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MHD convective flow through porous medium in a horizontal channel with insulated and impermeable bottom wall in the presence of viscous dissipation and Joule heating

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Abstract This paper deals with a steady MHD forced convective flow of a viscous fluid of finite depth in a saturated porous medium over a fixed horizontal channel with thermally insulated and impermeable bottom wall in the presence of viscous dissipation and joule heating. The governing equations are solved in the closed form and the exact solutions are obtained for velocity and temperature distributions when the temperatures on the fixed bottom and on the free surface are prescribed. The expressions for flow rate, mean velocity, temperature, mean temperature, mean mixed temperature in the flow region and the Nusselt number on the free surface have been obtained. The cases of large and small values of porosity coefficients have been obtained as limiting cases. Further, the cases of small depth (shallow fluid) and large depth (deep fluid) are also discussed. The results are presented and discussed with the help of graphs.

1. Introduction

The study of fluid flow problems associated with heat transfer is of widespread interest in almost all the fields of engineering as well as in astrophysics, biology, biomedicine, meteorology, physical chemistry, plasma physics, geophysics, oceanography and scores of other disciplines. Hydromagnetic flows and heat transfer in porous media have been considered extensively in

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recent years due to their occurrence in several engineering processes such as compact heat exchangers, metallurgy, casting, filtration of liquid metals, cooling of nuclear reactors and fusion control. Abdulla [1], investigated theoretically and experimentally of composite materials under the influence of thermal insulation. Ananda Reddy et al. [2], studied, thermal diffusion and chemical reaction effects with simultaneous thermal and mass diffusion in magneto hydrodynamic mixed convection flow with Joule heating. Analysis of combined forced and free flow in a vertical channel with viscous dissipation and isothermal-isoflux boundary conditions was studied by Barletta [3]. Effect of viscous dissipation on thermally developing forced convection duct flows was considered by Barletta et al. [4]. Barletta and Nield [5,6], considered the combined forced and free convective flow in a vertical porous channel in the presence of viscous dissipation and pressure work. Buonomo et al. [7] showed that for horizontal channels heated from below, the buoyancy force can induce secondary flow that enhances the heat transfer. The effect of local thermal non-equilibrium on forced convection boundary layer flow from a heated surface in porous media was studied by Celli et al. [8]. Chamkha [9–11], investigated MHD free convection from a vertical plate embedded in a thermally stratified porous medium. He considered similarity solutions for the laminar boundary layer equations describing steady hydromagnetic two dimensional flows and heat transfer in a stationary electrically conducting and heat generating fluid driven by a continuously moving porous surface immersed in a fluid saturated porous medium. Huang and Liu [12], experimentally studied convective instability in the thermal entrance region of a horizontal parallel plate channel heated from below. Effect of thermal insulation of the bottom of a melting tank on mass exchange of the glass was considered by Kuznetsov et al. [13]. Liu and Gau [14], studied onset of secondary flow and enhancement of heat transfer in horizontal convergent and divergent channels heated from below. Joule heating in magneto hydrodynamic flows in channels with thin conducting walls was investigated by Mao et al. [15]. Steady flow of a viscous fluid through a saturated porous medium of finite thickness, impermeable and thermally insulated bottom and the other side is stress free, at a constant temperature was studied by Mounuddin and Pattabhiramacharyulu [16]. Parvin and Hossain [17] investigated on the conjugate effect of Joule heating and magnetic field on combined convection in a Lid-driven cavity with undulated bottom surface. Magneto hydrodynamic mixed convection in a horizontal channel with an open cavity was considered by Rahman et al. [18]. Ravikumar et al. [19], studied MHD three dimensional Couette flow past a porous plate with heat transfer. Mixed convection inside a lid-driven parallelogram cavity with isoflux heating was considered by Sumon et al. [20]. Natural convection boundary layer flow over a continuously moving isothermal vertical surface immersed in a thermally stratified medium has been investigated by Takhar et al. [21]. Flow through a porous wall with convective acceleration was studied by Yamamoto and Yoshida [22]. MHD mixed convection from a vertical plate embedded in a porous medium with a convective boundary condition was investigated by Makinde and Aziz [24]. Makinde and Mhone [25] considered on temporal stability analysis for hydromagnetic flow in a channel filled with a saturated porous medium. In his study Makinde [26] considered MHD boundary layer flow and mass transfer past a vertical plate in a porous medium with constant heat flux. Recently, MHD mixed Convection in a lid-driven Cavity with various heat transfer effects was studied by Chatterjee et al. [27,28].

In spite of all the previous studies, MHD forced convective viscous flow through porous medium in a horizontal channel with insulated and impermeable bottom wall in the presence of viscous dissipation and Joule heating has received little attention. Motivated by the above referenced work and the numerous possible industrial applications of the problem, we have extended the work of Mounuddin and Pattabhiramacharyulu [16], in the presence of transverse magnetic field. Hence we have considered, the steady forced convective MHD flow of a viscous liquid of viscosity μ and of finite depth H through a porous medium of porosity coefficient 'K^{*}' over a fixed impermeable, thermally insulated bottom. The flow is generated by a constant horizontal pressure gradient parallel to the fixed bottom. The momentum equation considered is the generalized Darcy's law proposed by Yamamoto and Iwamura [23], which takes into account the convective acceleration and the Newtonian viscous stresses in addition to the classical Darcy force. The basic equations of momentum and energy are solved to give exact expressions for velocity and temperature distributions. Employing the flow rate, mean velocity, mean temperature, mean mixed temperature and the Nusselt number on the free surface have been obtained in the following cases (i) high porosity, (ii) low porosity, (iii) large depths (large H), and (iv) shallow depths (small H).

