Hydromagnetic peristaltic motion of a reacting and radiating couple stress fluid in an inclined asymmetric channel filled with a porous medium

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Abstract The aim of the present attempt was to investigate the hydromagnetic peristaltic motion of a reacting and radiating couple stress fluid in an inclined asymmetric channel filled with a porous medium. The governing equations for the flow with heat and mass transfer are presented. The Mathematical modeling is investigated by utilizing long wavelength and low Reynolds number assumptions. The exact solution has been evaluated for the stream function, velocity, temperature, concentration and pressure gradient. The pressure rise solution has been simplified using numerical integration. The effects of Hartmann number, Darcy number, thermal Grashof number, couple stress parameter, thermal radiation parameter, species Grashof number, heat generation parameter, inclination angle and chemical reaction parameter on the velocity characteristics, heat and mass transfer characteristics and pumping characteristics are presented through graphically and discussed in detail.

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1. Introduction

During the last few decades the Peristaltic motion gained considerable attention due to its various applications in physiological fluids such as vaso-motion of limited blood vessels, urine movement to bladder from kidney, absorption of food over esophagus, ovum movement in the female fallopian tube, blood pumps in the heart lung tool, chyme travel in the gastrointestinal tracts, sperm transport in the ductus efferentus of the male conception tract, movement of ovum in the fallopian tube, and absorption of food through esophagus. The principle of peristaltic transport is also exploited in many important industrial applications. The seminal work on the peristaltic motion is initially proclaimed by Latham\textsuperscript{1}. After his seminal work, there were many investigations on the peristaltic flows under different flow geometries and assumptions by employing analytical, numerical and experimental approaches which have been reported\textsuperscript{2–12}.

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particles such as polymeric suspensions, animal and human blood and lubrication. The Stokes’ problems for an incompressible couple stress fluid under the isothermal conditions have been investigated by Devakar and Iyengar [13]. Adesanya and Makinde [14] investigated the pulsatile flow of hydromagnetic non-Newtonian couple stress fluid with heat transfer between two permeable surfaces. The peristaltic hemodynamic flow of couple stress fluid through a porous medium was presented by Tripathi [15]. Abd elmaboud et al. [16] have been explored an analysis of the peristaltic flow of a couple stress fluid in a two-dimensional asymmetric channel. The literal solutions for elemental flows namely Couette, Poiseuille and generalized Couette flows of an incompressible couple stress fluid between parallel plates were studied by Devakar et al. [17]. Shit and Roy [18] have presented the impact of channel inclination on the peristaltic transport of couple stress fluids in the presence of externally applied magnetic field. Very recently, Adesanya and Makinde [19] numerically investigated the inherent irreversibility in a couple stress film flows along an inclined heated plate with adiabatic free surface. The simultaneous effects of Hall and convective conditions on peristaltic flow of couple stress fluid in an inclined asymmetric channel were studied by Hayat et al. [20]. Obaid Ullah Mehmood et al. [21] have explored simultaneous effects of dissipative heating and partial slip on peristaltic transport of Sisko fluid in asymmetric channel.

Radiative heat transfer is very decisive in the treatment of diseased tissues in cancer. Gnaneswara Reddy [22] accentuated thermal radiation and chemical reaction effects on MHD boundary layer slip flow in a porous medium with heat source and Ohmic heating. Unsteady radiative convective boundary layer flow of a Casson fluid with variable thermal conductivity was investigated by Gnaneswara Reddy [23]. Abbasi et al. [24] have studied the Peristaltic flow with convective mass condition and thermal radiation in a two-dimensional symmetric channel. Very recently, Kothandapani and Prakash [25] have conferred of thermal radiation and magnetic field on the peristaltic flow of Williamson nanofluids over a asymmetric channel. The same authors [26] studied radiation and magnetic field on peristaltic transport of nanofluids through a porous space. Effects of Joule heating on MHD Peristaltic flow of a Nanofluid with compliant walls were investigated by Gnaneswara Reddy and Venugopal Reddy [27]. Hayat et al. [28] have explored the slip effects on peristaltic transport in an asymmetric channel with transfer and chemical reaction. Hayat et al. [29] have investigated the exact solution for peristaltic transport of micropolar fluid in a channel with convective boundary conditions and heat source/sink. The Soret and Dufour effects in peristaltic transport of physiological fluids with chemical reaction were investigated by Hayat et al. [30]. Very recently, the influence of velocity slip and joule heating on MHD peristaltic flow through a porous medium with chemical reaction has been studied by Gnaneswara Reddy and Venugopal Reddy [31].

