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Boundedness and stability for Cohen–Grossberg neural network with time-varying delays ☆

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Abstract

In this paper, a model is considered to describe the dynamics of Cohen–Grossberg neural network with variable coefficients and time-varying delays. Uniformly ultimate boundedness and uniform boundedness are studied for the model by utilizing the Hardy inequality. Combining with the Halanay inequality and the Lyapunov functional method, some new sufficient conditions are derived for the model to be globally exponentially stable. The activation functions are not assumed to be differentiable or strictly increasing. Moreover, no assumption on the symmetry of the connection matrices is necessary. These criteria are important in signal processing and the design of networks.

Keywords: Ultimate boundedness; Lyapunov functional; Exponential stability; Hardy inequality; Halanay inequality; Cohen–Grossberg neural network

1. Introduction

In the past few decades, neural networks such as Hopfield neural network [1], cellular neural network [2,3], and bi-directional associative memory neural network [4–6,10,11,33] have attracted the attention of many mathematicians, physicists, and computer scientists

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(see [7–17]) due to their wide range of applications in, for example, pattern recognition, associative memory, and combinatorial optimization. Among them, the Cohen–Grossberg neural network [18] is an important one, which can be described by the system of ordinary differential equations

$$\frac{dx_i(t)}{dt} = -a_i\big(x_i(t)\big)\left[b_i\big(x_i(t)\big) - \sum_{j=1}^{n} c_{ij} g_j\big(x_j(t)\big)\right], \quad i = 1, 2, \ldots, n, \tag{1}$$

in which $n \geqslant 2$ is the number of neurons in the network; $x_i(t)$ denotes the state variable of the ith neuron at time t; $g_j(x_j(t))$ denotes the activation function of the jth neuron at time t; the feedback matrix $C = (c_{ij})_{n\times n}$ indicates the strength of the neuron interconnections within the network; $a_i(x_i(t))$ represents an amplification function; $b_i(x_i(t))$ is an appropriately behaved function such that the solutions of model (1) remain bounded. This model was firstly proposed and studied by Cohen and Grossberg (1983), it includes a lot of models from evolutionary theory, population biology and neurobiology. It should be pointed out that the Cohen–Grossberg neural network encompasses the Hopfield neural network [1] as a special case (when $a_i(x_i(t)) \equiv 1$, $b_i(x_i(t)) = \frac{C_i}{R_i} x_i(t) + I_i$), the latter could be described as

$$C_i \frac{dx_i(t)}{dt} = -\frac{x_i(t)}{R_i} + \sum_{j=1}^{n} c_{ij} g_j\big(x_j(t)\big) + I_i, \quad i = 1, 2, \ldots, n, \tag{2}$$

where C_i and R_i are positive constants representing the neuron amplifier input capacitance and resistance, respectively; I_i is the constant input from outside of the network.

In fact, due to the finite speeds of the switching and transmission of signals in a network, time delays do exist in a working network and thus should be incorporated into the model equations of the network. It was observed both experimentally and numerically in [19] that time delay could induce instability, causing sustained oscillations which may be harmful to a system. For the Cohen–Grossberg model (1), Ye et al. [20] also introduced delays by considering the following system of delayed differential equations:

$$\frac{dx_i(t)}{dt} = -a_i\big(x_i(t)\big)\left[b_i\big(x_i(t)\big) - \sum_{k=0}^{K}\sum_{j=1}^{n} c_{ij}^{k} g_j\big(x_j(t-\tau_k)\big)\right], \quad i = 1, 2, \ldots, n, \tag{3}$$

where the time delays τ_k $(k = 0, 1, \ldots, K)$ are arranged such that $0 = \tau_0 < \tau_1 < \cdots < \tau_K$. Further studies were taken by Wang and Zou [21,22], Lu and Chen [23], Chen and Rong [24] about the following model:

$$\frac{dx_i(t)}{dt} = -a_i\big(x_i(t)\big)\left[b_i\big(x_i(t)\big) - \sum_{j=1}^{n} c_{ij} g_j\big(x_j(t)\big) - \sum_{j=1}^{n} d_{ij} g_j\big(x_j(t-\tau_{ij})\big) + I_i\right], \tag{4}$$

in which $D = (d_{ij})_{n\times n}$ indicates the strength of the neuron interconnections within the network with time delay parameters τ_{ij}. In [22,24], several sufficient conditions were obtained to ensure model (4) to be asymptotically stable. A set of conditions ensuring global exponential stability of system (4) were derived in [21] when $c_{ij} \equiv 0$ and $d_{ij} \equiv 0$, respectively. And, by the property of Lyapunov diagonal stable matrix, absolutely global stability was studied in [23] for model (4) when $d_{ij} \equiv 0$. A more generalized model was studied by Hwang in [25],

$$\frac{dx_i(t)}{dt} = -a_i\big(x_i(t)\big)\Bigg[b_i\big(x_i(t)\big) - \sum_{j=1}^{n} c_{ij} g_j\big(x_j(t)\big) - \sum_{j=1}^{n} d_{ij} g_j\big(x_j\big(t-\tau_{ij}(t)\big)\big) + I_i\Bigg] \tag{5}$$

and exponential stability result was obtained.

The purpose of this paper is to study the dynamic behavior of the generalized Cohen–Grossberg neural network with variable coefficients and time-varying delays. The organization of this paper is as follows. In Section 2, we give a model description and some prerequisite results. In Section 3, boundedness of the model will be discussed. And the conditions ensuring the exponential stability of the Cohen–Grossberg neural network are obtained in Section 4. In Section 5, some examples and their numerical simulations are given to confirm and illustrate the analysis. Finally, in Section 6, we give concluding remarks of the derived results.

2. Model description and preliminaries

In this paper, we investigate the following delayed dynamical systems:

$$\frac{dx_i(t)}{dt} = -a_i\big(x_i(t)\big)\Bigg[b_i\big(x_i(t)\big) - \sum_{j=1}^{n} c_{ij}(t) g_j\big(x_j(t)\big) - \sum_{j=1}^{n} d_{ij}(t) g_j\big(x_j\big(t-\tau_{ij}(t)\big)\big) + I_i(t)\Bigg], \tag{6}$$

where $i = 1, 2, \ldots, n$; $0 \leqslant \tau_{ij}(t) \leqslant \tau$ and $\sup_{t\in[-\tau,+\infty)} \dot{\tau}_{ij}(t) < 1$ (where $\dot{\tau}_{ij}(t)$ represents the derivative of $\tau_{ij}(t)$); $c_{ij}(t)$, $d_{ij}(t)$ and $I_i(t)$ are continuous and bounded functions defined on $[-\tau, +\infty)$.

Define $x_t(s) = x(t+s)$, $s \in [-\tau, 0]$, $t \geqslant 0$. Let $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T$, its norm is defined as

$$\|x_t\|_r = \sup_{-\tau\leqslant s\leqslant 0} \big|x(t+s)\big|_r, \quad \text{where } \big|x(t+s)\big|_r = \Bigg[\sum_{i=1}^{n} \big|x_i(t+s)\big|^r\Bigg]^{\frac{1}{r}} \text{ and } r \geqslant 1. \tag{7}$$

Assume that the nonlinear model (6) has initial values of the type

$$x_i(t) = \varphi_i(t), \quad t \in [-\tau, 0],$$

in which $\varphi_i(t)$ $(i = 1, 2, \ldots, n)$ are continuous functions. By the fundamental theory of functional differential equations [29], model (6) has a unique solution $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T$ satisfying the initial condition in (7).

To establish the main results of the model given in (6), some of the following assumptions will apply:

(H_1) Each function $a_i(u)$ is bounded, positive and locally Lipschitz continuous. Furthermore, $0 < \underline{\alpha}_i \leqslant a_i(u) \leqslant \overline{\alpha}_i < +\infty$ for all $u \in \mathbb{R}$ and $i = 1, 2, \ldots, n$.
(H_2) Each function $b_i(u)$ is locally Lipschitz continuous and there exists $\beta_i > 0$ such that $ub_i(u) \geqslant \beta_i u^2$ for $u \in \mathbb{R}$, $i = 1, 2, \ldots, n$.
(H_2') Each function $b_i(u) \in C^1(\mathbb{R}, \mathbb{R})$ and $\dot{b}_i(u) \geqslant \beta_i > 0$; both $b_i(\cdot)$ and $b_i^{-1}(\cdot)$ are locally Lipschitz continuous.
(H_3) Each function $g_j : \mathbb{R} \to \mathbb{R}$ satisfies the Lipschitz condition with a Lipschitz constant $L_j > 0$, i.e., $|g_j(u) - g_j(v)| \leqslant L_j |u - v|$ for all $u, v \in \mathbb{R}$, $j = 1, 2, \ldots, n$.
(H_3') Each function $g_j(\cdot)$ is bounded and satisfies the Lipschitz condition with a Lipschitz constant $L_j > 0$.

Definition 1. System (6) is uniformly bounded if, for any constant $\delta > 0$, there is $B = B(\delta) > 0$ such that, for all $t_0 \in [0, +\infty)$, φ, and $\|\varphi|_r < \delta$, we have $|x(t, t_0, \varphi)|_r < B$ for all $t \geqslant t_0$.

Definition 2. System (6) is uniformly ultimately bounded if there is a $B > 0$ such that, for any $\delta > 0$, there is a constant $\tilde{t} = \tilde{t}(\delta) > 0$ such that $|x(t, t_0, \varphi)|_r < B$ for $t \geqslant t_0 + \tilde{t}$ for all $t_0 \in [0, +\infty)$, $\|\varphi\|_r < \delta$.