2. Mathematical formulation

We have considered a steady viscous, electrically conducting, forced convection flow through a saturated porous medium, over a fixed horizontal, impermeable and thermally insulated bottom of finite depth H. With reference to a rectangular Cartesian co-ordinates system, with the origin 'O' on the bottom, let X-axis is assumed to be in the direction of the flow and Y-axis is taken perpendicular to it (see Fig. 1 for flow configuration).The boundary at the bottom is represented as $Y = 0$ and the free surface as $Y = H$. The free surface is exposed to the atmosphere of constant temperature T_1. A transverse magnetic field of uniform strength B_0 is applied perpendicular to the flow. The flow is steady and the Grashof number is considered to be very small. The flow is due to a constant pressure gradient at the mouth of the channel. The magnetic Reynolds number is taken to be small enough to neglect the induced magnetic field in comparison with the applied magnetic field. Let the flow be characterized by a velocity U in the X direction. The fluid is assumed to be ionized and thereby is an electrical conductor. However with in any small but finite volume the

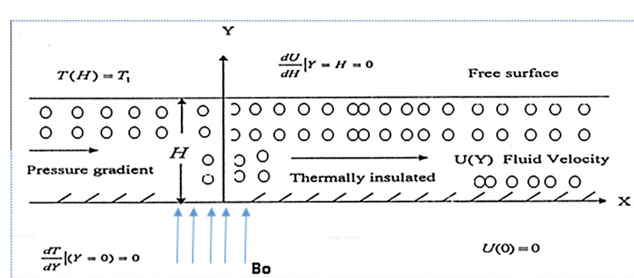

Figure 1 Flow configuration.

number of particles with positive and negative charges is nearly equal. Hence the total excess charge density and imposed electric field intensity is assumed to be zero. By considering the above assumptions the governing equations of the flow are given below.

$$-\frac{\partial P}{\partial X}+\mu\frac{d^2U}{dY^2}-\mu\frac{U}{K^*}-\sigma_e B_0^2 U=0 \tag{1}$$

$$\rho c U\frac{\partial T}{\partial X}=\kappa\frac{d^2T}{dY^2}+\mu\left(\frac{dU}{dY}\right)^2+\sigma_e B_0^2 U^2 \tag{2}$$

The corresponding boundary conditions are given by

$$U=0,\ \frac{dT}{dY}=0\ at\ Y=0 \tag{3}$$

$$\mu\frac{dU}{dY}=0, T=T_1\ at\ Y=H \tag{4}$$

where p is the pressure, μ is the kinematic viscosity, X and Y are the cartesian coordinates, K^* is dimensional porous parameter, σ is the electric conductivity, B_0 is applied magnetic field, ρ is the density, T is the dimensional temperature, His depth of the channel, and κ thermal conductivity.

Now we can employ the following dimensionless variables and parameters:

$$X=ax;\ Y=ay;\ H=ah;\ U=\frac{\mu u}{\rho a^2};\ P=\frac{\mu^2 p}{\rho a^2};\ M=\frac{\sigma_e B_0^2 a^2}{\mu};$$

$$\alpha_1^2=\alpha^2+M;\ \Pr=\frac{\mu c}{\kappa},\ K^*=\frac{a^2}{\alpha^2},$$

$$T=T_0+(T_1-T_0)\theta,\ Br=\frac{\mu^3}{\rho^2 a^2\kappa(T_1-T_0)},$$

$$-\frac{\partial P}{\partial X}=\frac{\mu^2 c_1}{\rho a^3}\left(c_1=-\frac{\partial p}{\partial x}\right)\ \text{and}$$

$$\frac{\partial T}{\partial X}=\frac{(T_1-T_0)}{a}c_2\ \text{where}\ c_2=\frac{\partial\theta}{\partial x}; \tag{5}$$

where M is the magnetic parameter, Pr is the Prandtl number, Br is the Brinkman number, T_1 and T_0 are the temperatures at bottom and upper surface respectively, and α is the permeability parameter in dimensionless form.

3. Solution of the problem

In view of Eq. (5), Eqs. (1) and (2) reduce to the following dimensionless form:

$$\frac{d^2u}{dy^2}-\alpha_1^2 u=-c_1 \tag{6}$$

$$\frac{d^2\theta}{dy^2}=Pr\ a\ c_2 u-Br\left(\frac{du}{dy}\right)^2-BrM\ u^2 \tag{7}$$

Together with the boundary conditions

$$\text{u}=0\ \text{and}\ \frac{d\theta}{dy}=0\ \text{at}\ y=0 \tag{8}$$

$$\frac{du}{dy}=0,\ \theta=1\ \text{at}\ y=h \tag{9}$$

The solutions of these equations together with the related boundary conditions yield the velocity distribution:

$$u(y)=\frac{c_1}{\alpha_1^2}\left[1-\frac{\cosh\alpha_1(h-y)}{\cosh\alpha_1 h}\right] \tag{10}$$

and the temperature distribution:

$$\begin{aligned}\theta(y)=1&+\frac{P_r c_1 c_2}{\alpha_1^2}\left\{\frac{(y^2-h^2)}{2}+\frac{(h-y)\tanh\alpha_1 h}{\alpha_1}+\frac{1}{\alpha_1^2\cos\alpha_1 h}\right.\\&\left.\times(1-\cos\alpha_1(h-y))\right\}+\frac{Brc_1^2}{2\alpha_1^2}\left\{\frac{(y^2-h^2)}{2\cosh^2\alpha_1 h}+\frac{(h-y)\tanh\alpha_1 h}{\alpha_1}\right.\\&\left.+\frac{(1-\cosh 2\alpha_1(h-y))}{4\alpha_1^2\cosh^2\alpha_1 h}\right\}-\frac{BrMc_1^2}{\alpha_1^4}\left\{\frac{(y^2-h^2)}{2}+\frac{(y^2-h^2)}{4\cosh^2\alpha_1 h}\right.\\&\left.+\frac{3(h-y)\tanh\alpha_1 h}{2\alpha_1}+\frac{2(1-\cosh\alpha_1(h-y))}{\alpha_1^2\cosh\alpha_1 h}-\frac{(1-\cosh 2\alpha_1(h-y))}{8\alpha_1^2\cosh^2\alpha_1 h}\right\}\end{aligned} \tag{11}$$

with the aid of the expressions for velocity and temperature distribution, we now derive the following important characteristics of the flow.

