman and Velikanov, respectively, in 1932 (YRBPPO 1979). The assumptions of the unit hydrograph method include uniform spatial distribution of rainfall and a linear watershed flow concentration process. The isochronal method originated from the area-time curve concept proposed by Ross in 1921, which becomes the isochronal area distribution curve under the assumption of uniform spatial distribution of water flow velocity (Rui 2004a). The area-time curve cannot be used directly to calculate watershed flow concentration because of the difficulty of apprehending the spatial distribution of water flow velocity, but the isochronal method can only be applied in the case that water flow velocity's spatial distribution is uniform and the course of water flow concentration is linear. Although the assumption of the linearity of watershed flow concentration has not yet been theoretically proven, the practical experience of many hydrologists and hydrological engineers has demonstrated that, in most

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cases, especially in a deluge, it is still acceptable (Pilgrim and Huff 1978). On the other hand, the assumption that the spatial distribution of rainfall is uniform can only be applied to small basins, and the assumption that the spatial distribution of velocity is uniform can only be applied to basins in which the slope and the roughness of the underlying surface are homogenous. For a much smaller basin, the Sherman unit hydrograph cannot be obtained directly because of the lack of observed discharge data. The Snyder synthetic unit hydrograph fails to achieve satisfying precision owing to the weak theoretical foundation of its transplant and extension. In addition, the slope and roughness of a natural basin's underlying surface are always heterogeneous. Thus, hydrologists need a reasonable watershed flow concentration calculation method that can be used in all kinds of conditions. As a step towards that goal, this paper presents a methodology based on the *grid drop* concept, the micromechanisms of watershed flow concentration, and utilization of a digital elevation model (DEM).

2 Formation of discharge at watershed outlet

Rainfall events can be regarded as the combination of many drops that fall at random in a watershed. After some are lost to groundwater and evaporation, the remaining drops will each follow a flow path to the watershed outlet. The time spent by a water drop along this path, until it reaches the outlet, is called the concentration time.

As shown in Figure 1, rainfall consists of n drops. The serial numbers of the drops are denoted as $1, 2, ..., i, ..., n$. If drop i has a flow path of length r_i and a kinematical velocity $v_i(x)$ correlated with the path, its concentration time is calculated as follows:

$$\tau_i = \int_{r_i} \frac{\mathrm{d}x}{v_i(x)} \qquad (i = 1, 2, ..., n) \tag{1}$$

where τ_i is the concentration time of drop i, and x is its flow path.

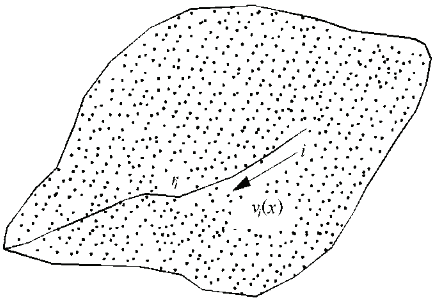


Figure 1 Spatial distribution of grid drops on a watershed

The spatial distribution of concentration times of all drops can be obtained after Eq. (1) is applied to each one. Supposing rainfall starts at time $t = 0$, only those drops whose concentration times are just t can reach the watershed outlet at time t and constitute the outlet discharge, so

$$q(t)=\sum_{i=1}^{n} I(\tau_i=t)h_i\Delta A_i \tag{2}$$

where $q(t)$ is the watershed outlet discharge at time t; $I(\tau_i=t)$ is the indicator function with a value of 1 when $\tau_i=t$, indicating drop i is one part of the outlet discharge at time t, or 0 when $\tau_i\neq t$, indicating drop i is not a part of the outlet discharge at time t; ΔA_i is the area occupied by drop i; and h_i is rainfall intensity on ΔA_i at time $t=0$. During rainfall, Eq. (2) is used repeatedly for different times, and the outlet discharge graph can be deduced using the superposition principle. Assuming that rainfall is distributed uniformly across the watershed and rainfall intensity is h, Eq. (2) can be transformed into

$$q(t)=h\sum_{i=1}^{n} I(\tau_i=t)\Delta A_i \tag{3}$$

According to the definition of the instantaneous unit hydrograph, the watershed instantaneous unit hydrograph of 1 mm of net rainfall is calculated as

$$u(t)=\frac{q(t)}{h}=\sum_{i=1}^{n} I(\tau_i=t)\Delta A_i \tag{4}$$

where $u(t)$ is the instantaneous unit hydrograph and the meanings of the other symbols are the same as before.

