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# Philosophical reflection on mathematics in Poland in the interwar period☆

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**Abstract**

In the paper the views and tendencies in the philosophical reflection on mathematics in Poland between the wars are analyzed. Views of most outstanding representatives of Lvov–Warsaw Philosophical School and of Polish Mathematical School are presented. Their influence on logical and mathematical researches is considered.


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*Keywords:* Philosophy of mathematics; Nominalism; Platonism; Psychologism; Lvov–Warsaw School; Polish Mathematical School

The aim of this paper is to indicate main trends and tendencies, main standpoints and views in the philosophical reflection on mathematics in Poland between the wars, i.e., between 1918 and 1939.

Why just this period? Because it was the time of intensive development of mathematics (Polish Mathematical School) and of logic (Warsaw Logical School, Lvov–Warsaw Philosophical School) in Poland. Hence a natural question arises whether this development of mathematics and logic was accompanied by philosophical reflection on those disciplines, whether the researches were founded on and stimulated by certain fixed philosophical presuppositions. On the other hand, philosophy of mathematics and logic is based on and uses certain results of metamathematics, of the foundations of mathematics and of logic. Did logical achievements influence the philosophical reflection?

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## 1. Before 1918

To discuss the philosophy of mathematics between the wars one should start earlier and consider predecessors.

In the 19th century there was no significant work on logic or on philosophy of mathematics in Poland. One can mention only Józef Maria Hoene-Wroński (1776–1853) but his ideas did not find any resonance. The reason was the ambiguity of his ideas and the unclear language in which they were formulated.

The situation began to change at the end of the 19th century. The main centers which played important role here were Lvov and Cracow. In Lvov the main figure was of course Kazimierz Twardowski (1866–1938), the founder of the Lvov–Warsaw Philosophical School. His role in the development of the analytic tradition of philosophy and of logic in Poland is well known. The atmosphere in Lvov stimulated also mathematicians. Here in 1908 Wacław Sierpiński (1882–1969) obtained his *habilitation* and started his lectures, in particular lectures in set theory which were probably the first lectures in set theory in the world. What is important from our point of view is the fact that Sierpiński has chosen as the subject of his *habilitation lecture* a problem from the philosophy of mathematics. His lecture was devoted to the problem of the role and meaning of the concept of a correspondence in mathematics (cf. Sierpiński [13]). He described it as one of the most important notions of mathematics, as "a source of all best ideas". The reason of that he saw in the fact that, as H. Poincaré wrote in *La Science et l'Hypothese*: "Mathematicians do not study objects but relations between them; hence it is not important for them when some objects are replaced by another provided the relations between them remain unchanged". (One can see here structuralistic ideas.) He also added that the source and base of all applications of mathematics was the existence of an ideal correspondence between the domain of abstract mathematical ideas and the domain of the reality.

Also another mathematician who played a crucial role in the rise of Polish Mathematical School, i.e., Zygmunt Janiszewski (1888–1920) showed interest in the philosophy of mathematics. He received his *habilitation* in Lvov in 1913. His habilitation thesis was in topology but the title of his *habilitation lecture* was "On realism and idealism in mathematics" (cf. Janiszewski [4]), hence it was devoted to the philosophy of mathematics. Janiszewski considered there the discussion between realists and idealists on the problem of existence in mathematics. This problem appeared with great power in connection with set theory and especially with Zermelo's well-ordering theorem (1904). Janiszewski presented in his lecture several opinions in this dispute and showed in the conclusion a scepticism towards perspective of finding a final solution to this problem because it is a part of the old dispute between nominalism and platonism.

Why did Sierpiński and Janiszewski choose subjects from the philosophy of mathematics for their habilitation lectures? One of the reasons could be the fact that the habilitation procedures were at the Philosophical Faculty and most of the members of the Faculty Councel were non-mathematicians. Therefore Sierpiński and Janiszewski have chosen general subjects. But on the other hand they could choose also popular subjects from mathematics itself. The fact that they have chosen subjects just from the philosophy of mathematics can indicate that there was a good atmosphere in Lvov

for foundations and philosophy of mathematics and that they both were interested not only in mathematics but also in its philosophy. Both were convinced that one needed a conception of mathematics on which the development of this discipline in Poland could be founded and suggested that set theory could be a basis of it.