(i) The flow rate:

$$q=\int_0^h u dy=\frac{c_1}{\alpha_1^2}\left[h-\frac{\tan\alpha_1 h}{\alpha_1}\right] \tag{12}$$

(ii) The mean of velocity:

$$\bar{u}=\frac{1}{h}\int_0^h u dy=\frac{c_1}{h\alpha_1^2}\left[h-\frac{\tan\alpha_1 h}{\alpha_1}\right] \tag{13}$$

(iii) The mean temperature:

$$\begin{aligned}\bar{\theta}=\frac{1}{h}\int_0^h\theta dy=1&+\frac{P_r c_1 c_2}{\alpha_1^2}\left[\frac{-h^2}{3}+\frac{h\tanh\alpha_1 h}{2\alpha_1}+\frac{1}{\alpha_1^2\cos\alpha_1 h}-\frac{\tanh\alpha_1 h}{h\alpha_1^3}\right]\\&+\frac{Brc_1^2}{2\alpha_1^2}\left[-\frac{h^2}{3\cosh^2\alpha_1 h}+\frac{h\tanh\alpha_1 h}{2\alpha_1}+\frac{1}{4\alpha_1^2\cosh^2\alpha_1 h}-\frac{\tanh\alpha_1 h}{4h\alpha_1^3}\right]\\&-\frac{Brc_1^2}{\alpha_1^4}\frac{\sigma_e B_0^2\alpha_1^2}{\mu}\left(\frac{h^2}{3}+\frac{h^2}{6\cosh^2\alpha_1 h}-\frac{3h\tanh\alpha_1 h}{4\alpha_1}-\frac{2}{\alpha_1^2\cosh\alpha_1 h}+\frac{1}{8\alpha_1^2\cosh^2\alpha_1 h}\right.\\&\left.+\frac{15\tanh\alpha_1 h}{8h\alpha_1^3}\right)\end{aligned} \tag{14}$$

(iv) The mean mixed temperature:

$$\frac{\int_0^h\theta u dy}{\int_0^h u dy}=\left\{\begin{array}{l}1+\frac{\Pr c_1 c_2}{\alpha_1(h\alpha_1-\tanh\alpha_1 h)}\left\{\begin{array}{l}\frac{-h^3}{3}+\frac{h^2\tanh\alpha_1 h}{\alpha_1}+\frac{2h}{\alpha_1^2\cosh\alpha_1 h}+\frac{h}{2\alpha_1^2\cosh^2\alpha_1 h}-\\\frac{\tanh\alpha_1 h}{2\alpha_1^3}-\frac{2\tanh\alpha_1 h}{\alpha_1^3\cosh\alpha_1 h}-\frac{h\tanh^2\alpha_1 h}{\alpha_1^2}\end{array}\right\}-\\\frac{Brc_1^2}{2\alpha_1(h\alpha_1-\tanh\alpha_1 h)}\left\{\begin{array}{l}\frac{-3\tanh\alpha_1 h}{4\alpha_1^3}+\frac{h^3}{3\cosh^2\alpha_1 h}-\frac{h^2\tanh\alpha_1 h}{2\alpha_1}+\frac{\tanh\alpha_1 h}{8\alpha_1^3\cosh^2\alpha_1 h}-\\\frac{h}{4\alpha_1^2\cosh^2\alpha_1 h}-\frac{\sinh 3\alpha_1 h}{24\alpha_1^3\cosh^3\alpha_1 h}-\frac{h}{\alpha_1^2\cosh^3\alpha_1 h}+\frac{\tanh\alpha_1 h}{\alpha_1^3\cosh\alpha_1 h}-\\\frac{h^2\tanh\alpha_1 h}{2\alpha_1\cosh^2\alpha_1 h}+\frac{h\tanh^2\alpha_1 h}{\alpha_1^2}\end{array}\right\}-\\\frac{BrMc_1^2}{\alpha_1^3(h\alpha_1-\tanh\alpha_1 h)}\left\{\begin{array}{l}\frac{-h^3}{3}-\frac{h^3}{6\cosh^2\alpha_1 h}+\frac{7h}{8\alpha_1^2\cosh^2\alpha_1 h}+\frac{3h}{\alpha_1^2\cosh\alpha_1 h}+\frac{h}{2\alpha_1^2\cosh^3\alpha_1 h}+\\\frac{5h^2\tanh\alpha_1 h}{4\alpha_1}-\frac{3\tanh\alpha_1 h}{8\alpha_1^3}-\frac{7\tanh\alpha_1 h}{2\alpha_1^3\cosh\alpha_1 h}+\frac{h^2\tanh\alpha_1 h}{4\alpha_1\cosh^2\alpha_1 h}-\\\frac{7\tanh\alpha_1 h}{16\alpha_1^3\cosh^2\alpha_1 h}-\frac{\sinh 3\alpha_1 h}{48\alpha_1^3\cosh^3\alpha_1 h}-\frac{3h\tanh^2\alpha_1 h}{2\alpha_1^2}\end{array}\right\}\end{array}\right\} \tag{15}$$