Under the assumptions (H_1), (H_2') and (H_3'), we know from [21] that system (5) has an equilibrium point $x^* = (x_1^*, x_2^*, \ldots, x_n^*)^T$. Let $y(t) = x(t) - x^*$, substitute $x(t) = y(t) + x^*$ in (5) and we have

$$\begin{aligned}\frac{dy_i(t)}{dt} = -a_i\big(y_i(t) + x_i^*\big)\Bigg[& b_i\big(y_i(t) + x_i^*\big) - b_i\big(x_i^*\big) \\ & - \sum_{j=1}^n c_{ij}\big(g_j\big(y_j(t) + x_j^*\big) - g_j\big(x_j^*\big)\big) \\ & - \sum_{j=1}^n d_{ij}\big(g_j\big(y_j\big(t - \tau_{ij}(t)\big) + x_j^*\big) - g_j\big(x_j^*\big)\big)\Bigg]. \end{aligned} \tag{8}$$

Denote $A_i(y_i(t)) = a_i(y_i(t) + x_i^*)$, $B_i(y_i(t)) = b_i(y_i(t) + x_i^*) - b_i(x_i^*)$, $f_j(y_j(t)) = g_j(y_j(t) + x_j^*) - g_j(x_j^*)$; then system (8) becomes

$$\frac{dy_i(t)}{dt} = -A_i\big(y_i(t)\big)\Bigg[B_i\big(y_i(t)\big) - \sum_{j=1}^n c_{ij} f_j\big(y_j(t)\big) - \sum_{j=1}^n d_{ij} f_j\big(y_j\big(t - \tau_{ij}(t)\big)\big)\Bigg]. \tag{9}$$

Definition 3. System (5) or (9) is globally exponentially stable if there exist constants $\varepsilon > 0$ and $M > 0$ such that

$$\|y_t\|_r = \|x_t - x^*\|_r \leqslant M\|\varphi - x^*\|_r e^{-\varepsilon t}$$

for all $t \geqslant 0$.

It is clear that x^* is globally exponentially stable for (5) if and only if the trivial solution of (8) or (9) is globally exponentially stable.

Throughout this paper, the following Hardy inequality and Halanay inequality are used.

Lemma 1 (Hardy inequality [26]). *Assume there exist constants $a_k \geqslant 0$, $p_k > 0$ $(k = 1, 2, \ldots, m+1)$, then the following inequality holds*:

$$\left(\prod_{k=1}^{m+1} a_k^{p_k}\right)^{\frac{1}{S_{m+1}}} \leqslant \left(\sum_{k=1}^{m+1} p_k a_k^r\right)^{\frac{1}{r}} S_{m+1}^{-\frac{1}{r}}, \tag{10}$$

where $r > 0$ and $S_{m+1} = \sum_{k=1}^{m+1} p_k$.

In (10), if we let $p_{m+1} = 1$, $r = S_{m+1} = \sum_{k=1}^{m} p_k + 1$, we will get

$$\left(\prod_{k=1}^{m} a_k^{p_k}\right) a_{m+1} \leqslant \frac{1}{r}\left(\sum_{k=1}^{m} p_k a_k^r\right) + \frac{1}{r} a_{m+1}^r. \tag{11}$$

Lemma 2 (Halanay inequality [27,28]). *Assume constant numbers k_1, k_2 satisfy $k_1 > k_2 > 0$, $V(t)$ is a nonnegative continuous function on $[t_0 - \tau, t_0]$, and as $t \geqslant t_0$, satisfy the following inequality*:

$$D^+ V(t) \leqslant -k_1 V(t) + k_2 \overline{V}(t),$$

where $\overline{V}(t) = \sup_{t-\tau \leqslant s \leqslant t}\{V(s)\}$, $\tau \geqslant 0$ is constant. Then as $t \geqslant t_0$, we have

$$V(t) \leqslant \overline{V}(t_0) e^{-\lambda(t-t_0)},$$

in which λ is the unique positive solution of the following equation: $\lambda = k_1 - k_2 e^{\lambda\tau}$.

3. Boundedness results

Consider the following equations:

$$\frac{dx_i(t)}{dt} = f_i(t, x_t), \quad i = 1, 2, \ldots, n, \tag{12}$$

where $f_i(t, \varphi) : [0, +\infty) \times C[-\tau, 0] \to \mathbb{R}$ is continuous with respect to (t, φ) and satisfies the Lipschitz condition with respect to φ $(i = 1, 2, \ldots, n)$. Let $W_j(s) : [0, +\infty) \to [0, +\infty)$ $(j = 1, 2, 3, 4)$ be continuous and increasing functions with $W_j(0) = 0$, $W_j(s) \to +\infty$ as $s \to +\infty$. Let the functional $V(t, \varphi) : [0, +\infty) \times C[-\tau, 0] \to [0, +\infty)$ be continuous with respect to (t, φ).

Lemma 3 [29]. *The solutions of system* (12) *are uniformly bounded and uniformly ultimately bounded if the functional $V(t, \varphi)$ and functions $W_j(s)$ $(j = 1, 2, 3, 4)$ satisfy the following conditions*:

(i) $W_1(|x(t)|_r) \leqslant V(t, x_t) \leqslant W_2(|x(t)|_r) + W_3(\int_{t-\tau}^t W_4(|x(s)|_r)\,ds)$;
(ii) $D^+V(t, x_t)|_{(12)} \leqslant -W_4(|x(t)|_r) + M$ *for some constant* $M > 0$.

Lemma 4 [30]. *For system* (12), *let* $f_i(t+T, \varphi) = f_i(t, \varphi)$ *and the solutions be uniformly bounded and uniformly ultimately bounded. Then system* (12) *has a* T*-periodic solution if, for any constant* $\delta > 0$, *there is a constant* $B = B(\delta) > 0$ *such that, for all* φ *with* $\|\varphi\|_r < \delta$, *we have* $|f_i(t, \varphi)| < B$ *for all* $t \in [-\tau, 0]$ $(i = 1, 2, \ldots, n)$.

For the boundedness of solutions for system (6), we have the following results.

Theorem 1. *Under assumptions* (H_1)–(H_3), *all solutions of model* (6) *are uniformly bounded and uniformly ultimately bounded if there exist constants* $\nu_k > 0$ $(k = 1, 2, \ldots, K_1)$, $\mu_k > 0$ $(k = 1, 2, \ldots, K_2)$, $\omega_i > 0$, $\sigma > 0$, $p_{ij}, p_{ij}^*, q_{ij}, q_{ij}^*, \xi_{ij}, \xi_{ij}^*, \eta_{ij}, \eta_{ij}^* \in \mathbb{R}$ *such that*

$$r\underline{\alpha}_i\beta_i\omega_i - \sum_{j=1}^n\sum_{k=1}^{K_1}\bar{\alpha}_i\omega_i\nu_k\left|c_{ij}(t)\right|^{\frac{r\xi_{ij}}{\nu_k}}L_j^{\frac{r\eta_{ij}}{\nu_k}} - \sum_{j=1}^n\sum_{k=1}^{K_2}\bar{\alpha}_i\omega_i\mu_k\left|d_{ij}(t)\right|^{\frac{rp_{ij}}{\mu_k}}L_j^{\frac{rq_{ij}}{\mu_k}}$$
$$- \sum_{j=1}^n\bar{\alpha}_j\omega_j\left|c_{ji}(t)\right|^{r\xi_{ji}^*}L_i^{r\eta_{ji}^*} - \sum_{j=1}^n\bar{\alpha}_j\omega_jL_i^{rq_{ji}^*}\frac{|d_{ji}(\psi_{ji}^{-1}(t))|^{rp_{ji}^*}}{1-\dot{\tau}_{ji}(\psi_{ji}^{-1}(t))} > r\sigma$$

holds for all $t \geqslant 0$; *where* $\psi_{ij}^{-1}(t)$ *is the inverse function of* $\psi_{ij}(t) = t - \tau_{ij}(t)$; $r = \sum_{k=1}^{K_1}\nu_k + 1 = \sum_{k=1}^{K_2}\mu_k + 1$ *and* $p_{ij}, p_{ij}^*, q_{ij}, q_{ij}^*, \xi_{ij}, \xi_{ij}^*, \eta_{ij}, \eta_{ij}^*$ *are any real constant numbers with* $K_1\xi_{ij} + \xi_{ij}^* = 1$, $K_1\eta_{ij} + \eta_{ij}^* = 1$, $K_2p_{ij} + p_{ij}^* = 1$, $K_2q_{ij} + q_{ij}^* = 1$ $(i, j = 1, 2, \ldots, n)$.

