Available online at www.sciencedirect.com
SCIENCE DIRECT®
Physics Letters B 570 (2003) 260–264
PHYSICS LETTERS B
www.elsevier.com/locate/npe

Universality of the linear potential in effective models for the low energy QCD coupled with the dilaton field

Andrzej Wereszczyński

Institute of Physics, Jagiellonian University, Reymonta 4, 30-059 Krakow, Poland

Received 12 June 2003; received in revised form 24 July 2003; accepted 25 July 2003

Editor: P.V. Landshoff

Abstract

QCD motivated effective models coupled with the cosmological dilaton field are analyzed. It is shown that all models possess confining solutions with the linear potential of confinement even thought such solutions are not observed in the original effective theory. In case of the Pagels–Tomboulis model analytical solutions are explicit found.

1. Introduction

The non-perturbative behavior of the quantum chromodynamics in the low energy sector is one of the main problem of the temporary theoretical physics. In spite of the many efforts such effects like the confinement of quarks and gluons are not satisfactorily understood. Because of the fact that we cannot use the well-known perturbative methods we have to find a new way analyzing the non-perturbative features. In fact, there are several methods of investigating of the low energy QCD. Roughly speaking, they can be divided into two groups. Namely, the lattice models and the effective Ginzburg–Landau-like models. The second group contains for example such popular models like the dual superconductor model [1], the color dielectric model [2] and the stochastic vacuum model [3]. In the present Letter we will focus on the second class of the effective models given by the following ansatz for the Lagrangian density in the Euclidean space–time [4]

$$L_{\text{eff}} = -\frac{1}{4}\frac{\mathcal{F}^{a\mu\nu}\mathcal{F}^{a}_{\mu\nu}}{\bar{g}^2(t)}, \tag{1}$$

where

$$t = \ln\frac{\mathcal{F}}{\mu^4} \tag{2}$$

and $\bar{g}$ is the running coupling constant and μ is a dimensional constant. Here $\mathcal{F} = \frac{1}{2}F^{a}_{\mu\nu}F^{a\mu\nu}$ and $F^{a}_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\epsilon^{abc}A^b_\mu A^c_\nu$ is the standard field tensor depends on the $SU(2)$ gauge fields. The model (1) was primary invented to reproduce, at the classical level, the trace anomaly known from the quantum chromodynamics [4]. Indead, one can find that trace of the energy–momentum tensor corresponding to (1) reads

$$T^{\mu}_{\mu} = \frac{\beta(\bar{g})}{2\bar{g}}\frac{\mathcal{F}^{a\mu\nu}\mathcal{F}^{a}_{\mu\nu}}{\bar{g}^2(t)}. \tag{3}$$

This is in agreement with result obtained in the full quantum theory. Particular examples of the La-

E-mail address: wereszcz@th.if.uj.edu.pl (A. Wereszczyński).

0370-2693 © 2003 Published by Elsevier B.V. Open access under CC BY license.
doi:10.1016/j.physletb.2003.07.054

grangian (1) are obtained by calculating $\bar{g}(t)$ from:

$$t = \int_{g_0}^{\bar{g}(t)} \frac{dg}{\beta(g)}. \tag{4}$$

For example, the usual perturbative one-loop β-function $\beta = -\frac{11}{32\pi} g^3$ gives the famous Savvidy–Adler model [5–7]. For $\beta = -\delta g$ one gets the Pagels–Tomboulis model [4]. Both models provide confinement of quarks and can be treated as first approximation to the true effective model. Here confinement is understood in the following way. Field configurations generated by an external electric source possess infinite energy due to the long range behavior of the fields. However, a dipole-like source with zero total charge has finite energy. Because of that the physical spectrum of the theory consists of the dipoles—mesons whereas charge solutions are excluded from it.

The main aim of this Letter is to analyze such effective models together with the cosmological dilaton [8]. There are many papers where classical solutions of the dilaton Yang–Mills theory has been considered [9–11]. However, because the classical Yang–Mills Lagrangian seems to have little to do with physics described by the low energy QCD it is probably more correct to investigate model where Yang–Mills part is substituted by (1).