(v) Heat transfer coefficient Nusselt number on the free surface:

$$\begin{aligned}Nu=\left.\frac{d\theta}{dy}\right|_{y=h}=&\frac{P_r c_1 c_2}{\alpha_1^2}\left\{h-\frac{\tanh\alpha_1 h}{\alpha_1}\right\}+\frac{BrMc^2}{2\alpha_1^2}\left\{\frac{h}{\cosh^2\alpha_1 h}-\frac{\tanh\alpha_1 h}{\alpha_1}\right\}\\&-\frac{BrMc_1^2}{\alpha_1^4}\left\{h+\frac{h}{2\cosh^2\alpha_1 h}-\frac{3\tanh\alpha_1 h}{2\alpha_1}\right\}\frac{BrMc_1^2}{\alpha_1^4}\left\{h+\frac{h}{2\cosh^2\alpha_1 h}-\frac{3\tanh\alpha_1 h}{2\alpha_1}\right\}\end{aligned} \tag{16}$$

4. Deductions

4.1. Fluid flow in a medium with high porosity

Flow for small values of α_1 i.e. large values of the porosity coefficient K* (neglecting powers of α higher than $o(\alpha_1^2)$), we get

(i) Velocity:

$$u(y) = c_1\left(\left(\frac{2hy - y^2}{2}\right) - \frac{\alpha_1^2}{24}\left(8h^3y - 4hy^3 + y^4\right)\right) \tag{17}$$

(ii) Mean velocity

$$\bar{u} = \frac{1}{h}\int_0^h u\,dy = \frac{c_1h^2}{15}[5 - 2\alpha^2h^2] \tag{18}$$

(iii) Temperature:

$$\theta(y) = 1 + \frac{P_rc_1c_2}{720}\{(120hy^3 - 30y^4 - 90h^4) - \alpha_1^2(y^6 - 6hy^5 + 40h^3y^3 - 35h^6)\}$$
$$+ \frac{Brc_1^2}{180}\{(45h^4 - 90h^2y^2 + 60hy^3 - 15y^4) - \alpha_1^2(35h^6 + 15h^4y^2$$
$$+20h^3y^3 + 15h^2y^4 - 12hy^5 + 2y^6)\}$$
$$-\frac{BrMc_1^2}{20{,}160}\left\{\begin{matrix}(-10{,}080h^6 + 1680h^2y^4 - 1008hy^5 + 168y^6) - \\ \alpha_1^2(-665h^8 + 1120h^4y^4 - 1008h^3y^5 - 224h^2y^6 + 952hy^7 + 17y^8)\end{matrix}\right\} \tag{19}$$

(iv) Mean temperature:

$$\bar{\theta} = \frac{1}{h}\int_0^h \theta dy1 - \frac{P_rc_1c_2h^4}{5040}\left(462 - 181\alpha_1^2h^2\right) + \frac{Brc_1^2h^4}{1260}\left(189 - 149\alpha_1^2h^2\right)$$
$$+ \frac{BrMc_1^2h^6}{60{,}480}\left(1944 - 1547\alpha_1^2h^2\right) \tag{20}$$

(v) Mean mixed temperature:

$$\frac{\int_0^h \theta u dy}{\int_0^h u dy} = 1 + \frac{P_rc_1c_2h^4}{1008(5 - 2\alpha^2h^2)}\left(-396 + 491\alpha_1^2h^2\right)$$
$$+ \frac{Brc_1^2h^4}{24{,}192(5 - 2\alpha^2h^2)}\left(14{,}256–28{,}997\alpha_1^2h^2\right)$$
$$+ \frac{BrMc_1^2h^5}{1680(5 - 2\alpha^2h^2)}\left(-1680 + 952\alpha_1^2h^2\right) \tag{21}$$

(vi) Nusselt number on the free surface:

$$\left.\frac{d\theta}{dy}\right|_{y=h} = \frac{P_rc_1c_2h^3}{15}\left(5 - 2\alpha_1^2h^2\right) + \frac{Brc_1^2h^3}{15}\left(-5 + 4\alpha_1^2h^2\right)$$
$$- \frac{BrMc_1^2h^5}{15}\left(42 - 34\alpha_1^2h^2\right) \tag{22}$$

4.2. For large values of α_1

For low porosity the asymptotic flow characteristics are: i.e, for large α_1 $\sinh\alpha_1h \approx \frac{e^{\alpha_1h}}{2}$; $\cosh\alpha_1h \approx \frac{e^{\alpha_1h}}{2}$; $\tanh\alpha_1h \approx 1$ and neglecting the terms of $o\left(\frac{1}{\alpha_1^3}\right)$, we get

(i) Velocity:

$$u(y) = \frac{c_1}{\alpha_1^2}(1 - e^{-\alpha y}) \tag{23}$$

(ii) Mean velocity:

$$\bar{u} = \frac{c_1}{\alpha_1^2} \tag{24}$$

(iii) Temperature:

$$\theta(y) = 1 + \frac{P_rc_1c_2}{2\alpha_1^2}(y^2 - h^2) \tag{25}$$

(iv) Mean temperature:

$$\bar{\theta} = 1 - \frac{P_rc_1c_2}{3\alpha_1^2}h^2 \tag{26}$$

(v) Mean mixed temperature:

$$\frac{\int_0^h \theta u dy}{\int_0^h u dy} = 1 - \frac{P_rc_1c_2}{3\alpha_1^2}h^2 \tag{27}$$

(vi) Nusselt number on the free surface:

$$\left.\frac{d\theta}{dy}\right|_{y=h} = \frac{P_rc_1c_2}{\alpha_1^2}h \tag{28}$$

4.3. Flow for large depth that is for large h

For large h $\sinh\alpha_1h \approx \frac{e^{\alpha_1h}}{2}$; $\cosh\alpha_1h \approx \frac{e^{\alpha_1h}}{2}$; $\tanh\alpha_1h \approx 1$ and neglecting the terms of $o\left(\frac{1}{h^3}\right)$, we get