Proof. Let $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T$ be any solution of model (6). Now consider the Lyapunov functional

$$V(t, x_t) = \frac{1}{r}\sum_{i=1}^n\omega_i\left[\left|x_i(t)\right|^r + \bar{\alpha}_i\sum_{j=1}^nL_j^{rq_{ij}^*}\int_{t-\tau_{ij}(t)}^t\left|x_j(s)\right|^r\frac{|d_{ij}(\psi_{ij}^{-1}(s))|^{rp_{ij}^*}}{1-\dot{\tau}_{ij}(\psi_{ij}^{-1}(s))}\,ds\right],$$

obviously, we have

$$V(t, x_t) \geqslant \frac{1}{r}\underline{\omega}\sum_{i=1}^n\left|x_i(t)\right|^r = \frac{1}{r}\underline{\omega}\left|x(t)\right|_r^r = W_1\left(\left|x(t)\right|_r\right), \tag{13}$$

$$V(t, x_t) \leqslant \frac{1}{r}\bar{\omega}\sum_{i=1}^n\left[\left|x_i(t)\right|^r + \sum_{j=1}^n\bar{\alpha}_iD_{ij}L_j^{rq_{ij}^*}\int_{t-\tau}^t\left|x_j(s)\right|^r\,ds\right]$$
$$\leqslant \frac{1}{r}\bar{\omega}\sum_{i=1}^n\left|x_i(t)\right|^r + \frac{1}{r}\bar{\omega}Ln\sum_{i=1}^n\int_{t-\tau}^t\left|x_i(s)\right|^r\,ds$$

$$= W_2\big(|x(t)|_r\big) + W_3\left(\int_{t-\tau}^{t} W_4\big(|x(s)|_r\big)\,ds\right) \tag{14}$$

for all $t \geqslant 0$, where

$$\underline{\omega} = \min_{1\leqslant i\leqslant n} \omega_i, \qquad \overline{\omega} = \max_{1\leqslant i\leqslant n} \omega_i,$$

$$D_{ij} = \sup_{s\in[-\tau,+\infty)} \frac{|d_{ij}(\psi_{ij}^{-1}(s))|^{rp_{ij}^*}}{1-\dot{\tau}_{ij}(\psi_{ij}^{-1}(s))}, \qquad L = \max_{1\leqslant i,j\leqslant n}\big(\overline{\alpha}_i D_{ij} L_j^{rq_{ij}^*}\big),$$

$$W_1(s) = \frac{1}{r}\underline{\omega}s^r, \quad W_2(s) = \frac{1}{r}\overline{\omega}s^r, \quad W_3(s) = \frac{2\overline{\omega}n}{r\sigma}Ls, \quad W_4(s) = \frac{1}{2}\sigma s^r.$$

Then

$$\begin{aligned}
D^+V(t,x_t)|_{(6)} = {} & \sum_{i=1}^n \omega_i\Bigg[\big|x_i(t)\big|^{r-1}D^+\big|x_i(t)\big| \\
& + \frac{1}{r}\overline{\alpha}_i\sum_{j=1}^n L_j^{rq_{ij}^*}\big|x_j(t)\big|^r\frac{|d_{ij}(\psi_{ij}^{-1}(t))|^{rp_{ij}^*}}{1-\dot{\tau}_{ij}(\psi_{ij}^{-1}(t))} \\
& - \frac{1}{r}\overline{\alpha}_i\sum_{j=1}^n L_j^{rq_{ij}^*}\big|x_j\big(t-\tau_{ij}(t)\big)\big|^r\frac{|d_{ij}(t)|^{rp_{ij}^*}}{1-\dot{\tau}_{ij}(t)}\big(1-\dot{\tau}_{ij}(t)\big)\Bigg] \\
= {} & \sum_{i=1}^n \omega_i\Bigg\{\big|x_i(t)\big|^{r-1}\operatorname{sgn}\big(x_i(t)\big) \\
& \times\Bigg[-a_i\big(x_i(t)\big)\Bigg(b_i\big(x_i(t)\big) - \sum_{j=1}^n c_{ij}(t)g_j\big(x_j(t)\big) \\
& - \sum_{j=1}^n d_{ij}(t)g_j\big(x_j\big(t-\tau_{ij}(t)\big)\big) + I_i(t)\Bigg)\Bigg] \\
& + \frac{1}{r}\overline{\alpha}_i\sum_{j=1}^n L_j^{rq_{ij}^*}\big|x_j(t)\big|^r\frac{|d_{ij}(\psi_{ij}^{-1}(t))|^{rp_{ij}^*}}{1-\dot{\tau}_{ij}(\psi_{ij}^{-1}(t))} \\
& - \frac{1}{r}\overline{\alpha}_i\sum_{j=1}^n L_j^{rq_{ij}^*}\big|x_j\big(t-\tau_{ij}(t)\big)\big|^r\big|d_{ij}(t)\big|^{rp_{ij}^*}\Bigg\} \\
\leqslant {} & \sum_{i=1}^n \omega_i\Bigg[-\underline{\alpha}_i\beta_i\big|x_i(t)\big|^r + \overline{\alpha}_i\sum_{j=1}^n\big|c_{ij}(t)\big|\big|x_i(t)\big|^{r-1}L_j\big|x_j(t)\big| \\
& + \overline{\alpha}_i\sum_{j=1}^n\big|d_{ij}(t)\big|\big|x_i(t)\big|^{r-1}L_j\big|x_j\big(t-\tau_{ij}(t)\big)\big|
\end{aligned}$$

$$
\begin{aligned}
&+\bar{\alpha}_i\left|x_i(t)\right|^{r-1}\left(\sum_{j=1}^{n}\left|c_{ij}(t)\right|\left|g_j(0)\right|+\sum_{j=1}^{n}\left|d_{ij}(t)\right|\left|g_j(0)\right|+\left|I_i(t)\right|\right)\\
&+\frac{1}{r}\bar{\alpha}_i\sum_{j=1}^{n}L_j^{rq_{ij}^*}\left|x_j(t)\right|^r\frac{|d_{ij}(\psi_{ij}^{-1}(t))|^{rp_{ij}^*}}{1-\dot{\tau}_{ij}(\psi_{ij}^{-1}(t))}\\
&-\frac{1}{r}\bar{\alpha}_i\sum_{j=1}^{n}L_j^{rq_{ij}^*}\left|d_{ij}(t)\right|^{rp_{ij}^*}\left|x_j\big(t-\tau_{ij}(t)\big)\right|^r\Bigg].
\end{aligned}
$$

From Lemma 1, we have

$$
\begin{aligned}
&\left|c_{ij}(t)\right|L_j\left|x_i(t)\right|^{r-1}\left|x_j(t)\right|=\prod_{k=1}^{K_1}\Big(\left|c_{ij}(t)\right|^{\frac{\xi_{ij}}{\nu_k}}L_j^{\frac{\eta_{ij}}{\nu_k}}\left|x_i(t)\right|\Big)^{\nu_k}\left|c_{ij}(t)\right|^{\xi_{ij}^*}L_j^{\eta_{ij}^*}\left|x_j(t)\right|\\
&\leqslant\frac{1}{r}\sum_{k=1}^{K_1}\nu_k\left|c_{ij}(t)\right|^{\frac{r\xi_{ij}}{\nu_k}}L_j^{\frac{r\eta_{ij}}{\nu_k}}\left|x_i(t)\right|^r+\frac{1}{r}\left|c_{ij}(t)\right|^{r\xi_{ij}^*}L_j^{r\eta_{ij}^*}\left|x_j(t)\right|^r
\end{aligned}
$$

and

$$
\begin{aligned}
&\left|d_{ij}(t)\right|L_j\left|x_i(t)\right|^{r-1}\left|x_j\big(t-\tau_{ij}(t)\big)\right|\\
&=\prod_{k=1}^{K_2}\Big(\left|d_{ij}(t)\right|^{\frac{p_{ij}}{\mu_k}}L_j^{\frac{q_{ij}}{\mu_k}}\left|x_i(t)\right|\Big)^{\mu_k}\left|d_{ij}(t)\right|^{p_{ij}^*}L_j^{q_{ij}^*}\left|x_j\big(t-\tau_{ij}(t)\big)\right|\\
&\leqslant\frac{1}{r}\sum_{k=1}^{K_2}\mu_k\left|d_{ij}(t)\right|^{\frac{rp_{ij}}{\mu_k}}L_j^{\frac{rq_{ij}}{\mu_k}}\left|x_i(t)\right|^r+\frac{1}{r}\left|d_{ij}(t)\right|^{rp_{ij}^*}L_j^{rq_{ij}^*}\left|x_j\big(t-\tau_{ij}(t)\big)\right|^r.
\end{aligned}
$$

So

$$
\begin{aligned}
D^+V(t,x_t)|_{(6)}\leqslant\sum_{i=1}^{n}\omega_i\Bigg\{\Bigg[&-\underline{\alpha}_i\beta_i+\frac{1}{r}\bar{\alpha}_i\sum_{j=1}^{n}\sum_{k=1}^{K_1}\nu_k\left|c_{ij}(t)\right|^{\frac{r\xi_{ij}}{\nu_k}}L_j^{\frac{r\eta_{ij}}{\nu_k}}\\
&+\frac{1}{r}\bar{\alpha}_i\sum_{j=1}^{n}\sum_{k=1}^{K_2}\mu_k\left|d_{ij}(t)\right|^{\frac{rp_{ij}}{\mu_k}}L_j^{\frac{rq_{ij}}{\mu_k}}\Bigg]\left|x_i(t)\right|^r\\
&+\Bigg[\frac{1}{r}\bar{\alpha}_i\sum_{j=1}^{n}\left|c_{ij}(t)\right|^{r\xi_{ij}^*}L_j^{r\eta_{ij}^*}\\
&+\frac{1}{r}\bar{\alpha}_i\sum_{j=1}^{n}L_j^{rq_{ij}^*}\frac{|d_{ij}(\psi_{ij}^{-1}(t))|^{rp_{ij}^*}}{1-\dot{\tau}_{ij}(\psi_{ij}^{-1}(t))}\Bigg]\left|x_j(t)\right|^r\Bigg\}\\
&+\sum_{i=1}^{n}\omega_i\bar{\alpha}_i\Bigg(\sum_{j=1}^{n}\left|c_{ij}(t)\right|\left|g_j(0)\right|\\
&+\sum_{j=1}^{n}\left|d_{ij}(t)\right|\left|g_j(0)\right|+\left|I_i(t)\right|\Bigg)\left|x_i(t)\right|^{r-1}
\end{aligned}
$$