On the other hand, the non-minimal coupling between scalar and gauge fields has been often used to reproduce the non-trivial quantum phenomena in the framework of classical field theories (c.f. color dielectric model). The scalar field represents unusual properties of the non-perturbative vacuum in which the gauge fields propagate. It can suggest that the correct effective model should contain more than only gauge fields. It is in agreement with lattice gauge theory where, in the low energy limit, many additional effective fields (scalar, vector, tensor) appear [12].

2. The linear potential

Let us now couple an effective model for the low energy QCD with the cosmological scalar field, i.e., dilaton. Then the action, in the Minkowski space–time, takes the following form:

$$L_{\text{gen}} = -\frac{e^{b\frac{\phi}{\Lambda}}}{4} \frac{F^{a\mu\nu} F^a_{\mu\nu}}{\bar{g}^2(t)} + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi, \tag{5}$$

where t is given by

$$t = \frac{1}{2} \ln \frac{F^2}{\Lambda^8}$$

and b is a dimensionless constant. Here Λ is a dimensional constant and F corresponds to $\mathcal{F}$ in the Minkowski space–time. For convenience we introduce a new function

$$f(F) := \frac{1}{\bar{g}^2(t)}. \tag{6}$$

The pertinent equations of motion read:

$$D_\mu \left[e^{b\phi/\Lambda} \frac{\partial (f(F)F)}{\partial F} F^{a\mu\nu} \right] = j^{a\nu}, \tag{7}$$

$$\partial_\mu \partial^\mu \phi = \frac{b}{2\Lambda} e^{b\phi/\Lambda} f(F) F. \tag{8}$$

In the present Letter we are mainly interested in analyzing of the field configurations generated by external electric static sources. Due to that the external current is

$$j^{a\mu} = 4\pi q \delta(r) \delta^{0\mu} \delta^{a3}, \tag{9}$$

where q is an external charge. We would like to notice that restriction to the Abelian current is not essential. Using the results presented in [11] one can find the solution of these equations in the general non-Abelian case as well. However, they differ from the Abelian solutions only by a multiplicative color-dependent constant. The dependence on spatial coordinates remains unchanged. One could argue that the small difference between Abelian and non-Abelian case is in the contradiction with the well-known fact that the non-perturbative features of QCD originate in the non-Abelian character of the gauge fields. But one should remember that there is no simple correspondence between the quantum gauge fields in QCD and the fields in the presented classical model. The non-Abelian character of the original quantum theory is implemented by taking into account the QCD motivated running coupling constant. The particular form of the Lie group on which the classical gauge fields in the effective model are based seems to play less important role (at least in the problem of the confinement of external sources).

Because of that we will consider only Abelian degrees of freedom. For example, one can set $A_\mu^a = A_\mu \delta^{a3}$. The equations of motion can be rewritten as:

$$\left[r^2 e^{b\phi/\Lambda} \frac{\partial(f(E^2)E^2)}{\partial E^2} E\right]' = 4\pi q \delta(r), \tag{10}$$

$$\nabla_r^2 \phi = -\frac{b}{2\Lambda} f\left(E^2\right) E^2 e^{b\phi/\Lambda}. \tag{11}$$

Here, we have assumed the spherical symmetry of the problem $\vec{E} = E\hat{r}$. The prime denotes differentiation in respect to r. The solutions take the following form:

$$\frac{\phi(r)}{\Lambda} = -\frac{2}{b} \ln r\Lambda + \phi_0, \tag{12}$$

$$E = E_0 \Lambda^2 = \text{const}. \tag{13}$$

Where ϕ_0 and E_0 are yet to be determined. This corresponds to linear electric potential

$$U = E_0 r \Lambda^2. \tag{14}$$

Inserting these solutions into the field equations we can obtain the algebraic equations for the constants ϕ_0 and E_0

$$e^{b\phi_0} \frac{\partial f(E^2)E^2}{\partial E^2}\bigg|_{E=E_0} E_0 = q,$$

$$e^{b\phi_0} f\left(E_0^2\right) E_0^2 = \frac{4}{b^2}. \tag{15}$$

After eliminating ϕ_0 we find that the constant E_0 is given by the following (in general non-linear) algebraic equation

$$E \frac{\partial}{\partial E^2} \ln\left(f\left(E^2\right)E^2\right)\bigg|_{E=E_0} = \frac{qb^2}{4}. \tag{16}$$

Unfortunately, we are not able to solve this equation in the general case. However, as it is shown below, for particular forms of the function f one can find the value of constants E_0 and ϕ_0.