(i) Velocity:

$$u(y) = \frac{c_1}{\alpha_1^2}(1 - e^{-\alpha y}) = \frac{c_1}{\alpha_1^2}\left(1 - e^{-\frac{y}{\delta}}\right) \tag{29}$$

where $y \gg \delta = \frac{1}{\alpha_1} = \frac{\sqrt{k^*}}{a} \cong \frac{c_1}{\alpha_1^2}\left(y \gg \frac{\sqrt{k^*}}{a}\right)$porosity effect on the velocity is confined to narrow region of thickness of the order $\sqrt{K^*}$ above the bottom. Beyond which we have a plug flow with velocity $\frac{c_1}{\alpha_1^2} = \frac{c_1a^2}{K^*}$

This is the velocity when viscous term is not there i.e. $\mu\$^2u = 0$Therefore

$$c_1 - \alpha_1^2u = 0 \Rightarrow u = \frac{c_1}{\alpha_1^2}\ \text{(Darcian velocity)} \tag{30}$$

(ii) Mean velocity:

$$\bar{u} = \frac{c_1}{h\alpha_1^2}\left(h - \frac{1}{\alpha_1}\right) \tag{31}$$

(iii) Temperature:

$$\theta(y) = 1 + \frac{P_rc_1c_2}{\alpha_1^2}\left\{\frac{(y^2 - h^2)}{2} + \frac{(h - y)}{\alpha_1} + \frac{e^{-\alpha_1y}}{\alpha_1^2}\right\} + \frac{Brc_1^2}{2\alpha_1^2}\left\{\frac{(h - y)}{\alpha_1} + \frac{e^{-2\alpha y}}{\alpha_1^2}\right\}$$
$$- \frac{BrMc_1^2}{\alpha_1^4}\left\{\frac{(y^2 - h^2)}{2} + \frac{3(h - y)}{2\alpha_1} - \frac{2e^{-\alpha y}}{\alpha_1^2} - \frac{e^{-2\alpha y}}{4\alpha_1^2}\right\} \tag{32}$$

(iv) Mean temperature:

$$\bar{\theta} = 1 + \frac{P_rc_1c_2}{\alpha_1^2}\left[\frac{-h^2}{3} + \frac{h}{2\alpha_1} - \frac{1}{h\alpha_1^3}\right] - \frac{Brc_1^2}{2\alpha_1^2}\left[-\frac{h}{2\alpha_1} + \frac{1}{4h\alpha_1^3}\right]$$
$$- \frac{BrMc_1^2}{\alpha_1^4}\left(\frac{h^2}{3} - \frac{3h}{4\alpha_1} + \frac{15}{8h\alpha_1^3}\right) \tag{33}$$

(v) Mean mixed temperature:

$$\frac{\int_0^h \theta u dy}{\int_0^h u dy} = 1 + \frac{P_rc_1c_2}{\alpha_1(h\alpha_1 - 1)}\left\{\frac{-h^3}{3} + \frac{h^2}{\alpha_1} - \frac{1}{2\alpha_1^3} - \frac{h}{\alpha_1^2}\right\}$$
$$+ \frac{Brc_1^2}{\alpha_1(h\alpha_1 - 1)}\left\{\frac{11}{24\alpha_1^3} - \frac{h^2}{4\alpha_1} - \frac{h}{2\alpha_1^2}\right\} + \frac{BrMc_1^2}{\alpha_1^3(h\alpha_1 - 1)}$$
$$\times\left\{\frac{h^3}{3} - \frac{5h^2}{4\alpha_1} + \frac{11}{24\alpha_1^3} + \frac{3h}{2\alpha_1^2}\right\} \tag{34}$$

(vi) Nusselt number on the free surface:

$$\left.\frac{d\theta}{dy}\right|_{y=h} = \left.\frac{d\theta}{dy}\right|_{y=h} = \frac{P_r c_1 c_2}{\alpha_1^2}\left\{h - \frac{\tanh \alpha_1 h}{\alpha_1}\right\} - \frac{BrMc^2}{2\alpha_1^3} - \frac{BrMc_1^2}{\alpha_1^4}\left\{h - \frac{3}{2\alpha_1}\right\} \tag{35}$$

4.4. Flow for shallow fluids that is 'h' small (retaining terms up to the $o(h^2)$)

(i) Velocity:

$$u(y) = \frac{c_1}{24}((24hy - 12y^2) - \alpha_1^2(y^4 - 4hy^3)) \tag{36}$$

(ii) Mean velocity:

$$\bar{u} = \frac{1}{h}\int_0^h u dy = \frac{c_1 h^2}{3} \tag{37}$$

(iii) Temperature:

$$\begin{aligned}\theta(y) = 1 &+ \frac{P_r c_1 c_2}{720}\{(120hy^3 - 30y^4) - \alpha_1^2(y^6 - 6hy^5)\} \\ &+ \frac{Brc_1^2}{180}\{(90h^2y^2 + 60hy^3 - 15y^4) - \alpha_1^2(15h^2y^4 - 12hy^5 + 2y^6)\} \\ &- \frac{BrMc_1^2}{20,160}\{(1680h^2y^4 - 1008hy^5 + 168y^6) - \alpha_1^2(-224h^2y^6 + 952hy^7 \\ &+ 17y^8)\}\end{aligned} \tag{38}$$

(iv) Mean temperature:

$$\bar{\theta} = \frac{1}{h}\int_0^h \theta dy = 1 \tag{39}$$

(v) Mean mixed temperature:

$$\frac{\int_0^h \theta u dy}{\int_0^h u dy} = 1 \tag{40}$$

(vi) Nusselt number on the free surface: $\left.\frac{d\theta}{dy}\right|_{y=h} = \rightarrow 0$ as terms of $o(h^3)$ are neglected.