The interests of Polish mathematicians in logic and philosophy of mathematics as well as their conviction of the importance of those domains for mathematics can be seen also in the interesting book published in 1915 *Poradnik dla samouków* (Handbook (Guide) for Autodidacts) devoted to mathematics. One finds there chapters by Jan Łukasiewicz (introductory chapter "About science"), W. Sierpiński (about set theory) and some chapters by Z. Janiszewski who was the main contributor and the soul of the whole enterprise. Janiszewski wrote chapters on the foundations of geometry, on mathematical logic and on philosophical problems of mathematics (cf. [3]). The last one was an extensive chapter presenting main problems and views of the philosophy of mathematics with an extensive bibliography in which the current positions of the literature were well represented. In Vol. III of the guide (published in 1923) there was also a chapter "On the meaning of logic for mathematics" [15] by Jan Sleszyński. It is worth mentioning here that Sleszyński wrote there that mathematical logic cannot be reduced to the methodology of mathematics, because it is an autonomous discipline. He defended also logic against various objections formulated by mathematicians and philosophers (among others by Poincaré).

The above remarks indicate that connections between mathematicians and logicians in Warsaw were in fact very good and that they collaborated closely. We return to that later.

The second center which should be considered here was Cracow. Main figures there were Stanisław Zaremba (1863–1942) and Jan Sleszyński (1854–1931) (whom we mentioned already). Zaremba was working mainly in analysis and applications of mathematics but was interested also in the philosophy and methodology of mathematics. He published several papers in the last domains, in particular "Pogląd na te kierunki w badaniach matematycznych, które mają znaczenie teoretyczno-poznawcze" (Remarks on those trends in mathematical investigations which have epistemological meaning) [28] and "Uwagi o metodzie w matematyce i fizyce" (Remarks on the methods of mathematics and physics) [31]. In the first one he considered the meaning of the studies in the foundations of geometry and in set theory for the philosophy, especially for the epistemology. In the second one he claimed that the investigations of the nature are the most important source of new mathematical discoveries and that the role of mathematics in physics consists in providing a tool to deduce corollaries from hypotheses obtained by observation and experience.

Talking about Zaremba one should also mention the controversy about the concept of a magnitude. In 1916 Jan Łukasiewicz (1878–1956) devoted one of his courses at Warsaw University to the methodology of deductive sciences. During his lectures he discussed the book by Zaremba [29] *Arytmetyka teoretyczna* (Theoretical Arithmetic) and analyzed it from the methodological point of view challenging some principles adopted by Zaremba as well as his definition of a magnitude (in particular he criticized the usage of sentences with no contents). Łukasiewicz [7] published his remarks in a paper. This was the beginning of a dispute in which several persons took part,

among others Kazimierz Kuratowski, Tadeusz Czeżowski, Leon Chwistek and of course Zaremba. The essence of the dispute concerned in fact not the concept of a magnitude but the role of logic in mathematics. Zaremba [30] represented the view that logic should be "in mathematics", should be *ancilla mathematicae* (cf. the title of his work *La logique en mathématique*) whetheras Łukasiewicz saw the (mathematical) logic as an autonomous discipline providing the foundations and methodology of mathematics. The latter idea was also accepted by leaders and founders of Polish Mathematical School in Warsaw who stressed the role of set theory, of the foundations of mathematics and of mathematical logic and saw the logic in the center of mathematics (by Zaremba its place was in the periphery of mathematics).

Zaremba and Sleszyński influenced the interest in logic and the philosophy of mathematics of some young mathematicians in Cracow, among others of Witold Wilkosz (1891–1941). He wrote some papers in which he discussed the meaning of mathematical logic to mathematics and the process of abstraction (trying to base it on the abstraction principle) (cf. Wilkosz [26,27]).

With Cracow was connected also Edward Stamm (1886–1940) who studied in Switzerland and Austria. He was a teacher of mathematics in a small town and the author of some papers devoted to the algebra of logic and to the philosophy of mathematics (cf. [16–18]). Especially interesting is the paper "Czem jest i czem będzie matematyka?" (What is and what will be mathematics?) [17] in which analyzing the development of mathematics he comes to the conclusion that "mathematics is not a science but a method, it is the ideal deductive-symbolic stage of a science in general".