Let us analyze the energy of these solutions. The energy component of the energy–momentum tensor corresponding to the Lagrangian (1) has the following form:

$$T_{00} = \left[e^{b\phi/\Lambda}\left(E^2 \frac{\partial(f(F)F)}{\partial F} - \frac{1}{4} f(F)F\right) + \frac{1}{2}(\partial_0\phi)^2 + \frac{1}{2}(\partial_i\phi)^2\right]. \tag{17}$$

For the static, pure electric solutions we obtain

$$T_{00} = \left[e^{b\phi/\Lambda}\left(E^2 \frac{\partial(f(E^2)E^2)}{\partial E^2} - \frac{1}{2} f(E^2)E^2\right) + \frac{1}{2}(\partial_i\phi)^2\right]. \tag{18}$$

Then after substituting the solutions (12), (13) and using relations (15) one gets

$$T_{00} = \frac{\Lambda^2}{r^2}\left[e^{b\phi_0/\Lambda}\left(E_0^2 \frac{\partial(f(E^2)E^2)}{\partial E^2}\bigg|_{E=E_0} - \frac{1}{2} f\left(E_0^2\right)E_0^2\right) + \frac{2}{b^2}\right] = \frac{\Lambda^2}{r^2} E_0 q. \tag{19}$$

The energy stored in the ball with radius R around the external static, point-like electric charge diverges linearly with R

$$E(R) = \int_0^R T_{00}\, d^3r = 4\pi E_0 q R \Lambda^2. \tag{20}$$

The field configurations generated by external electric sources have infinite total energy. In contradictory to the Maxwell theory energy diverges due to the long range behavior of the fields. In that sense electric charges are confined. It should be underlined that appearance of the linear potential (constant energy density) is independent on the form of the original effective Lagrangian. One can find many examples of physically interesting models which originally do not have linear potential. However, because of the interaction with the dilaton field, the potential becomes linear.

3. An example

In order to find explicit solutions one has to choose a particular form of the function f. Here we will take the Pagels–Tomboulis model [4]

$$L = -\frac{1}{4}\left(\frac{F_{\mu\nu}^a F^{a\mu\nu}}{2\Lambda^4}\right)^{2\delta} F_{\mu\nu}^a F^{a\mu\nu}, \tag{21}$$

where δ is a dimensionless parameter. It was shown that the Pagels–Tomboulis model can serve as a candidate for the low energy effective action for $\delta \in (\frac{1}{4}, \infty)$. In particular, it assures confinement of electric charges and gives the following confining potential

[13]

$$U_{\mathrm{PT}} = a_0 |q|^{\frac{2+4\delta}{1+4\delta}} \Lambda^{\frac{8\delta}{1+4\delta}} r^{\frac{4\delta-1}{4\delta+1}}, \tag{22}$$

which, after fitting the values of the parameter, is in agreement with the experimental data. Here a_0 is a numerical constant. One can easily see that the linear potential is achieved only in the limit $\delta \to \infty$. Obviously, such a limit cannot be implemented in the Lagrangian. That means that it is not possible to realize the linear confinement in the Pagels–Tomboulis model.

Let us turn to the model with the dilaton field. Then the Lagrangian density reads

$$L = -\frac{e^{b\phi/\Lambda}}{4} \left(\frac{F^a_{\mu\nu} F^{a\mu\nu}}{2\Lambda^4} \right)^{2\delta} F^a_{\mu\nu} F^{a\mu\nu} + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi. \tag{23}$$

Then equations of motion are:

$$D_\mu \left[(2\delta + 1) e^{b\phi/\Lambda} \left(\frac{F}{\Lambda^4} \right)^{2\delta} F^{a\mu\nu} \right] = j^{a\nu}, \tag{24}$$

$$\partial_\mu \partial^\mu \phi = \frac{b}{2\Lambda} e^{b\phi/\Lambda} \left(\frac{F}{\Lambda^4} \right)^{2\delta} F. \tag{25}$$