5. Results and discussion

In order to determine the physical nature of the problem, the results discussed in the previous section are presented through graphs from Figs. 2–13. The effect of various physical parameters such as α, h, M, and Pr on the convective flow, the numerical values of velocity and temperature fields when the temperatures on the fixed bottom and on the free surface are obtained. In addition to this, the flow rate, mean velocity, mean temperature, mean mixed temperature in the flow region and the Nusselt number on the free surface have been discussed. In Fig. 2 velocity profiles are displayed with the variations in porosity parameter α. From this figure, it is noticed that the velocity of the fluid increases from bottom to free surface with the increase in the values of the porosity parameter α. physically, an increase in the permeability of porous medium leads the rise in the flow of fluid through it. When the holes of the porous medium become large, the resistance of the medium may be neglected. So that velocity at the insulated bottom is observed to be zero and gradually it increases as it reaches the free surface and attains a maximum there in.

In Figs. 3–5, mean velocity profiles are displayed. From these figures it is observed that the mean velocity of the fluid increases with the increasing values of h and decreases with the increase in the values of the porosity parameter α and magnetic parameter M as expected. This is due to the application of a magnetic field to an electrically conducting fluid produces a dragline force which causes reduction in the fluid velocity. For the cases of small α and large α, it is clear that the mean velocity increases with the increasing pressure gradient c_1 and decreases for the increasing values of α. In the case of large depths that is large 'h' mean velocity decreases with the increasing values of the porosity parameter α and as well as magnetic parameter M. In the absence of magnetic parameter these results are in good agreement with the results of Moinuddin and Pattabhiramacharyulu [16]. Effects of porosity parameter α and magnetic parameter M are studied on temperature through Figs. 6 and 7. From these figures it is noticed that the temperature of the fluid decreases with the increase in the values of the porosity parameter α. This is due to the presence of Joule heating that reduces the temperature because of free expansion which causes a decrease in temperature. In the case of small α temperature of the flow region decreases with the increasing values of α and for the case of large α temperature reaches unity. In the case of large depth that is large 'h' temperature of the fluid increases with increasing values of α and for shallow depths that is small 'h', the temperature of the fluid decreases with the increasing α's above $y = 0.07$ and remains constant for $y < 0.07$ for different porosity parameters α. But reverse effect is observed in the case of magnetic parameter M, because the magnetic field retards the velocity of fluid and therefore temperature of the fluid is higher.

Figs. 8 and 9, depict that effects of Prandtl number Pr and magnetic parameter M on mean temperature distribution. It is observed that the mean temperature decreases as the Prandtl number 'Pr' increases. For smaller values of Prandtl number 'Pr' the mean temperature increases as 'Pr' increases above $y = 0.7$. But this trend reverses for $y < 0.7$. For larger values of α mean temperature reaches unity the boundary value on the free surface. In the case of large depths that is large 'h' mean temperature decreases as 'Pr' increases. A similar effect is noticed in the case of magnetic parameter M. Mean mixed temperature distributions are presented in Figs. 10 and 11, with the variations in magnetic parameter M and porosity

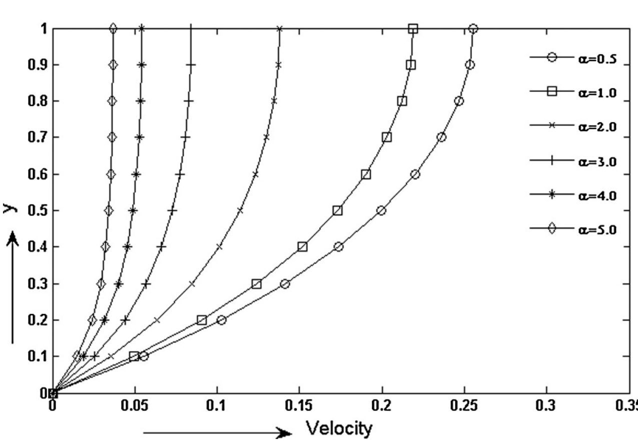

Figure 2 Effects of α on velocity.

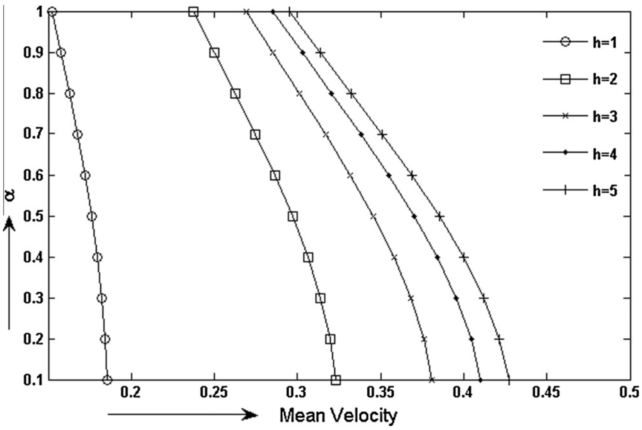

Figure 3 Effects of h on mean velocity.

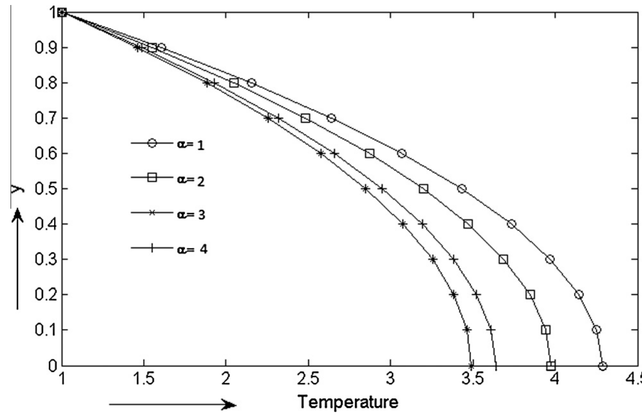

Figure 6 Effects of α on temperature.