## 2. The period 1918–1939

We should start by stressing the fact of close collaboration and mutual influences of logicians and mathematicians in Warsaw in the considered period. Both groups saw the mathematical logic and the methodology of mathematics as disciplines which are autonomous with respect to both mathematics and philosophy on the one hand and on the other they were convinced that those disciplines play a fundamental role in developing mathematics. According to them mathematics and mathematical logic should be neutral towards various philosophical controversies, they should be developed independent of any philosophical presuppositions. This attitude can be illustrated, for example, by the conviction of Polish mathematicians that the philosophy of the axiom of choice must be separated from its role in mathematics. Sierpiński wrote in [14, p. 95]:

> Still, apart from our personal inclination to accept the axiom of choice, we must take into consideration, in any case, its role in the set theory and in the calculus. On the other hand, since the axiom of choice has been questioned by some mathematicians, it is important to know which theorems are proved with its aid and to realize the exact point at which the proof has been based on the axiom of choice; for it has frequently happened that various authors have made use of the axiom of choice in their proofs without being aware of it. And after all, even no

> one questioned the axiom of choice, it would not be without interest to investigate which proofs are based on it and which theorems are proved without its aid—this, as we know, is also done with regard to other axioms.

This means simply that one should disregard philosophical controversies (and treat them as a "private" matter) and investigate (controversial) axioms as purely mathematical constructions using any fruitful methods.

One of the consequences of the described attitude of Polish logicians and mathematicians was the fact that they did not attempt to develop a comprehensive philosophy of mathematics and logic (Stanisław Leśniewski and Leon Chwistek were here the exceptions!). They formulated their philosophical opinions concerning mathematics or logic only occassionally and only on problems which were just interesting for them or on which they actually worked. Consequently, there were no genuine philosophers of mathematics in Poland. Philosophical remarks were formulated by logicians and mathematicians only on the margin of their proper mathematical or logical works (and had no meaning for the results themselves).

The current trends and views in the philosophy of mathematics, i.e., logicism, intuitionism and formalism, were of course well known (and there appeared papers discussing those tendencies, their meaning and development). But none of them was represented in Warsaw School. Moreover, it did not represent any other trend, it had no official philosophy of logic and mathematics. This followed from the belief of the autonomy of logic and mathematics with respect to philosophy. Opinions in the field of the philosophy of logic and mathematics were treated as "private" problems and philosophical declarations were made relunctantly and seldom. If they were made then it was stressed, directly or indirectly, that these were personal opinions.

Though some of logical investigations were motivated by philosophical problems (e.g. the many-valued logics by Łukasiewicz) but the formal, logical constructions were always separated from their philosophical interpretations. This attitude was still strengthened by Alfred Tarski (1901–1983) and Andrzej Mostowski (1913–1975) who claimed that a logician or a mathematician can have philosophical views or sympathies quite different from those which could be suggested by the scope of problems he is working on. They also provided good examples of this attitude by their own work. Mostowski wrote about Tarski (cf. Mostowski [11, p. 81]):

> Tarski, in oral discussions, has often indicated his sympathies with nominalism. While he never accepted the 'reism' of Tadeusz Kotarbiński, he was certainly attracted to it in the early phase of his work. However, the set-theoretical methods that form the basis of his logical and mathematical studies compel him constantly to use the abstract and general notions that a nominalist seeks to avoid. In the absence of more extensive publications by Tarski on philosophical subjects, this conflict appears to have remained unresolved.

Mostowski on the other hand was a sympathizer of constructivism but in his logical and foundational investigations did not take into account the methodological limitations put by it.

Another example are the investigations on intuitionistic logic carried out among others by Tarski without accepting intuitionism as the philosophy of mathematics. Programme of Janiszewski and Polish Mathematical School created set-theoretical foundations of mathematics in the methodological and not philosophical sense.