Similar as in the previous section we will consider an Abelian static and point-like external electric charge. Thus the field equations take the form:

$$\left[(2\delta + 1) r^2 e^{b\phi/\Lambda} \frac{E^{4\delta+1}}{\Lambda^{8\delta}} \right]' = q \delta(r), \tag{26}$$

$$\nabla_r^2 \phi = -\frac{b}{2\Lambda} \frac{E^{4\delta+2}}{\Lambda^{8\delta}} e^{b\phi/\Lambda}. \tag{27}$$

One can solve Eq. (26) and find the electric field

$$E = \Lambda^2 \left(\frac{q}{(1+2\delta) r^2 \Lambda^2} \right)^{\frac{1}{1+4\delta}} e^{-\frac{b}{1+4\delta} \frac{\phi}{\Lambda}}. \tag{28}$$

Then Eq. (27) reads

$$\nabla_r^2 \phi = -\Lambda^3 \frac{b}{2} \left(\frac{q^2}{(1+2\delta)^2 r^4 \Lambda^4} \right)^{\frac{1+2\delta}{1+4\delta}} e^{-\frac{b}{1+4\delta} \frac{\phi}{\Lambda}}. \tag{29}$$

After some calculation one can find that the solution is

$$\frac{\phi(r)}{\Lambda} = -\frac{2}{b} \ln r\Lambda + \phi_0, \tag{30}$$

where the constant

$$\phi_0 = -\frac{1+4\delta}{b} \ln \left[\frac{4}{b^2} \left(\frac{1+2\delta}{q} \right)^{2\frac{1+2\delta}{1+4\delta}} \right]. \tag{31}$$

The corresponding electric field is given in the following form:

$$E(r) = \frac{4(1+2\delta)}{qb^2} \Lambda^2. \tag{32}$$

Finally, we obtain linear confining potential

$$U = \frac{4(1+2\delta)}{qb^2} r\Lambda^2. \tag{33}$$

As it was said before it is really remarkable that the potential takes the linear form for all δ. The dependence on the parameter δ is, unlikely the original Pagels–Tomboulis model, visible only in the 'string tension'. The functional dependence is always linear.

One can notice that the case $\delta = 0$ is a little bit special. Then the additional symmetry appears in the equations of motion. Namely, if one defines a new variable $x = \frac{1}{r}$ then the translation $x \to x + x_0$ remains equations unchanged. Because of that we find a whole family of the solutions which depends on the translation parameter x_0 [11]. These solutions have finite energy. One should remember that this effect is present only for $\delta = 0$ and does not occur in the general case.

4. Conclusions

In the present Letter we have shown that the cosmological dilaton field coupled with the effective model (1), originally dependent only on the $SU(2)$ gauge fields, provides the linear confinement of the electric charges. It is striking that this behavior is observed for all models based on gauge fields with the $U(1)$ subgroup. Even though the original gauge model does not possess confining solutions then interaction with the dilaton causes that the electric charges are confined. The confining potential is always linear and only string tension is model dependent. In other words, the particular form of the effective model is not important if one would like to model the confinement of the quarks. The essential is the form of the coupling between the scalar field and the gauge field. The linear confinement is not observed if one takes, for example, power-like coupling in stead of exponential one [14,15]. Then the potential strongly depends on the particular form of the gauge part of the model. Only exponential coupling gives indifferently the same functional form of the potential.

Of course, this result can be applied not only to QCD effective models but to all model with at least $U(1)$ gauge field (for instance, the non-linear electrodynamics).

There are several directions in which the present work can be continued. First of all, one can analyze more general than the tree level approximated dilaton coupling [16]. It would be interesting to know how this more realistic interaction inflects on the electric solutions of the QCD induced effective models. Similar, one should consider the non-perturbative dilaton potential [17] and/or the mass term [18]. Moreover, one could also ask about another cosmological scalar field, i.e., the modulus field [16,19]. However, in our opinion the most important problem is to analyze dipole sources. As we have mentioned it before, disappearance of the electric charge from physical spectrum is not sufficient to have confinement in the theory. One has to show that fields generated by a dipole-like source possess finite energy. It would be interesting to know whether the functional form of the total energy is also in this case independent on the gauge part of the model. We plan to address this last problem in out next paper.

Acknowledgements

We would like to thank Professor H. Arodź for reading the manuscript and many helpful remarks.