$$
\begin{aligned}
&\leqslant \sum_{i=1}^{n}\Bigg[-\underline{\alpha}_i\beta_i\omega_i+\frac{1}{r}\overline{\alpha}_i\sum_{j=1}^{n}\sum_{k=1}^{K_1}\omega_i\nu_k\big|c_{ij}(t)\big|^{\frac{r\xi_{ij}}{\nu_k}}L_j^{\frac{r\eta_{ij}}{\nu_k}}\\
&\quad+\frac{1}{r}\overline{\alpha}_i\sum_{j=1}^{n}\sum_{k=1}^{K_2}\omega_i\mu_k\big|d_{ij}(t)\big|^{\frac{rp_{ij}}{\mu_k}}L_j^{\frac{rq_{ij}}{\mu_k}}\\
&\quad+\frac{1}{r}\sum_{j=1}^{n}\overline{\alpha}_j\omega_j\big|c_{ji}(t)\big|^{r\xi^*_{ji}}L_i^{r\eta^*_{ji}}\\
&\quad+\frac{1}{r}\sum_{j=1}^{n}\overline{\alpha}_j\omega_jL_i^{rq^*_{ji}}\frac{|d_{ji}(\psi_{ji}^{-1}(t))|^{rp^*_{ji}}}{1-\dot{\tau}_{ji}(\psi_{ji}^{-1}(t))}\Bigg]\big|x_i(t)\big|^r\\
&\quad+M\sum_{i=1}^{n}\big|x_i(t)\big|^{r-1}\\
&\leqslant -\sigma\sum_{i=1}^{n}\big|x_i(t)\big|^r+M\sum_{i=1}^{n}\big|x_i(t)\big|^{r-1}\\
&=-\sigma\big|x(t)\big|_r^r+M\big|x(t)\big|_{r-1}^{r-1}
\end{aligned}
$$

in which

$$
M=\sup_{t\in[0,+\infty)}\left[\max_{1\leqslant i\leqslant n}\left(\omega_i\overline{\alpha}_i\left(\sum_{j=1}^{n}\big|c_{ij}(t)\big|\big|g_j(0)\big|+\sum_{j=1}^{n}\big|d_{ij}(t)\big|\big|g_j(0)\big|+\big|I_i(t)\big|\right)\right)\right].
$$

By the equivalence of the norms in $\mathbb{R}^n$, there is a constant $\theta>0$ such that $|x(t)|_{r-1}\leqslant\theta|x(t)|_r$, so we obtain

$$
\begin{aligned}
D^+V(t,x_t)|_{(6)}&\leqslant-\frac{1}{2}\sigma\big|x(t)\big|_r^r+\big|x(t)\big|_r^{r-1}\left(M\theta^{r-1}-\frac{1}{2}\sigma\big|x(t)\big|_r\right)\\
&\leqslant-\frac{1}{2}\sigma\big|x(t)\big|_r^r+M^*,
\end{aligned}
\tag{15}
$$

where

$$
M^*=\sup_{s\in[0,+\infty)}s^{r-1}\left(M\theta^{r-1}-\frac{1}{2}\sigma s\right).
$$

From (13)–(15) and Lemma 3 we know all solutions of model (6) are uniformly bounded and uniformly ultimately bounded. □

In Theorem 1, when $r=1$, define

$$
V(t,x_t)=\sum_{i=1}^{n}\omega_i\left[\big|x_i(t)\big|+\overline{\alpha}_i\sum_{j=1}^{n}L_j\int_{t-\tau_{ij}(t)}^{t}\big|x_j(s)\big|\frac{|d_{ij}(\psi_{ij}^{-1}(s))|}{1-\dot{\tau}_{ij}(\psi_{ij}^{-1}(s))}\,ds\right],
$$

not using Hardy inequality and by direct computation, we have the following corollary.

Corollary 1. *Under assumptions* (H_1)–(H_3)*, all solutions of model* (6) *are uniformly bounded and uniformly ultimately bounded if there exist constants* $\omega_i > 0$ $(i = 1, 2, \ldots, n)$, $\sigma > 0$, *such that*

$$\underline{\alpha}_i \beta_i \omega_i - \sum_{j=1}^{n} \overline{\alpha}_j \omega_j \left|c_{ji}(t)\right| L_i - \sum_{j=1}^{n} \overline{\alpha}_j \omega_j L_i \frac{|d_{ji}(\psi_{ji}^{-1}(t))|}{1 - \dot{\tau}_{ji}(\psi_{ji}^{-1}(t))} > \sigma$$

holds for all $t \geqslant 0$.

From Theorem 1 and Lemma 4, we can easily derive the following results.

Theorem 2. *Under assumptions* (H_1)–(H_3)*, let* $c_{ij}(t), d_{ij}(t), \tau_{ij}(t), I_i(t)$ $(i, j = 1, 2, \ldots, n)$ *be periodic functions with periodic* $T > 0$*, then model* (6) *has a unique* T*-periodic solution if there exist constants* $\nu_k > 0$ $(k = 1, 2, \ldots, K_1)$, $\mu_k > 0$ $(k = 1, 2, \ldots, K_2)$, $\omega_i > 0$, $\sigma > 0$, $p_{ij}, p_{ij}^*, q_{ij}, q_{ij}^*, \xi_{ij}, \xi_{ij}^*, \eta_{ij}, \eta_{ij}^* \in \mathbb{R}$ *such that*

$$\begin{aligned} & r\underline{\alpha}_i \beta_i \omega_i - \sum_{j=1}^{n} \sum_{k=1}^{K_1} \overline{\alpha}_i \omega_i \nu_k \left|c_{ij}(t)\right|^{\frac{r\xi_{ij}}{\nu_k}} L_j^{\frac{r\eta_{ij}}{\nu_k}} - \sum_{j=1}^{n} \sum_{k=1}^{K_2} \overline{\alpha}_i \omega_i \mu_k \left|d_{ij}(t)\right|^{\frac{rp_{ij}}{\mu_k}} L_j^{\frac{rq_{ij}}{\mu_k}} \\ & \quad - \sum_{j=1}^{n} \overline{\alpha}_j \omega_j \left|c_{ji}(t)\right|^{r\xi_{ji}^*} L_i^{r\eta_{ji}^*} - \sum_{j=1}^{n} \overline{\alpha}_j \omega_j L_i^{rq_{ji}^*} \frac{|d_{ji}(\psi_{ji}^{-1}(t))|^{rp_{ji}^*}}{1 - \dot{\tau}_{ji}(\psi_{ji}^{-1}(t))} > r\sigma \end{aligned}$$

holds for all $t \geqslant 0$; *where* $r = \sum_{k=1}^{K_1} \nu_k + 1 = \sum_{k=1}^{K_2} \mu_k + 1$ *and* $K_1 \xi_{ij} + \xi_{ij}^* = 1$, $K_1 \eta_{ij} + \eta_{ij}^* = 1$, $K_2 p_{ij} + p_{ij}^* = 1$, $K_2 q_{ij} + q_{ij}^* = 1$ $(i, j = 1, 2, \ldots, n)$.

Corollary 2. *Under assumptions* (H_1)–(H_3)*, let* $c_{ij}(t), d_{ij}(t), \tau_{ij}(t), I_i(t)$ $(i, j = 1, 2, \ldots, n)$ *be periodic functions with periodic* $T > 0$*, then model* (6) *has a unique* T*-periodic solution if there exist constants* $\omega_i > 0$ $(i = 1, 2, \ldots, n)$, $\sigma > 0$, *such that*

$$\underline{\alpha}_i \beta_i \omega_i - \sum_{j=1}^{n} \overline{\alpha}_j \omega_j \left|c_{ji}(t)\right| L_i - \sum_{j=1}^{n} \overline{\alpha}_j \omega_j L_i \frac{|d_{ji}(\psi_{ji}^{-1}(t))|}{1 - \dot{\tau}_{ji}(\psi_{ji}^{-1}(t))} > \sigma$$

holds for all $t \geqslant 0$.

Corollary 3. *Under assumptions* (H_1)–(H_3)*, let* $c_{ij}(t), d_{ij}(t), \tau_{ij}(t), I_i(t)$ $(i, j = 1, 2, \ldots, n)$ *be periodic functions with periodic* $T > 0$*, then model* (6) *has a unique* T*-periodic solution if there exist a constant* $\sigma > 0$*, such that*

$$\underline{\alpha}_i \beta_i - \sum_{j=1}^{n} \overline{\alpha}_i \left|c_{ij}(t)\right| L_j - \sum_{j=1}^{n} \overline{\alpha}_i L_j \left|d_{ij}(t)\right| > \sigma$$

holds for all $t \geqslant 0$ *and* $i = 1, 2, \ldots, n$.