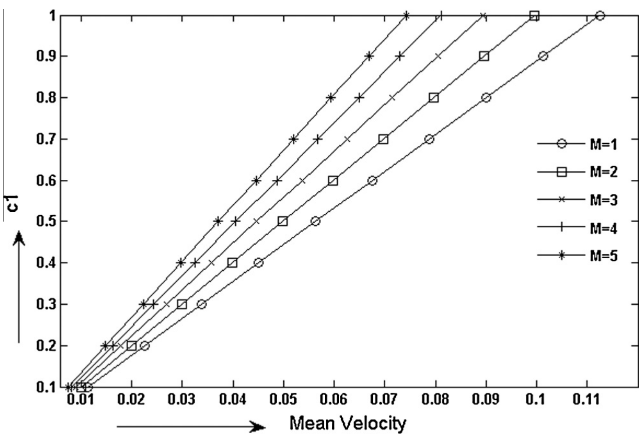

Figure 4 Effects of M on mean velocity.

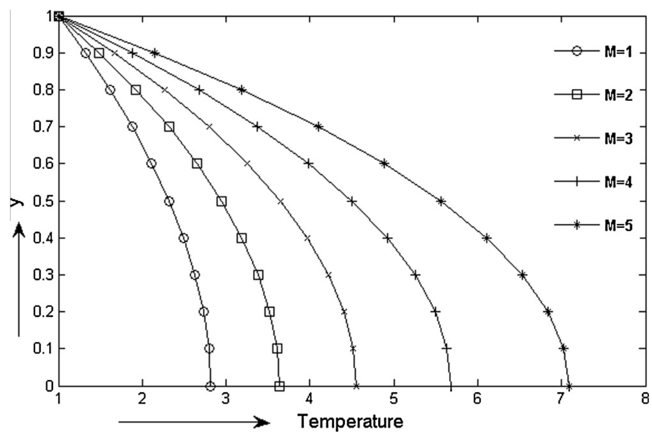

Figure 7 Effects of M on temperature.

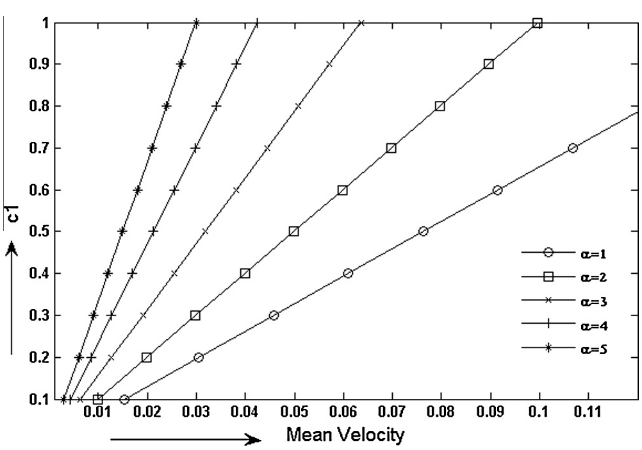

Figure 5 Effects of α on mean velocity.

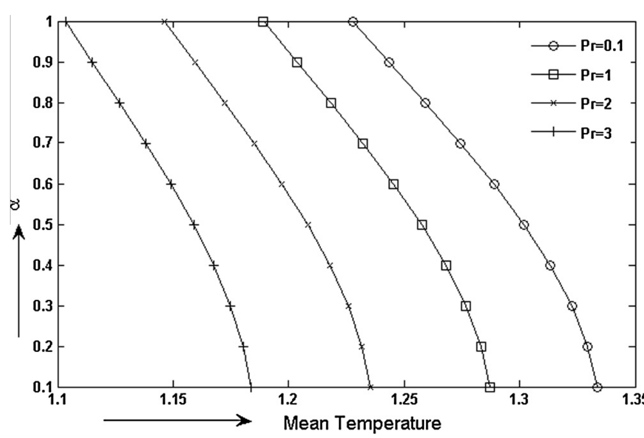

Figure 8 Effects of Pr on mean temperature.

parameter α. From these figures, it is noticed that mean mixed temperature decreases with the increase in the values of the porosity parameter α, and also magnetic parameter M. In the case of small α, mean temperature increases with the increase in the Prandtl number 'Pr'. In the case of large α, mean mixed temperature increases with the increase in the values of α. In the case of large depth that is large 'h' mean mixed temperature decrease with the increasing Prandtl number 'Pr' and as well as magnetic parameter M. From Figs. 12 and 13, it is observed that the heat transfer rate increases with the increase in the values of the Prandtl number 'Pr'. In the case of small α, Nusselt number increases with increase in the values of the Prandtl number 'Pr' and decreases for the increasing values of α. In the case of large depths that are large 'h' the rate of heat transfer Nusselt number increases with the increasing values of the Prandtl number 'Pr'.

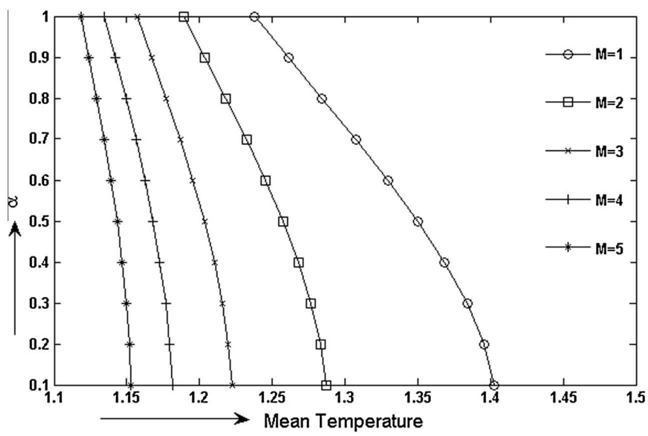

Figure 9 Effects of M on mean temperature.