References

[1] M. Baker, J.S. Ball, F. Zachariasen, Phys. Rep. 209 (1991) 73;
M. Baker, N. Brambilla, H.G. Dosch, A. Vairo, Phys. Rev. D 58 (1998) 034010.
[2] R. Friedberg, T.D. Lee, Phys. Rev. D 15 (1977) 1694;
R. Friedberg, T.D. Lee, Phys. Rev. D 18 (1978) 2623.
[3] H.G. Dosch, Y.A. Simonov, Phys. Lett. B 205 (1988) 339;
Y.A. Simonov, Nucl. Phys. B 307 (1988) 512.
[4] H. Pagels, E. Tomboulis, Nucl. Phys. B 143 (1978) 485.
[5] G. Matinyan, G.K. Savvidy, Nucl. Phys. B 134 (1978) 539.
[6] S.L. Adler, Phys. Rev. D 23 (1981) 2905;
S.L. Adler, Phys. Lett. B 110 (1981) 302;
S.L. Adler, T. Piran, Phys. Lett. B 113 (1982) 405;
S.L. Adler, T. Piran, Phys. Lett. B 117 (1982) 91.
[7] P.M. Fishbane, S. Gasiorowicz, P. Kaus, Phys. Rev. D 36 (1987) 251;
P.M. Fishbane, S. Gasiorowicz, P. Kaus, Phys. Rev. D 43 (1991) 933;
R.R. Mendel, et al., Phys. Rev. D 30 (1984) 621;
R.R. Mendel, et al., Phys. Rev. D 33 (1986) 2666;
R.R. Mendel, et al., Phys. Rev. D 40 (1989) 3708;
R.R. Mendel, et al., Phys. Rev. D 42 (1990) 911.
[8] G.W. Gibbons, K. Maeda, Nucl. Phys. B 298 (1988) 741;
D. Garfinkle, G.T. Horowitz, A. Strominger, Phys. Rev. D 43 (1991) 3140;
G.T. Horowitz, The dark side of string theory: black holes and black strings, in: Proceedings of the 1992 Trieste Spring School on String Theory and Quantum Gravity.
[9] G. Lavrelashvili, D. Maison, Phys. Lett. B 295 (1992) 67.
[10] P. Bizon, Phys. Rev. D 47 (1993) 1656.
[11] R. Dick, Phys. Lett. B 397 (1996) 193;
R. Dick, Phys. Lett. B 409 (1997) 321.
[12] H.B. Nielsen, A. Patcos, Nucl. Phys. B 195 (1982) 137;
H.-J. Pirner, J. Wroldsen, M. Ilgenfritz, Nucl. Phys. B 294 (1987) 905;
J.-F. Mathiot, G. Chanfray, H.-J. Pirner, Nucl. Phys. A 500 (1989) 605;
H. Arodz, H.-J. Pirner, Acta Phys. Pol. B 30 (1999) 3895.
[13] H. Arodź, M. Ślusarczyk, A. Wereszczyński, Acta Phys. Pol. B 32 (2001) 2155.
[14] R. Dick, L.P. Fulcher, Eur. Phys. J. C 9 (1999) 271;
R. Dick, Eur. Phys. J. C 6 (1999) 701.
[15] M. Ślusarczyk, A. Wereszczyński, hep-ph/0307213, Eur. Phys. J. C, in press;
M. Ślusarczyk, A. Wereszczyński, Eur. Phys. J. C 28 (2003) 151;
M. Ślusarczyk, A. Wereszczyński, Eur. Phys. J. C 23 (2002) 145;
M. Ślusarczyk, A. Wereszczyński, Acta Phys. Pol. B 32 (2001) 2911.
[16] M. Cvetič, A.A. Tseytlin, Nucl. Phys. B 416 (1994) 137.
[17] J. Garcia-Bellido, M. Quirós, Nucl. Phys. B 385 (1992) 558;
N.R. Stewart, Mod. Phys. Lett. A 7 (1992) 983;
R. Brustein, P. Steinhardt, Phys. Lett. B 302 (1993) 196.
[18] J.H. Horne, G.T. Horowitz, Nucl. Phys. B 399 (1993) 169.
[19] E. Witten, Phys. Lett. B 155 (1985) 151.