Proof. In Theorem 2, if we take $K_1 = K_2 = 1$, $\nu_k = \mu_k = r - 1$, $\omega_i = 1$, $\xi_{ij} = \eta_{ij} = p_{ij} = q_{ij} = \frac{r-1}{r}$ and $\xi_{ij}^* = \eta_{ij}^* = p_{ij}^* = q_{ij}^* = \frac{1}{r}$ $(i, j = 1, 2, \ldots, n)$, a condition to ensure system (6) has a T-periodic solution is obtained as

$$\underline{\alpha}_i\beta_i - \frac{r-1}{r}\sum_{j=1}^{n}\bar{\alpha}_i\left|c_{ij}(t)\right|L_j - \frac{1}{r}\sum_{j=1}^{n}\bar{\alpha}_j\left|c_{ji}(t)\right|L_i - \frac{r-1}{r}\sum_{j=1}^{n}\bar{\alpha}_i\left|d_{ij}(t)\right|L_j$$
$$- \frac{1}{r}\sum_{j=1}^{n}\bar{\alpha}_j L_i \frac{|d_{ji}(\psi_{ji}^{-1}(t))|}{1-\dot{\tau}_{ji}(\psi_{ji}^{-1}(t))} > \sigma.$$

Let r tends to $+\infty$, and the result follows. □

Remark 1. For system (6), when $a_i(x_i(t)) \equiv 1$, $b_i(x_i(t)) = b_i(t)x_i(t)$ (in which $b_i(t)$ is not only differentiable but also bounded on interval $[-\tau, +\infty)$, and its maximal lower bound is denoted as $\beta_i > 0$) then system (6) turns out to be a recurrent neural network model with variable coefficients and time-varying delays. In this case, Theorems 1 and 2 turn out to be a generalized result for those in [31,32], that is, the results in [31,32] are special cases of ours.

4. Stability results

In this section, we will obtain some criteria for global exponential stability of (5) or (9). Moreover, the uniqueness of the equilibrium point follows directly from its global exponential stability.

Theorem 3. *Under assumptions* (H_1), (H_2') *and* (H_3'), *model* (9) *is globally exponentially stable if there exist constants* $\nu_k > 0$ $(k = 1, 2, \ldots, K_1)$, $\mu_k > 0$ $(k = 1, 2, \ldots, K_2)$, p_{ij}, $p_{ij}^*, q_{ij}, q_{ij}^*, \xi_{ij}, \xi_{ij}^*, \eta_{ij}, \eta_{ij}^* \in \mathbb{R}$ *such that*

$$\sigma_1 > \sigma_2 > 0,$$

where $r = \sum_{k=1}^{K_1}\nu_k + 1 = \sum_{k=1}^{K_2}\mu_k + 1$; $K_1\xi_{ij} + \xi_{ij}^* = 1$, $K_1\eta_{ij} + \eta_{ij}^* = 1$, $K_2 p_{ij} + p_{ij}^* = 1$, $K_2 q_{ij} + q_{ij}^* = 1$ *and*

$$\sigma_1 = \min_{1\leqslant i\leqslant n}\left\{ r\underline{\alpha}_i\beta_i - \bar{\alpha}_i\sum_{j=1}^{n}\sum_{k=1}^{K_1}\nu_k|c_{ij}|^{\frac{r\xi_{ij}}{\nu_k}}L_j^{\frac{r\eta_{ij}}{\nu_k}}\ \bar{\alpha}_i\sum_{j=1}^{n}\sum_{k=1}^{K_2}\mu_k|d_{ij}|^{\frac{rp_{ij}}{\mu_k}}L_j^{\frac{rq_{ij}}{\mu_k}} \right.$$
$$\left. - \sum_{j=1}^{n}\bar{\alpha}_j|c_{ji}|^{r\xi_{ji}^*}L_i^{r\eta_{ji}^*} \right\},$$

$$\sigma_2 = \max_{1\leqslant i\leqslant n}\sum_{j=1}^{n}\bar{\alpha}_j|d_{ji}|^{rp_{ji}^*}L_i^{rq_{ji}^*}.$$

Proof. Define

$$V(t, y_t) = \frac{1}{r}\sum_{i=1}^{n}\left|y_i(t)\right|^r,$$

it can easily be verified that $V(t, y_t)$ is a nonnegative function over $[-\tau, +\infty)$ and that it is radically unbounded, that is, $V(t, y_t) \to +\infty$ as $|y(t)|_r \to +\infty$.

Next, evaluating the Dini derivative of V along the trajectory of (9) gives

$$\begin{aligned}
D^+V(t,y_t) &= \sum_{i=1}^{n}\big|y_i(t)\big|^{r-1}\operatorname{sgn}\big(y_i(t)\big)\Bigg[-A_i\big(y_i(t)\big)\Bigg(B_i\big(y_i(t)\big) \\
&\quad - \sum_{j=1}^{n} c_{ij} f_j\big(y_j(t)\big) - \sum_{j=1}^{n} d_{ij} f_j\big(y_j\big(t-\tau_{ij}(t)\big)\big)\Bigg)\Bigg] \\
&\leqslant \sum_{i=1}^{n}\Bigg[-\underline{\alpha}_i\beta_i\big|y_i(t)\big|^r + \overline{\alpha}_i\sum_{j=1}^{n}|c_{ij}|L_j\big|y_i(t)\big|^{r-1}\big|y_j(t)\big| \\
&\quad + \overline{\alpha}_i\sum_{j=1}^{n}|d_{ij}|L_j\big|y_i(t)\big|^{r-1}\big|y_j\big(t-\tau_{ij}(t)\big)\big|\Bigg] \\
&\leqslant \sum_{i=1}^{n}\Bigg[-\underline{\alpha}_i\beta_i\big|y_i(t)\big|^r \\
&\quad + \overline{\alpha}_i\sum_{j=1}^{n}\Bigg(\frac{1}{r}\sum_{k=1}^{K_1}\nu_k|c_{ij}|^{\frac{r\xi_{ij}}{\nu_k}}L_j^{\frac{r\eta_{ij}}{\nu_k}}\big|y_i(t)\big|^r + \frac{1}{r}|c_{ij}|^{r\xi^*_{ij}}L_j^{r\eta^*_{ij}}\big|y_j(t)\big|^r\Bigg) \\
&\quad + \overline{\alpha}_i\sum_{j=1}^{n}\Bigg(\frac{1}{r}\sum_{k=1}^{K_2}\mu_k|d_{ij}|^{\frac{rp_{ij}}{\mu_k}}L_j^{\frac{rq_{ij}}{\mu_k}}\big|y_i(t)\big|^r \\
&\quad + \frac{1}{r}|d_{ij}|^{rp^*_{ij}}L_j^{rq^*_{ij}}\big|y_j\big(t-\tau_{ij}(t)\big)\big|^r\Bigg)\Bigg] \\
&= -\frac{1}{r}\sum_{i=1}^{n}\Bigg[r\underline{\alpha}_i\beta_i - \overline{\alpha}_i\sum_{j=1}^{n}\sum_{k=1}^{K_1}\nu_k|c_{ij}|^{\frac{r\xi_{ij}}{\nu_k}}L_j^{\frac{r\eta_{ij}}{\nu_k}} \\
&\quad - \sum_{j=1}^{n}\overline{\alpha}_j|c_{ji}|^{r\xi^*_{ji}}L_i^{r\eta^*_{ji}} - \overline{\alpha}_i\sum_{j=1}^{n}\sum_{k=1}^{K_2}\mu_k|d_{ij}|^{\frac{rp_{ij}}{\mu_k}}L_j^{\frac{rq_{ij}}{\mu_k}}\Bigg]\big|y_i(t)\big|^r \\
&\quad + \frac{1}{r}\sum_{i=1}^{n}\Bigg(\sum_{j=1}^{n}\overline{\alpha}_j|d_{ji}|^{rp^*_{ji}}L_i^{rq^*_{ji}}\big|y_i\big(t-\tau_{ji}(t)\big)\big|^r\Bigg) \\
&\leqslant -\sigma_1V(t,y_t) + \sigma_2\overline{V}(t),
\end{aligned}$$

and from Lemma 2, it can be drawn that if $\sigma_1 > \sigma_2 > 0$, then

$$V(t,y_t) \leqslant \Big(\sup_{-\tau\leqslant s\leqslant 0} V(s)\Big)e^{-\lambda s},$$

where λ is the unique positive solution of equation: $\lambda = \sigma_1 - \sigma_2 e^{\lambda\tau}$.

Therefore, $V(t,y_t)$ converges to zero exponentially, which in turn implies that $y(t)$ also converges globally and exponentially to zero with a convergence rate $\frac{\lambda}{r}$, and this completes the proof. □

In Theorem 3, if we take $K_1 = K_2 = 1$, $\nu_k = \mu_k = r - 1$, $\xi_{ij} = \eta_{ij} = p_{ij} = q_{ij} = \frac{r-1}{r}$ and $\xi^*_{ij} = \eta^*_{ij} = p^*_{ij} = q^*_{ij} = \frac{1}{r}$ $(i, j = 1, 2, \ldots, n)$, we have the following result.