What were the separated philosophical opinions formulated by Polish logicians, philosophers and mathematicians? Let us start by the problem of psychologism. Psychologism was popular in the philosophy of logic and mathematics in the late 19th century. According to it, the objects studied by logic and mathematics exist as psychic entities and come to be known just like other psychic facts. Already Twardowski took a step towards antipsychologism. Next step was a paper by Łukasiewicz "Logika a psychologia" (Logic and psychology) [5] where he declared himself firmly for antipsychologism in logic. His arguments were as follows: (1) logical laws are certain and psychological ones (being empirical in fact) only probable, (2) laws of logic and laws of psychology differ in content because the former concern the connections between the truth and falsehood of judgements and the latter state relationships between psychic phenomena, (3) the terms 'thinking' and 'judgement' have different meaning in psychology and in logic. Łukasiewicz stated finally in [5]:

> The clarification of the relationship between logic and psychology may prove to the advantage of both disciplines. Logic will be purified of the weeds of psychologism and empiricism, which hamper its true development, and the psychology of cognition will rid itself of elements of apriorism, behind which the genuine light of its truth could not fully show itself. It must be borne in mind that logic is an a priori science, like mathematics, while psychology, like any natural science, is, and must be, based on experience.

Łukasiewicz's arguments against psychologism were similar to those of Husserl and Meinong. They were universally accepted in Poland. Their consequence was the conviction that the certainty of theorems of logic cannot be explained by psychological arguments. This was in fact a negative solution to the problem of certainty of logic. Since almost all Polish logicians were sympathizers of genetic empiricism hence any aprioristic solution of this problem could not be accepted.

Polish logicians did not accept the concept of logic as pure syntax. This view was popular at that time, it was developed under the influence of Hilbert's metamathematics and the philosophy of language of Vienna Circle. An exception was here Chwistek who treated his semantical systems as formal systems of expressions. Warsaw School represented the semantical point of view. In this context one should see the semantical foundations of logic founded by Tarski in the 1930s. An original approach to the problem of the nature of logic was represented by Leśniewski. One can call it intuitive formalism. He attempted the complete formalization of logical systems but claimed that formal expressions always code a fixed intuitive contents.

The semantical point of view implied the rejection of analytical concept of logic, i.e., the rejection of the thesis that logic is a collection of tautologies that are contentually empty (this is a thesis on logic and mathematics versus reality). Leśniewski (and

Kotarbiński) claimed that logic describes the most general features of being, logic plays a role of a general theory of the real world.

Tarski described the concept of tautology as vague and did not see any objective basis for the division of terms into logical and extra-logical. Consequently he did not treat the borderline between formal and empirical disciplines as sharply marked.

Andrzej Mostowski wrote in [12, p. 42] that various metamathematical results:

> obtained by the mathematical method confirm, therefore, the assertion of materialistic philosophy that mathematics is in the last resort a natural science, that its notions and methods are rooted in experience and that attempts at establishing the foundations of mathematics without taking into account its originating in natural sciences are bound to fail.

And added:

> An explanation of the nature of mathematics does not belong to mathematics but to philosophy, and is possible only within the limits of a broadly conceived philosophical view treating mathematics not as detached from other sciences but taking into account its being rooted in natural sciences, its applications, its associations with other sciences and, finally, its history.

Łukasiewicz's views concerning the considered problem of relations between logic and mathematics on the one hand and reality on the other were changing. In [6] he claimed that logical and mathematical judgements are a priori truths about the world of ideal entities. Hence he treated both disciplines as unrelated to experience. The discovery of many-valued logics implied that Łukasiewicz maintained that logic systems can be given an ontological interpretation and that experience will help to decide which of systems of logic is fulfilled in the reality (cf. [8]). Later he tended to the conventionalism and relativism. In [10] he wrote:

> We have no means to decide which of the $n$-valued systems of logic [...] is true. Logic is not a science of the laws of thought or of any real object; it is, in my opinion, only an instrument which enables us to draw asserted conclusions from asserted premises. [...] The more useful and richer a logical system is, the more valuable it is.

Kazimierz Ajdukiewicz (1890–1963) was also a sympathizer of conventionalism (in a radical version). He claimed that logic is something implied by the meaning rules (rules of sense), both axiomatic and deductive ones. Later he abandoned the radical conventionalism and claimed that laws of logic refer indirectly to the experience and that they should be treated as rules of inference, hence they belong to the metascience and are mainly of a methodological character.

Next problem which was discussed and commented was the problem of nominalism. A declared nominalist was Leśniewski. Hence he denied the existence of any general objects. Consequently, the systems he created consisted of a finite number of individual

inscriptions. Tarski had nominalistic leanings (inherited from Leśniewski) but the needs of metamathematics made him to abandon these sympathies. In particular he referred to formula types, that is classes of equiform formulas but these classes of formulas were treated by him as consisting of formulas interpreted as physical bodies.