Although Eq. (4) reveals the unit hygrograph’s micromechanism, it is just a special case of Eq. (3), when rainfall is distributed uniformly in space. Compared with Eq. (4), Eq. (2) is the watershed flow concentration calculation method that can be more broadly used.

The key to deriving the watershed outlet discharge graph formed by rainfall is calculating the scale, path and velocity of the drops.

3 Scale and flow path of a drop

The digital elevation model (DEM), which first appeared in the 1950s, supplies the technical foundation for a watershed flow concentration calculation method based on concentration micromechanisms (Rui and Shi 2004). The DEM is a discrete digital description of surface elevation varying with spatial position, and it is an important tool in plotting topographical grids and quantitatively analyzing the watershed’s topographical and geomorphic features. Using the DEM, it becomes easier not only to divide a watershed into grid cells but also to automatically extract its elevation, slope, flow direction, water system, shape, area, Horton’s geomorphological parameters, width function, topographical index, and other properties.

The runoff formed by rainfall on a grid cell can be considered a drop whose size is determined by the cell’s size. Such a drop may be called the *grid drop*, and its flow path can be obtained using the DEM. In a gridded watershed, there are 8 other cells around each grid cell, so the runoff in each cell may flow to any of the adjacent 8, as shown in Figure 2. Fairfield and Leymarie (1991) assumed that the actual water flows along the path with the maximum slope. Therefore, of the 8 calculated gradients, the one with the maximum slope is the direction in which the drop flows. This is called the D8 method.

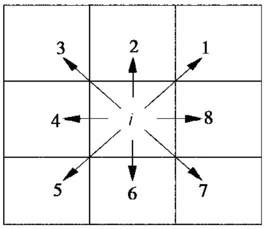


Figure 2 Flow direction of a grid drop

The slopes from each cell to its 8 adjacent cells can be calculated as follows:

$$S_{ij} = \begin{cases} \dfrac{Z_i - Z_j}{d} & \left(j = 2, 4, 6, 8 \right) \\ \dfrac{Z_i - Z_j}{\sqrt{2}d} & \left(j = 1, 3, 5, 7 \right) \end{cases} \tag{5}$$

where S_{ij} is the slope from grid cell i to its adjacent cell j, Z_i is the central elevation of cell i, Z_j is the central elevation of cell j $\left(j = 1, 2, \ldots, 8 \right)$, and d is the side length of a cell.

The flow path of each grid drop and its geometric length are acquired after flow direction is determined.

The D8 method is one of the single flow direction methods. The so-called multi-flow direction method is also available. Experience and practice show that the D8 method generates satisfactory results with adequate precision.

4 Flow velocity

The cells into which the watershed is divided may be located within the channel or on the slope. For this reason, the determination of the velocity at which grid drops flow depends on the choice of the velocity formula.

4.1 Formula of channel flow velocity

Flow in a channel is mainly dominated by the hydrodynamic interaction in the flow. Manning’s formula is widely used to calculate the flow velocity in channels:

$$v_{\mathrm{r}} = \frac{1}{n} R^{2/3} S_{\mathrm{r}}^{1/2} \tag{6}$$

where v_{r} is the mean flow velocity of the discharge section, R is the hydraulic radius, S_{r} is the water surface slope, and n is the channel roughness. R is associated with the discharge from upstream of the cross section. Using the water catchment area of a grid to represent the amount of water it is collecting, the flow velocity formula can be written as

$$v_{\mathrm{r}} = \alpha A^{\beta} S_{\mathrm{r}}^{1/2} \tag{7}$$

where A is the catchment area of a grid, α is the parameter reflecting channel roughness, β is the empirical index, and the meanings of the other symbols are defined as before.

Since A in Eq. (7) can be extracted easily from the DEM, it is very convenient to use Eq. (7) to calculate the flow velocity in a channel.