Corollary 4. *Under assumptions* (H_1), (H_2') *and* (H_3'), *model* (9) *is globally exponentially stable if*

$$\sigma_1 > \sigma_2 > 0,$$

where

$$\sigma_1 = \min_{1\leqslant i\leqslant n}\Biggl\{ r\underline{\alpha}_i\beta_i - (r-1)\bar{\alpha}_i\sum_{j=1}^{n}|c_{ij}|L_j - (r-1)\bar{\alpha}_i\sum_{j=1}^{n}|d_{ij}|L_j - \sum_{j=1}^{n}\bar{\alpha}_j|c_{ji}|L_i\Biggr\},$$

$$\sigma_2 = \max_{1\leqslant i\leqslant n}\sum_{j=1}^{n}\bar{\alpha}_j|d_{ji}|L_i$$

and $r > 1$ *is a constant number.*

Remark 2. In [25] the authors gave a condition $L(\|C\| + \|D\|)\eta < 1$ to ensure system (5) to be globally exponentially stable. To obtain the result, firstly system (5) was written into a matrix form

$$\frac{dx(t)}{dt} = -A\bigl(x(t)\bigr)\bigl[B\bigl(x(t)\bigr) - Cg\bigl(x(t)\bigr) - Dg\bigl(x\bigl(t-\tau(t)\bigr)\bigr) + I\bigr];$$

secondly norm of matrix was utilized. However, in the term $g_j(x_j(t - \tau_{ij}(t)))$, $\tau_{ij}(t)$ is not only related to index j but also to index i, it is thus impossible to write system (5) into the required matrix form as suggested in [25].

Theorem 4. *Under assumptions* (H_1), (H_2') *and* (H_3'), *model* (9) *is globally exponentially stable if there exist constants* $\nu_k > 0$ $(k = 1, 2, \ldots, K_1)$, $\mu_k > 0$ $(k = 1, 2, \ldots, K_2)$, $\omega_i > 0$, $p_{ij}, p^*_{ij}, q_{ij}, q^*_{ij}, \xi_{ij}, \xi^*_{ij}, \eta_{ij}, \eta^*_{ij} \in \mathbb{R}$ *such that*

$$r\underline{\alpha}_i\beta_i\omega_i - \sum_{j=1}^{n}\sum_{k=1}^{K_1}\bar{\alpha}_i\omega_i\nu_k|c_{ij}|^{\frac{r\xi_{ij}}{\nu_k}}L_j^{\frac{r\eta_{ij}}{\nu_k}} - \sum_{j=1}^{n}\sum_{k=1}^{K_2}\bar{\alpha}_i\omega_i\mu_k|d_{ij}|^{\frac{rp_{ij}}{\mu_k}}L_j^{\frac{rq_{ij}}{\mu_k}} - \sum_{j=1}^{n}\bar{\alpha}_j\omega_j|c_{ji}|^{r\xi^*_{ji}}L_i^{r\eta^*_{ji}} - \sum_{j=1}^{n}\bar{\alpha}_j\omega_jL_i^{rq^*_{ji}}\frac{|d_{ji}|^{rp^*_{ji}}}{1 - \dot{\tau}_{ji}(\psi^{-1}_{ji}(t))} > 0$$

holds for all $t \geqslant 0$; *where* $r = \sum_{k=1}^{K_1}\nu_k + 1 = \sum_{k=1}^{K_2}\mu_k + 1$ *and* $K_1\xi_{ij} + \xi^*_{ij} = 1$, $K_1\eta_{ij} + \eta^*_{ij} = 1$, $K_2p_{ij} + p^*_{ij} = 1$, $K_2q_{ij} + q^*_{ij} = 1$ $(i, j = 1, 2, \ldots, n)$.

Proof. Suppose $y(t) = (y_1(t), y_2(t), \ldots, y_n(t))^T$ is a solution of model (9) with $\varphi - x^*$ as its initial function, that is,

$$y_i(t) = \varphi_i(t) - x^*_i, \quad t \in [-\tau, 0],\ i = 1, 2, \ldots, n.$$

Since

$$r\underline{\alpha}_i\beta_i\omega_i - \sum_{j=1}^{n}\sum_{k=1}^{K_1}\bar{\alpha}_i\omega_i\nu_k|c_{ij}|^{\frac{r\xi_{ij}}{\nu_k}}L_j^{\frac{r\eta_{ij}}{\nu_k}} - \sum_{j=1}^{n}\sum_{k=1}^{K_2}\bar{\alpha}_i\omega_i\mu_k|d_{ij}|^{\frac{rp_{ij}}{\mu_k}}L_j^{\frac{rq_{ij}}{\mu_k}}$$
$$- \sum_{j=1}^{n}\bar{\alpha}_j\omega_j|c_{ji}|^{r\xi_{ji}^*}L_i^{r\eta_{ji}^*} - \sum_{j=1}^{n}\bar{\alpha}_j\omega_j L_i^{rq_{ji}^*}\frac{|d_{ji}|^{rp_{ji}^*}}{1-\dot{\tau}_{ji}(\psi_{ji}^{-1}(t))} > 0,$$

we can choose a small $\varepsilon > 0$ such that

$$\omega_i(\varepsilon - r\underline{\alpha}_i\beta_i) + \sum_{j=1}^{n}\sum_{k=1}^{K_1}\bar{\alpha}_i\omega_i\nu_k|c_{ij}|^{\frac{r\xi_{ij}}{\nu_k}}L_j^{\frac{r\eta_{ij}}{\nu_k}} + \sum_{j=1}^{n}\sum_{k=1}^{K_2}\bar{\alpha}_i\omega_i\mu_k|d_{ij}|^{\frac{rp_{ij}}{\mu_k}}L_j^{\frac{rq_{ij}}{\mu_k}}$$
$$+ \sum_{j=1}^{n}\bar{\alpha}_j\omega_j|c_{ji}|^{r\xi_{ji}^*}L_i^{r\eta_{ji}^*} + e^{\varepsilon\tau}\sum_{j=1}^{n}\bar{\alpha}_j\omega_j L_i^{rq_{ji}^*}\frac{|d_{ji}|^{rp_{ji}^*}}{1-\dot{\tau}_{ji}(\psi_{ji}^{-1}(t))} < 0.$$

Now we consider the Lyapunov functional

$$V(t, y_t) = \frac{1}{r}\sum_{i=1}^{n}\omega_i\Bigg[\big|y_i(t)\big|^r e^{\varepsilon t}$$
$$+ \bar{\alpha}_i\sum_{j=1}^{n}L_j^{rq_{ij}^*}\int_{t-\tau_{ij}(t)}^{t}\big|y_j(s)\big|^r\frac{|d_{ij}|^{rp_{ij}^*}}{1-\dot{\tau}_{ij}(\psi_{ij}^{-1}(s))}e^{\varepsilon(s+\tau_{ij}(\psi_{ij}^{-1}(s)))}\,ds\Bigg],$$

calculating the upper right Dini derivative of $V(t, y_t)$, we obtain

$$D^+V(t, y_t)|_{(9)} = \frac{1}{r}\sum_{i=1}^{n}\omega_i\Bigg[\varepsilon e^{\varepsilon t}\big|y_i(t)\big|^r + re^{\varepsilon t}\big|y_i(t)\big|^{r-1}D^+\big|y_i(t)\big|$$
$$+ \bar{\alpha}_i\sum_{j=1}^{n}L_j^{rq_{ij}^*}e^{\varepsilon(t+\tau_{ij}(\psi_{ij}^{-1}(t)))}\big|y_j(t)\big|^r\frac{|d_{ij}|^{rp_{ij}^*}}{1-\dot{\tau}_{ij}(\psi_{ij}^{-1}(t))}$$
$$- \bar{\alpha}_i\sum_{j=1}^{n}L_j^{rq_{ij}^*}|d_{ij}|^{rp_{ij}^*}\big|y_j\big(t-\tau_{ij}(t)\big)\big|^r e^{\varepsilon t}\Bigg]$$
$$\leqslant e^{\varepsilon t}\sum_{i=1}^{n}\omega_i\Bigg[\frac{1}{r}\varepsilon\big|y_i(t)\big|^r - \underline{\alpha}_i\beta_i\big|y_i(t)\big|^r$$
$$+ \bar{\alpha}_i\sum_{j=1}^{n}|c_{ij}|\big|y_i(t)\big|^{r-1}L_j\big|y_j(t)\big|$$
$$+ \bar{\alpha}_i\sum_{j=1}^{n}|d_{ij}|\big|y_i(t)\big|^{r-1}L_j\big|y_j\big(t-\tau_{ij}(t)\big)\big|$$
$$+ \frac{1}{r}\bar{\alpha}_i\sum_{j=1}^{n}L_j^{rq_{ij}^*}e^{\varepsilon\tau}\big|y_j(t)\big|^r\frac{|d_{ij}|^{rp_{ij}^*}}{1-\dot{\tau}_{ij}(\psi_{ij}^{-1}(t))}$$

$$-\frac{1}{r}\bar{\alpha}_i\sum_{j=1}^{n}L_j^{rq_{ij}^*}|d_{ij}|^{rp_{ij}^*}\left|y_j\left(t-\tau_{ij}(t)\right)\right|^r\Bigg].$$