Łukasiewicz's view towards nominalism was different. He maintained that arguments of Tarski defending nominalism were not sufficient. He thought that logicians merely use nominalistic terminology but in fact they are not nominalists. He inclined to interpret logic in an outright neo-platonic spirit. In [9] he wrote:

> In concluding these remarks I should like to outline an image which is connected with the most profound intuitions which I always experience in the face of logistic. That image will perhaps shed more light on the true background of that discipline, at least in my case, than all discursive description could. Now, whenever I work on even the least significant logistic problem—for instance, when I search for the shortest axiom of the implicational propositional calculus—I always have the impression that I am facing a powerful, most coherent and most resistant structure. I sense that structure as if it were a concrete, tangible object, made of the hardest metal, a hundred times stronger than steel and concrete. I cannot change anything in it; I do not create anything of my own will, but by strenuous work I discover in it ever new details and arrive at unshakable and eternal truths. Where is and what is that ideal structure? A believer would say that it is in God and His thought.

Note that Łukasiewicz stressed that logic itself cannot solve the philosophical controversy over universals. Hence any claims that logic is nominalistic or not are groundless. Similar opinions towards the neutrality of logic with respect to the problem of universals held Ajdukiewicz and Czeżowski. Add that Kotarbiński in the early and radical version of his reism held views similar to those of Leśniewski.

As remarked above Leon Chwistek (1884–1944) and Stanisław Leśniewski (1886–1939) were two exceptions in the described attitude towards philosophical problems and issues, more exactly they were interested only in those logical problems which were implied by their own philosophical views in the foundations of mathematics, their philosophical views generated their interest in particular problems, their logical investigations were motivated by their philosophical views.

Leśniewski had the chair of the philosophy of mathematics at the University of Warsaw (since 1919 till his death in 1939). He represented a philosophical approach to logic though he was convinced that philosophical investigations are hopeless and lead to no definite solutions. His aim was to construct a system of logic that would satisfy two general requirements: it should serve as the foundations for mathematics and should be constructed in a manner free of any ambiguities. He did it by constructing three systems: protothetic, ontology and mereology. Leśniewski meant logical systems in a nominalistic way. Language was for him a collection of concrete individual inscriptions. There existed only those expressions which have been actually written, he admitted no 'potential' existence. This was called by him 'constructive nominalism'. It was

connected with intuitive formalism. Consequently he rejected the interpretation of logic and mathematics as games using symbols devoid of meaning. According to him every language system says 'something' 'about something', is a way to express what is intuitively true, is an indispensable way of encoding and transmitting logical intuitions. Add also that Leśniewski maintained that logic should be meant as extensional and bivalence.

Chwistek is known mainly for his logical works, i.e., for his simplification of the theory of types of Russell and Whitehead (he did it in a nominalistic spirit). His aim in logical investigations was to create a comprehensive system of logic and mathematics based on a theory of expressions (called by him rational metamathematics). His results and ideas had rather limited influence. The reason was the complicated and nonstandard notation used by him as well as his way of presenting the results.

He represented rationalism (called by him critical rationalism) and rejected irrationalism. According to him there are two sources of knowledge: experience and deduction. Methods used in science and in the philosophy should be constructive. The aim of science is to describe by mathematical expressions objects given in an experience. Mathematical formulas are only descriptions of an experience and cannot be treated as laws concerning objects which are not given by experience.

One of the most known philosophical conceptions of Chwistek was his theory of the plurality of the reality. It was published for the first time in his paper from [1] "Trzy odczyty odnoszące się do pojęcia istnienia" (Three lectures concerning the concept of existence) and found its final presentation in his book *Granice nauki* (Limits of Science) [2]. He postulated, according to various types of experience, four types of reality: reality of impressions, reality of images, reality of things, and physical reality (constructed in science). He attempted also to characterize properties of those types of realities by suitable sets of axioms.

Chwistek represented in logic and mathematics nominalism and was against formalism. He claimed that objects of deductive systems are expressions and one cannot accept any other objects. According to him geometry is an experimental discipline. The development of non-euclidean geometries was considered by him as the most important achievement in science. It rejected the Kantian idealism and the view that geometry is given a priori. Geometry and other mathematical theories as well as theories of the science should be developed constructively, i.e., one should base them on such axioms and definitions that the theorems deduced from them should be in accordance with experience. This would suggest that he would accept conventionalism. But in *Granice nauki* he rejected it. Moreover he claimed that conventionalism was incorrect not only in science but it was also a source of wrong views in social problems (it reduced truth to the usefulness and efficiency and in this way led to the reinforcement of the ruling class).