4.2 Formula for overland flow velocity

Owing to the complexity of micro-relief and ground cover, the grade surface is just like an uneven carpet. If the water depth on the grade surface is less than the carpet's thickness, water will pass through the carpet almost like macroporous flow (Rui et al. 2007). The overland flow is neither like the stream flow, which is dominated by hydrodynamic interaction in the flow, nor the soil pore flow, which is mainly influenced by its solid boundary; it is instead an intermediate flow state. Consequently, the formula for overland flow velocity can be conceived by combining Manning's formula, in which flow velocity is directly proportional to $R^{2/3} S_{\mathrm{r}}^{1/2}$, and Darcy's formula, in which flow velocity is directly proportional to the hydraulic gradient, and described as

$$v_{\mathrm{s}} = a S_{\mathrm{s}}^{1/2} \tag{8}$$

where v_{s} is the overland flow velocity, S_{s} is the overland slope, and a is an empirical coefficient reflecting overland roughness.

In this paper, Eq. (8) is only applied to the calculation of the overland flow velocity.

5 Case study

Yandu River Basin is located on the north bank of the Three Gorges, and has an area of 601 km^2. The DEM was generated from a 1:50 000 topographic map and the basin was divided into 60 100 grid cells (100 m × 100 m). The flow direction and gradient distributions automatically abstracted from the DEM are displayed in Figure 3 and Figure 4, respectively.

The topography of Yandu River Basin undulates visibly and the river is narrow. The majority of each grid cell (100 m × 100 m) is land surface. This means that each grid drop's velocity can be calculated by Eq. (8), and its concentration time can be obtained subsequently. The result of the flow concentration calculation for Yandu River Basin shows adequate precision when a is calibrated to a value of 2.25 m/s. The spatial distribution of grid drop concentration time in Yandu River Basin is shown in Figure 5 (a).

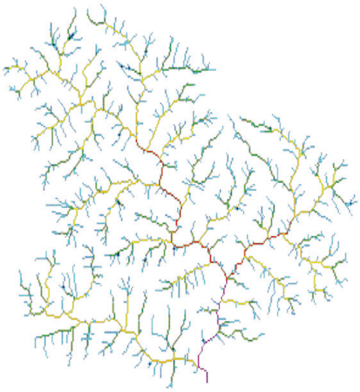
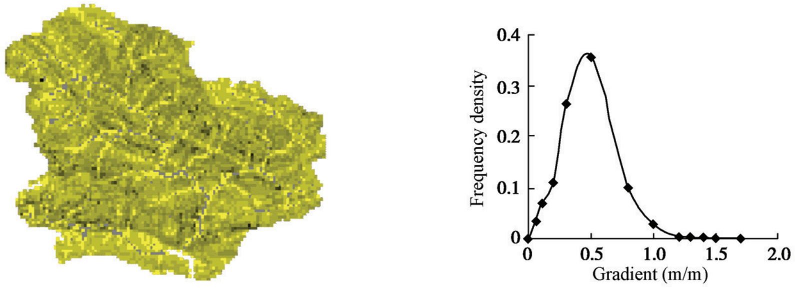


Figure 3 Flow direction of Yandu River Basin



(a) Gradient distribution (b) Gradient frequency distrbution

Figure 4 Gradient distribution of Yandu River Basin

If watershed flow concentration is calculated with Eq. (3), the concentration time's probability density, that is, the instantaneous unit hydrograph of the grid drops (Figure 5(b)), must first be extracted according to Figure 5(a) (Rui 2004b). The time-interval Δt is designated as 1 hour, and the watershed instantaneous unit hydrograph therefore needs to be converted into a 1-hour unit hydrograph $u(1,t)$ (Table 1). Accordingly, the runoff concentration for Yandu River Basin can be calculated, and the results are listed in Table 2.