Estimating the right of inequality above by the Hardy inequality, we have

$$\begin{aligned}
D^+V(t,y_t)|_{(9)} &\leqslant e^{\varepsilon t}\sum_{i=1}^{n}\omega_i\Bigg\{\frac{1}{r}\varepsilon\left|y_i(t)\right|^r-\underline{\alpha}_i\beta_i\left|y_i(t)\right|^r \\
&\quad+\bar{\alpha}_i\sum_{j=1}^{n}\Bigg[\frac{1}{r}\sum_{k=1}^{K_1}\nu_k|c_{ij}|^{\frac{r\xi_{ij}}{\nu_k}}L_j^{\frac{r\eta_{ij}}{\nu_k}}\left|y_i(t)\right|^r+\frac{1}{r}|c_{ij}|^{r\xi_{ij}^*}L_j^{r\eta_{ij}^*}\left|y_j(t)\right|^r\Bigg] \\
&\quad+\bar{\alpha}_i\sum_{j=1}^{n}\Bigg[\frac{1}{r}\sum_{k=1}^{K_2}\mu_k|d_{ij}|^{\frac{rp_{ij}}{\mu_k}}L_j^{\frac{rq_{ij}}{\mu_k}}\left|y_i(t)\right|^r \\
&\quad+\frac{1}{r}|d_{ij}|^{rp_{ij}^*}L_j^{rq_{ij}^*}\left|y_j\left(t-\tau_{ij}(t)\right)\right|^r\Bigg] \\
&\quad+\frac{1}{r}\bar{\alpha}_i\sum_{j=1}^{n}L_j^{rq_{ij}^*}e^{\varepsilon\tau}\left|y_j(t)\right|^r\frac{|d_{ij}|^{rp_{ij}^*}}{1-\dot{\tau}_{ij}(\psi_{ij}^{-1}(t))} \\
&\quad-\frac{1}{r}\bar{\alpha}_i\sum_{j=1}^{n}L_j^{rq_{ij}^*}|d_{ij}|^{rp_{ij}^*}\left|y_j\left(t-\tau_{ij}(t)\right)\right|^r\Bigg\} \\
&=\frac{1}{r}e^{\varepsilon t}\sum_{i=1}^{n}\Bigg[\omega_i(\varepsilon-r\underline{\alpha}_i\beta_i)+\sum_{j=1}^{n}\sum_{k=1}^{K_1}\bar{\alpha}_i\omega_i\nu_k|c_{ij}|^{\frac{r\xi_{ij}}{\nu_k}}L_j^{\frac{r\eta_{ij}}{\nu_k}} \\
&\quad+\sum_{j=1}^{n}\sum_{k=1}^{K_2}\bar{\alpha}_i\omega_i\mu_k|d_{ij}|^{\frac{rp_{ij}}{\mu_k}}L_j^{\frac{rq_{ij}}{\mu_k}}+\sum_{j=1}^{n}\bar{\alpha}_j\omega_j|c_{ji}|^{r\xi_{ji}^*}L_i^{r\eta_{ji}^*} \\
&\quad+e^{\varepsilon\tau}\sum_{j=1}^{n}\bar{\alpha}_j\omega_jL_i^{rq_{ji}^*}\frac{|d_{ji}|^{rp_{ji}^*}}{1-\dot{\tau}_{ji}(\psi_{ji}^{-1}(t))}\Bigg]\left|y_i(t)\right|^r \\
&\leqslant 0,
\end{aligned}$$

and so

$$V(t)\leqslant V(0),\quad t\geqslant 0,$$

since

$$V(t)\geqslant\frac{1}{r}e^{\varepsilon t}\underline{\omega}\sum_{i=1}^{n}\left|y_i(t)\right|^r=\frac{1}{r}\underline{\omega}e^{\varepsilon t}\left|y(t)\right|_r^r,\quad t\geqslant 0,$$

$$
\begin{aligned}
V(0) = {}& \frac{1}{r}\sum_{i=1}^{n}\omega_i\Bigg[\big|\varphi_i(0)-x_i^*\big|^r \\
&+ \bar{\alpha}_i\sum_{j=1}^{n}L_j^{rq_{ij}^*}\int_{-\tau_{ij}(0)}^{0}\big|y_j(s)\big|^r\frac{|d_{ji}|^{rp_{ij}^*}}{1-\dot{\tau}_{ji}(\psi_{ji}^{-1}(s))}e^{\varepsilon(s+\tau_{ij}(\psi_{ij}^{-1}(s)))}\,ds\Bigg] \\
\leqslant {}& \frac{1}{r}\bar{\omega}\Bigg[\sum_{i=1}^{n}\big|\varphi_i(0)-x_i^*\big|^r + Lne^{\varepsilon\tau}\sum_{i=1}^{n}\int_{-\tau}^{0}\big|y_i(s)\big|^r\,ds\Bigg] \\
\leqslant {}& \frac{1}{r}\bar{\omega}(1+Ln\tau e^{\varepsilon\tau})\sup_{s\in[-\tau,0]}\Bigg(\sum_{i=1}^{n}\big|\varphi_i(s)-x_i^*\big|^r\Bigg) \\
= {}& \frac{1}{r}\bar{\omega}(1+Ln\tau e^{\varepsilon\tau})\|\varphi-x^*\|_r^r,
\end{aligned}
$$

then it easily follows that

$$
\|y_t\|_r^r = \|x_t - x^*\|_r^r \leqslant \frac{\bar{\omega}}{\underline{\omega}}(1+Ln\tau e^{\varepsilon\tau})\|\varphi-x^*\|_r^r e^{-\varepsilon t},
$$

and this means

$$
\|y_t\|_r = \|x_t - x^*\|_r \leqslant M\|\varphi - x^*\|_r e^{-\varepsilon^* t}
$$

for all $t \geqslant 0$, where $M \geqslant 1$ is a constant, $\varepsilon^* = \frac{\varepsilon}{r} > 0$. This implies the solutions of model (5) or (9) is globally exponentially stable. □

Remark 3. Notice that (5) becomes (4) when $\tau_{ij}(t) = \tau_{ij}$. For this model, it has been reported in [24] that if

(i) $g_j(\cdot)$ $(j = 1, 2, \ldots, n)$ are nondecreasing and

(ii)
$$
\min_{1\leqslant i\leqslant n}\Bigg\{\beta_i - a_{ii}^+L_i - \frac{1}{2}\sum_{j\neq i,\, j=1}^{n}\big(|c_{ij}|L_j + |c_{ji}|L_i\big) - \frac{1}{2}\sum_{j=1}^{n}|d_{ij}| - \frac{1}{2}\sum_{j=1}^{n}|d_{ji}|L_i^2\Bigg\} > 0,
$$

then (4) has a unique and globally *asymptotically* stable equilibrium point. In Theorem 4 above, by taking $K_1 = K_2 = \nu_k = \mu_k = 1$, $\xi_{ij} = \eta_{ij} = p_{ij} = \xi_{ij}^* = \eta_{ij}^* = p_{ij}^* = \frac{1}{2}$, $\omega_i = 1$, $q_{ij}^* = 1$ and $q_{ij} = 0$, a similar result can be derived (we obtained a stronger result of *exponential* stability). In other words, [24, Theorem 1] is a special case of ours.

Remark 4. In the theoretical development in [24, Theorem 3], when $y(t) = x(t) - x^* \neq 0$, $f(y(t)) = g(y(t) + x^*) - g(x^*) = 0$ and $f(y(t-\tau)) = g(y(t-\tau) + x^*) - g(x^*) = 0$, system (4) becomes

$$
\frac{dy_i(t)}{dt} = -A_i\big(y_i(t)\big)B_i\big(y_i(t)\big)
$$

(see [24, Eq. (38)]). The authors then concluded that

$$\frac{dy_i(t)}{dt} \leqslant -A_i\big(y_i(t)\big)\beta_i y_i(t)$$

which is incorrect and should be modified to

$$\begin{aligned} D^+\big|y_i(t)\big| &= \operatorname{sgn}\big(y_i(t)\big)\frac{dy_i(t)}{dt} = -A_i\big(y_i(t)\big)\operatorname{sgn}\big(y_i(t)\big)B_i\big(y_i(t)\big) \\ &= -A_i\big(y_i(t)\big)\big|B_i\big(y_i(t)\big)\big| \leqslant -A_i\big(y_i(t)\big)\beta_i\big|y_i(t)\big|. \end{aligned}$$

Nevertheless, the conclusion that $|y_i(t)| \to 0$ (as $t \to +\infty$) remains valid.

When $r = 1$, in the proof of Theorem 4, if one defines

$$\begin{aligned} V(t, y_t) = \sum_{i=1}^{n} \omega_i \Bigg[& e^{\varepsilon t}\big|y_i(t)\big| \\ & + \overline{\alpha}_i \sum_{j=1}^{n} L_j \int_{t-\tau_{ij}(t)}^{t} \big|y_j(s)\big| \frac{|d_{ij}|}{1 - \dot{\tau}_{ij}(\psi_{ij}^{-1}(s))} e^{\varepsilon(s+\tau_{ij}(\psi_{ij}^{-1}(s)))}\, ds \Bigg], \end{aligned}$$

and not employing the Hardy inequality, direct computation leads to the following results.

Corollary 5. *Under assumptions (H_1), (H_2') and (H_3'), all solutions of model (9) are globally exponentially stable if there exist constants $\omega_i > 0$ ($i = 1, 2, \ldots, n$), such that*

$$\underline{\alpha}_i \beta_i \omega_i - \sum_{j=1}^{n} \overline{\alpha}_j \omega_j \big|c_{ji}(t)\big| L_i - \sum_{j=1}^{n} \overline{\alpha}_j \omega_j L_i \frac{|d_{ji}(\psi_{ji}^{-1}(t))|}{1 - \dot{\tau}_{ji}(\psi_{ji}^{-1}(t))} > 0$$

holds for all $t \geqslant 0$.

Corollary 6. *Under assumptions (H_1), (H_2') and (H_3'), all solutions of model (9) are globally exponentially stable if*

$$\underline{\alpha}_i \beta_i - \sum_{j=1}^{n} \overline{\alpha}_i |c_{ij}| L_j - \sum_{j=1}^{n} \overline{\alpha}_i L_j |d_{ij}| > 0$$

holds for all $t \geqslant 0$ and $i = 1, 2, \ldots, n$.