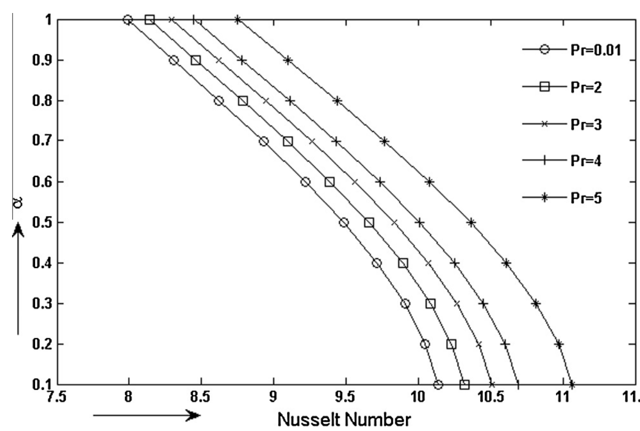

Figure 12 Effects of α on Nusselt number.

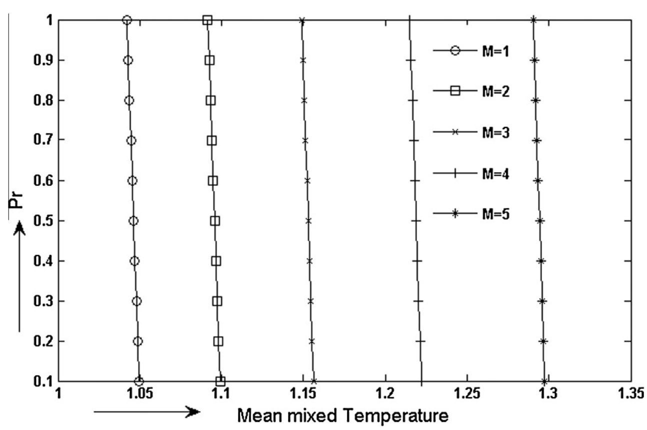

Figure 10 Effects of M on mean mixed temperature.

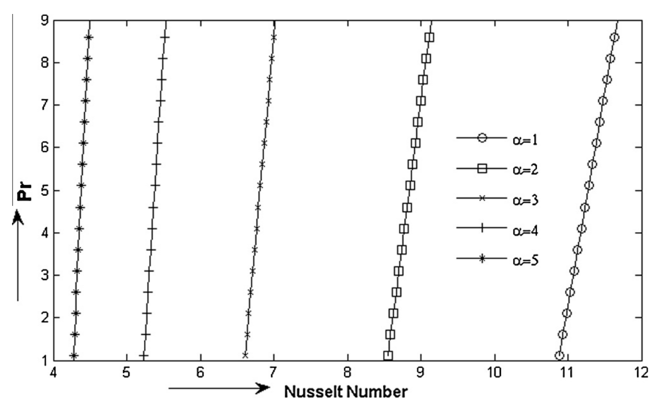

Figure 13 Effect of Pr on Nusselt number.

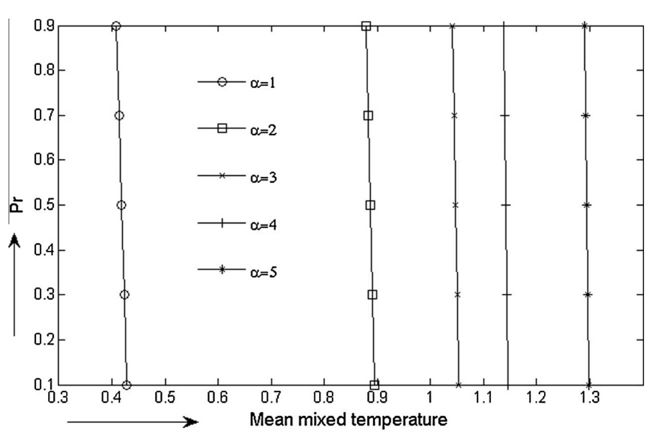

Figure 11 Effects of α on mean mixed temperature.

6. Concluding remarks

We have considered, the steady forced convective MHD flow of a viscous liquid of viscosity μ and of finite depth H through a porous medium of porosity coefficient 'K^*' over a fixed impermeable, thermally insulated bottom. The flow is generated by a constant horizontal pressure gradient parallel to the fixed bottom. The momentum equation considered is the generalized Darcy's law proposed by Yama Moto and Iwamura [23], which takes into account the convective acceleration and the Newtonian viscous stresses in addition to the classical Darcy force. The basic equations of momentum and energy are solved to give exact expressions for velocity and temperature distributions. Employing the flow rate, mean velocity, mean temperature, mean mixed temperature and the Nusselt number on the free surface have been obtained in the following cases (i) high porosity (ii) low porosity and (iii) Large depths (large H) and (iv) Shallow depths (small H). In this study the following conclusions are made.

a. Velocity and mean velocity distributions are observed to decrease with the increase in magnetic parameter M, whereas it shows reverse effect in the case of h and porosity parameter α.
b. Temperature distribution and mean temperature distributions increase with the increasing values of M and Pr, but reverse effect is seen in the case of α.
c. Mean temperature and mean mixed temperature distributions decrease with an increase in M, α and as well as Pr.
d. Nusselt number increases with increasing values of Pr and decreases with an increase in α.

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