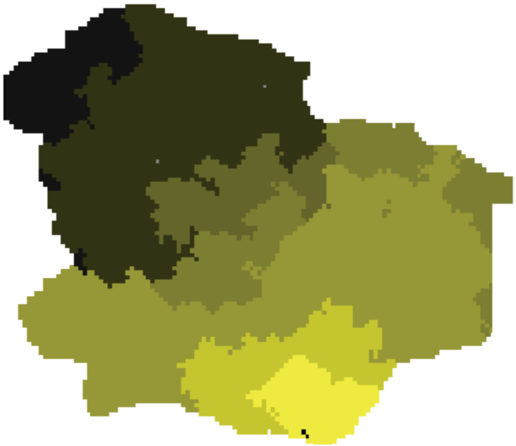
Table 1 1-hour geomorphologic unit hydrograph of Yandu River Basin (a = 2.25 m/s)

t (h)	0	1	2	3	4	5	6	7	8	9
$u(1, t)$	0	0.058	0.163	0.182	0.158	0.122	0.100	0.077	0.050	0.032
t (h)	10	11	12	13	14	15	16	17	18	19
$u(1, t)$	0.016	0.011	0.011	0.005	0.005	0.004	0.003	0.002	0.001	0

If Eq. (2) is applied to watershed flow concentration calculation, the influence of rainfall's spatial distribution on flow concentration can be taken into account. Results such as those listed in Table 2 can be obtained based on Figure 5 (a).

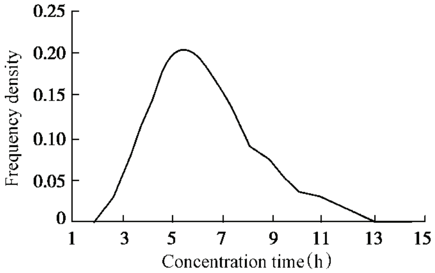
Table 2 Comparison of flood characteristics generated by Eq. (2) and Eq. (3)

Flood No.	Observation			Results of Eq. (3)			Results of Eq. (2)		
	Rainfall (mm)	Flood volume (mm)	Peak discharge (m^3/s)	Flood peak error (%)	Time difference of peak appearance (h)	Deter-ministic coefficient	Flood peak error (%)	Time difference of peak appearance (h)	Deter-ministic coefficient
810623	168.2	128.7	1130	−3.56	0	0.97	−0.50	0	0.97
810810	157.7	127.3	628	6.10	1	0.80	−1.50	0	0.92
810824	95.2	82.9	509	3.67	−1	0.66	1.40	−1	0.93
820820	202.8	171.3	573	3.15	0	0.89	4.60	−1	0.89
830623	219.0	213.0	1520	−25.56	1	0.87	−11.30	0	0.87
830906	268.7	262.3	897	−5.10	0	0.85	1.60	−1	0.92
831004	181.4	176.7	575	−0.30	−25	0.66	0.03	0	0.94
831017	126.2	114.7	378	−19.52	−12	0.65	−6.90	−1	0.93
840612	144.0	134.8	632	−23.76	−1	0.74	0	−1	0.87
840723	176.1	144.4	1060	−17.56	0	0.89	0.20	0	0.97
840909	115.0	73.9	356	−19.05	0	0.69	−0.50	−1	0.95
850621	123.5	100.7	476	−3.39	−1	0.86	0.90	−1	0.92
860615	83.4	66.4	482	−25.58	1	0.73	−1.40	1	0.93
860909	194.2	181.9	844	−11.62	0	0.94	−0.37	−1	0.96
870719	115.7	98.1	819	24.29	2	0.82	−0.09	2	0.91
Mean value						0.80			0.93
Mean absolute value				12.81			2.14		