5. Illustrative examples

Example 1. Consider the following system:

$$\begin{aligned} \frac{dx_1(t)}{dt} = &-\big(2 + \cos x_1(t)\big)\big[8x_1(t) - \sin t \times f\big(x_1(t - 0.5\sin t - 1)\big) \\ &- \cos t \times f\big(x_2(t - 0.5\sin t - 1)\big) + \sin t\big], \\ \frac{dx_2(t)}{dt} = &-\big(2 + \sin x_2(t)\big)\big[8x_2(t) - \cos t \times f\big(x_1(t - 0.5\sin t - 1)\big) \\ &- \sin t \times f\big(x_2(t - 0.5\sin t - 1)\big) + \cos t\big], \end{aligned} \tag{16}$$

where $f(x) = 0.5(|x+1| - |x-1|)$.

The system satisfies all assumptions in this paper with $\underline{\alpha}_1 = \underline{\alpha}_2 = 1$, $\bar{\alpha}_1 = \bar{\alpha}_2 = 3$, $L_1 = L_2 = 1$, $\beta_1 = \beta_2 = 8$, $0 \leqslant \tau_{ij}(t) = 0.5 \sin t + 1 \leqslant 1.5$ and $\sup_{t \in [-1.5, +\infty)} \dot{\tau}_{ij}(t) = 0.5 < 1$ $(i, j = 1, 2)$. In Corollary 3, if we take $\sigma = 1$, then

$$\underline{\alpha}_1 \beta_1 - \bar{\alpha}_1 \left| d_{11}(t) \right| L_1 - \bar{\alpha}_1 \left| d_{12}(t) \right| L_2 = 8 - 3|\sin t| - 3|\cos t| > \sigma = 1,$$
$$\underline{\alpha}_2 \beta_2 - \bar{\alpha}_2 \left| d_{21}(t) \right| L_1 - \bar{\alpha}_2 \left| d_{22}(t) \right| L_2 = 8 - 3|\cos t| - 3|\sin t| > \sigma = 1,$$

therefore we can deduce that system (16) has a 2π-periodic solution and it is uniformly bounded, uniformly ultimately bounded.

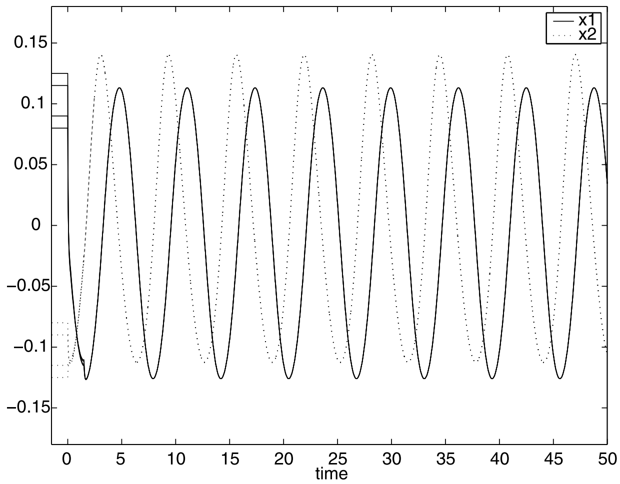

Fig. 1. Transient response of state variables $x_1(t)$ and $x_2(t)$ for Example 1.

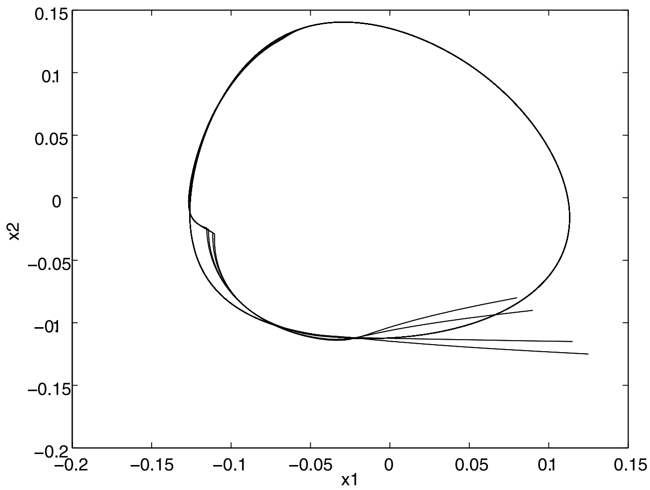

Fig. 2. Phase plots of state variables $x_1(t)$ and $x_2(t)$ for Example 1.

For numerical simulation, the following four cases are given:
Case 1 with the initial state $(\varphi_1(t), \varphi_2(t)) = (0.09, -0.09)$ for $t \in [-1.5, 0]$;
Case 2 with the initial state $(\varphi_1(t), \varphi_2(t)) = (0.115, -0.115)$ for $t \in [-1.5, 0]$;
Case 3 with the initial state $(\varphi_1(t), \varphi_2(t)) = (0.08, -0.08)$ for $t \in [-1.5, 0]$;
Case 4 with the initial state $(\varphi_1(t), \varphi_2(t)) = (0.125, -0.125)$ for $t \in [-1.5, 0]$.

Figure 1 depicts the time responses of state variables of $x_1(t)$ and $x_2(t)$ with step $h = 0.01$, and Fig. 2 depicts the phase plots of state variables $x_1(t)$ and $x_2(t)$. It confirms that the proposed condition leads to the unique 2π-periodic solution for the model.

Example 2. Consider

$$\begin{aligned}\frac{dx_1(t)}{dt} &= -\big(7 + \sin x_1(t)\big)\big[3x_1(t) - \tanh x_1(t-1) - \tanh x_2(t-1) + 2\big],\\ \frac{dx_2(t)}{dt} &= -\big(4 + \cos x_2(t)\big)\big[4x_2(t) - \tanh x_1(t-1) - \tanh x_2(t-1) + 3\big]. \qquad (17)\end{aligned}$$

This system satisfies all assumptions in this paper with $\underline{\alpha}_1 = 6$, $\underline{\alpha}_2 = 3$, $\bar{\alpha}_1 = 8$, $\bar{\alpha}_2 = 5$, $L_1 = L_2 = 1$, $\beta_1 = 3$, $\beta_2 = 4$, $\tau_{ij}(t) \equiv 1$ $(i, j = 1, 2)$, then

$$\begin{aligned}&\underline{\alpha}_1\beta_1 - \bar{\alpha}_1|d_{11}|L_1 - \bar{\alpha}_1|d_{12}|L_2 = 18 - 8 - 8 = 2 > 0,\\ &\underline{\alpha}_2\beta_2 - \bar{\alpha}_2|d_{21}|L_1 - \bar{\alpha}_2|d_{22}|L_2 = 12 - 5 - 5 = 2 > 0,\end{aligned}$$

from Corollary 6 we know the solutions of system (17) are globally exponentially stable.

For numerical simulation, the following two cases are given:
Case 1 with the initial state $(\varphi_1(t), \varphi_2(t)) = (-0.1 - |\sin t|, -0.1 - |\cos t|)$ for $t \in [-2, 0]$;
Case 2 with the initial state $(\varphi_1(t), \varphi_2(t)) = (-0.1 - |\sin 2t|, -0.1 - |\cos 2t|)$ for $t \in [-2, 0]$.

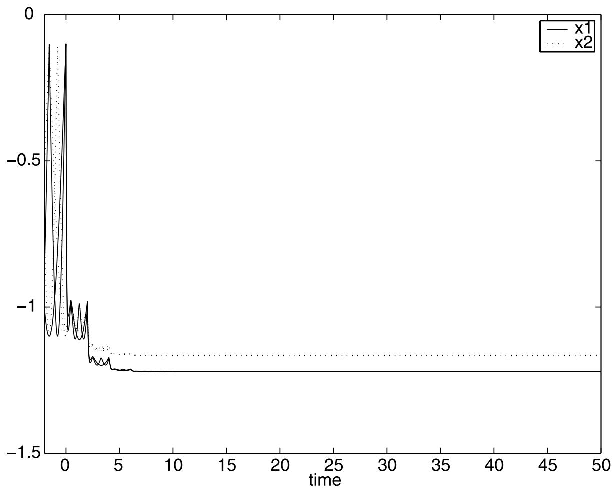

Fig. 3. Transient response of state variables $x_1(t)$ and $x_2(t)$ for Example 2.

Figure 3 depicts the time responses of state variables of $x_1(t)$ and $x_2(t)$ with step $h = 0.01$. It confirms that the proposed condition leads to the unique and globally exponentially stable solution for the model.

6. Conclusions

The dynamics of the Cohen–Grossberg neural network is studied in this paper with variable coefficients and time-varying delays. By employing the Hardy inequality, several sufficient conditions have been obtained which guarantee the model to be uniformly bounded and ultimately uniformly bounded under appropriate assumptions. The Halanay inequality and Lyapunov functional method are also used in this paper to derive some new sufficient conditions ensuring the model to be globally exponentially stable. It is noted that the criteria derived in this paper are less restrictive than those reported in [21,22,24]. Several examples and their numerical simulations are also given to illustrate the effectiveness. The results obtained in this paper are delay-independent, which implies the strong self-regulation is dominant in the networks, and moreover they are useful in the design and applications of Cohen–Grossberg neural network. In addition, the methods given in this paper may be extended for more complex systems.

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