## 3. Alfred Tarski

In previous sections we mentioned some views of Tarski concerning the philosophy of logic and mathematics. Let us say now more about his philosophy (cf. [23–25]).

Start by noting that Tarski was interested in philosophical problems and very actively participated in the philosophical life of his time. He was convinced of philosophical significance of his works, in particular of his work on truth. In [20] he wrote:

> I shall be satisfied if this paper convince the reader that the method used above already now is an indispensable apparatus which may be helpful in considerations of purely philosophical problems. [...] The central problem of this paper—construction of a definition of a true sentence and founding a scientific basis of a theory of truth—belongs to epistemology and is one of the main problems in this domain of philosophy. Hence I expect that just specialists in epistemology will take an interest in it, that—not becoming discouraged by difficult notions and methods, which so far have not been applied in this field—will analyze in a critical way results contained in it and will be able to use them in their further studies.

He described himself as (cf. Tarski [21]):

> Being a mathematician (as well as a logician, perhaps a philosopher of a sort) [...]

Tarski's philosophical attitude was anti-metaphysical, he supported the idea of scientific philosophy. He accepted a programme of "small philosophy" which aims at detailed and systematic analysis of the concepts used in philosophy. Such a philosophy is minimalistic, anti-speculative and sceptical towards many fundamental problems of traditional philosophy. This attitude was inherited by Tarski from the Lvov–Warsaw School and strengthened by contacts with the Vienna Circle. He also maintained empiricism and abondoned the analytic/synthetic distinction and stressed that logical and empirical truths belong to the same generic category. Influenced by Leśniewski and Kotarbiński he was inclined to rather a strongly nominalistic understanding of expressions. According to this sentences are treated as concrete physical objects and languages as consisting of token-expressions. Needs of metalogical studies forced the understanding of them as expressions-types. Tarski sharply contrasted colloquial, natural language and formalized language.

Tarski was inclined to identify mathematics with the deductive method. He maintained that there is no hard borderline between formal and empirical sciences. He admitted the rejection of logical and mathematical theories on empirical grounds. He claimed also that there is no sharp demarcation between logical and factual truth and that the concept of tautology is unclear.

One must stress that all those were his "private" philosophical views which did not influence his logical and mathematical researches, in other words, his researches were independent of any philosophical presuppositions. In the paper "Fundamentale Begriffe der Methodologie der deduktiven Wissenschaften" [19] he expicitly wrote:

> [...] it should be noted that no particular philosophical standpoint regarding the foundations of mathematics is presupposed in the present work.

This was typical for him and for the whole Warsaw School in logic. This independence of logical and mathematical studies and philosophical views explains the cognitive conflict and discrepancy between Tarski's nominalistic and empiricistic sympathies and his "platonic" mathematical and logical practice. Note that his attitude enabled him to contribute to various important foundational streams without the necessity of accepting their philosophical assumptions and attempting to reconcile the philosophy and the research practice. His programme of metamathematics can be summarized by his words from the paper [22] where he wrote:

> As an essential contribution of the Polish school to the development of metamathematics one can regard the fact that from the very beginning it admitted into metamathematical research all fruitfull methods, whether finitary or not.

Note that this attitude was in full accordance with the attitude of Polish mathematicians indicated above. According to it one should study the problems using any fruitfull methods and making no philosophical presuppositions. There is no need to announce one's philosophical views concerning the investigated problems because this does not belong to scientific duties, this is a "private" affair.

## 4. Conclusions

As we showed above Polish logicians and mathematicians believed that philosophical problems of logic and mathematics are important. They knew quite well the current views and trends in this field, commented upon them and formulated several own opinions concerning the philosophy of mathematics. But on the other hand they treated logic and mathematics as autonomous disciplines independent of the philosophical reflection on them, independent of any philosophical presuppositions. They sharply separated mathematical and logical research practice and philosophical discussions concerning logic and mathematics. Philosophical views and opinions were treated as "private" matter that should not influence the mathematical and metamathematical investigations where all correct methods can and should be used.

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