0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	14.18
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	13.43	13.42	0.00	13.38	13.37	13.37	14.15
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	13.84	13.42	13.39	13.42	13.36	13.34	13.36	13.41
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	13.80	13.82	13.76	13.42	13.33	13.30	13.34	13.38	13.16
0.00	0.00	0.00	0.00	0.00	13.78	13.75	13.76	13.79	13.72	13.75	13.81	13.36	13.36	13.17	13.13	13.18
0.00	0.00	0.00	0.00	13.87	13.86	13.73	13.76	13.67	13.70	13.77	13.81	13.82	13.20	13.20	13.20	13.05
0.00	0.00	0.00	13.82	13.82	13.84	13.86	13.68	13.64	13.70	13.75	13.78	13.81	13.86	13.24	13.23	13.23
0.00	0.00	13.81	13.79	13.80	13.82	13.64	13.61	13.53	13.70	13.74	13.40	13.82	13.85	13.24	13.24	12.95
0.00	0.00	13.78	13.76	13.73	13.70	13.73	13.67	13.64	13.43	13.36	13.41	13.43	13.84	13.22	13.24	13.19
0.00	13.80	13.79	13.78	13.76	13.76	13.73	13.67	13.47	13.66	13.47	13.29	13.45	13.26	13.21	13.17	12.98
0.00	13.83	13.81	13.81	13.80	13.79	13.80	13.70	13.70	13.51	13.50	13.51	13.22	13.17	13.12	13.08	13.15
0.00	0.00	13.76	13.74	13.72	13.75	13.70	13.74	13.77	13.53	13.54	13.29	13.33	13.27	13.21	13.15	13.11
0.00	0.00	13.76	13.74	13.70	13.68	13.71	13.74	13.77	13.50	13.36	13.56	13.32	13.31	13.34	13.38	13.40
0.00	0.00	13.77	13.73	13.71	13.66	13.62	13.59	13.46	13.42	13.53	13.39	13.48	13.42	13.46	13.42	13.45
0.00	0.00	0.00	13.76	13.70	13.74	13.68	13.64	13.62	13.49	13.44	13.48	13.46	13.48	13.46	13.50	13.45
0.00	0.00	0.00	0.00	13.74	13.74	13.71	13.71	13.56	13.59	13.52	13.48	13.49	13.48	13.49	13.48	13.54
0.00	0.00	0.00	0.00	0.00	0.00	13.75	13.73	13.58	13.58	13.62	13.52	13.52	13.53	13.36	13.37	13.37
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	13.61	13.62	13.63	13.45	13.43	13.45	13.34	13.31	13.24
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	13.45	13.39	13.41	13.36	13.34

(a) Distribution of watershed flow concentration time and partial display



(b) Instantaneous unit hydrograph

Figure 5 Flow concentration curves of Yandu River Basin

Table 2 shows that the results calculated with Eq. (2) are more precise than those calculated with Eq. (3). In other words, the method that is shown in Figure 5(a) and considers rainfall's varied spatial distribution has much higher precision than the unit hydrograph method, which is based on uniform spatial distribution of rainfall. This demonstrates that the influence of spatial heterogeneity of rainfall on watershed flow concentration cannot be ignored.

6 Conclusions

In this paper, a watershed flow concentration calculation method based on the grid drop concept and a DEM is proposed. This method not only considers the influence of the basin's shape and river systems, but also the effect of the spatial velocity gradient distribution on watershed flow concentration. It not only shares the merits of the area-time curve method, but also overcomes the weakness of the unit hydrograph method, the assumption that rainfall distribution is spatially uniform. Thus, it is a general watershed flow concentration method with a solid theoretical foundation, encompassing two popular methods – the unit hydrograph and isochronal methods – as special cases.

On the basis of a preliminary analysis of grade surface structure and flow features, Eq. (8) is recommended as a basic formula for calculating overland flow velocity. The watershed flow concentration calculation method proposed in this paper thus includes only one parameter, a, that needs to be calibrated and has an enhanced physical conceptualization. Hydrologists generally claim that the flood routing process based on the Saint Venant equations has a rigorous hydromechanics foundation (Rui 1995), but, in fact, it also includes a parameter that needs to be calibrated: roughness, n. In the authors' opinion, the watershed flow concentration calculation method proposed in this paper has similar theoretical characteristics to the flood routing process based on the Saint Venant equations.

How can watershed flow concentration be calculated without hydrological data? It seems to be a permanent research question for every hydrologist. The authors suggest that the key to solving this problem is to understand the theoretical relationship between watershed flow concentration and topographical/geomorphologic features. The method proposed in this paper, which is different from Snyder's empirical equation (Snyder 1938) and the geomorphologic unit hydrograph proposed by Rodríguez-Iturbe and Valdés (1979) and Gupta et al. (1980), is an important step towards this goal. This method can be used as long as the parameter a can be calibrated and a DEM is available, so it has promising prospects for application in un-gauged basins.

Unfortunately, this method does not consider the interaction of grid drops. At present, there is no reasonable theory to describe that interaction, but a solution to the problem will be pursued in the near future.

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