The mass transfer in a non-Newtonian fluid at a peristaltic surface with temperature-dependent viscosity was investigated by Eldabe et al. [32]. Srinivas and Kothandapani [33] have discussed the consequence of heat and mass transfer on peristaltic transport in a porous space. Mixed convective heat and mass transfer in a vertical wavy channel with traveling thermal waves and porous medium was studied by Muthuraj and Srinivas [34]. Nadeem et al. [35] have proposed the influence of heat and mass transfer on the peristaltic flow of a third order fluid in a diverging tube. Nadeem and Akbar [36] have presented the impact of heat and mass transfer on the peristaltic flow of Johnson–Segalman fluid in a vertical asymmetric channel with induced magnetic field. Hayat and Hina [37] have studied the effects of heat and mass transfer on peristaltic flow of Williamson fluid in a non uniform channel with slip conditions. The wall properties effect on peristaltic transport of micropolar fluid in the presence of heat and mass transfer has been presented by Eldabe and Abu-Zied [38].

Heat and mass transfer flows with chemical reaction are significant in many processes such as drying, dehydrogenation at the surface of a water body, geothermal pool, thermal insulation, enhanced oil recovery, cooling of nuclear reactors and the flow in a desert cooler. These types of flows have many applications in industries. Many practical diffusive operations contain the molecular diffusion of species in the presence of chemical reaction within or at the boundary. Heat and mass transfer effects are also detected in the chemical industry, in reservoirs, in thermal recovery mechanism and in the investigation of hot salty springs in the sea. Peristalsis with heat and mass transfer is employed in medical operations to increase the mass transport (blood oxygenator). Nadeem and Akbar [39] have conferred the influence of chemical reactions on peristaltic flow of Newtonian fluid in a deviating tube. Heat and chemical reaction on peristaltic flow and chemical reaction in compliant walls were investigated by Hina et al. [40]. The consequence of inclined magnetic field and mass transfer in peristaltic flow with chemical reaction was studied by Noreen et al. [41].

The aim of present investigation was to study the effect of thermal radiation and chemical reaction on peristaltic MHD slip flow of a couple stress fluid through a porous medium in an asymmetric channel. This paper runs in the following arrangement. The problem is first modeled, non-dimensional governing equations under the long wavelength and low Reynolds number approximation and governing equations are formulated, and the corresponding boundary conditions are prescribed in Section 2. The exact solution of stream function, velocity, temperature, concentration and pressure gradient is obtained in Section 3. Section 4 includes the results for the velocity, temperature, concentration, pressure gradient and pressure rise which have been discussed for various values of the physical parameters. Finally, the important findings are summarized in Section 5.

2. Mathematical formulation

Consider the flow of an incompressible electrically conducting couple stress fluid in a two-dimensional inclined asymmetric channel of width $d_1 + d_2$ filled with a porous medium. The Cartesian coordinate system $(X, Y)$ is chosen in such way that the direction of wave propagation is in $X$-direction and $Y$-coordinate is perpendicular to the flow direction. Asymmetry in the channel is produced by selecting the peristaltic wave trains propagating with constant speed $c$ along the walls ($H_1$ is the lower wall and $H_2$ is the upper wall) with different amplitudes and phases. The upper wall of the peristaltic channel is preserved at temperature $T_1$ and concentration $C_1$ while the lower wall is preserved at temperature $T_2$ and concentration $C_0$. The fluid is subject to a constant transverse magnetic field $B_0$ as shown in Fig. 1. A very small magnetic Reynolds number
is assumed and hence the induced magnetic field is neglected in comparison with the applied magnetic field. Further, it is also assumed that the internal dissipation is neglected. The geometry of the asymmetric channel wall surface expressions is of the following form:

\[ Y = H_1(X, Y, t) = d_1 + a_1 \cos \left( \frac{2\pi}{\lambda} (X - ct) \right) \] (1)

\[ Y = H_2(X, Y, t) = -d_2 - a_2 \cos \left( \frac{2\pi}{\lambda} (X - ct) + \phi \right) \] (2)

where \( a_1 \) and \( a_2 \) are the wave amplitudes, \( \lambda \) is the wavelength, \( d_1 + d_2 \) is the channel width, \( c \) is the velocity of propagation, \( t \) is the time and \( X \) is the direction of wave propagation. \( \phi \) (\( 0 \leq \phi \leq \pi \)) is the phase difference, in which \( \phi = 0 \) and \( \phi = \pi \) correspond to symmetric channel with waves out of phase and in phase respectively and further \( a_1, a_2, d_1, d_2 \) and \( \phi \) satisfy the relation of the inlet of divergent channel:

\[ a_1^2 + a_2^2 + 2a_1a_2 \cos \phi \leq (d_1 + d_2)^2 \] (3)

In laboratory frame, in the absence of body forces and the body couples, the governing equations of balance of mass, momentum, temperature and concentration for the peristaltic transport of an incompressible radiating and reacting couple stress fluid through a porous medium in an inclined irregular channel are

\[ \rho \left( \frac{\partial V}{\partial t} + U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} \right) = -\frac{\partial P}{\partial Y} + \mu \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) - \eta \left( \frac{\partial^2 V}{\partial X^2} + 2 \frac{\partial^2 V}{\partial X \partial Y} + \frac{\partial^2 V}{\partial Y^2} \right) \]

\[ -\frac{\mu}{k_o} V + pg\beta_r(T - T_0) \cos x \]

\[ + \rho g \beta_c (C - C_0) \sin x \] (5)

\[ \rho c_p \left( \frac{\partial T}{\partial t} + U \frac{\partial T}{\partial X} + V \frac{\partial T}{\partial Y} \right) = k' \left( \frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial Y^2} \right) + Q_o - \frac{\partial q_r}{\partial Y} \] (6)

\[ \left( \frac{\partial C}{\partial t} + U \frac{\partial C}{\partial X} + V \frac{\partial C}{\partial Y} \right) = D \left( \frac{\partial^2 C}{\partial X^2} + \frac{\partial^2 C}{\partial Y^2} \right) + \frac{Dk_T}{T_m} \left( \frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial Y^2} \right) - k_1 (C - C_m) \] (7)

\[ (4) - (8) \] can be written as

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \] (11)
\[ \rho \left( \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = - \frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - \eta \left( \frac{\partial^2 u}{\partial x^2} + 2 \frac{\partial^4 u}{\partial x^2 \partial y^2} + \frac{\partial^2 u}{\partial y^2} \right) - \sigma B_i(u + c) + \rho g \beta_r(T - T_0) + \frac{\partial}{\partial y} \left( \frac{\partial^2 \psi}{\partial y^2} \right) - \delta \frac{\partial^2 \psi}{\partial y^2} \right) = - \frac{\partial p}{\partial x} + \left( \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} \right) - \frac{1}{\gamma} \left( \frac{\partial^2 \psi}{\partial x^2} + 2 \frac{\partial^4 \psi}{\partial x^2 \partial y^2} + \frac{\partial^2 \psi}{\partial y^2} \right) \]
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\[ \psi = - F \frac{\partial \psi}{\partial y} - L \frac{\partial^2 \psi}{\partial y^2} = -1, \frac{\partial^2 \psi}{\partial y^2} = 0, \theta = 1, \phi = 1 \text{ at } y = 0, \]
\[ h_2 = -d - b \cos(2\pi x + \phi) \]  
(27)

where \( L \) is the velocity slip parameter and \( F \) is the flux in the wave frame and the constants \( a, b, \phi \) and \( d \) should satisfy the relation
\[ a^2 + b^2 + 2ab \cos \phi \leq (1 + d)^2. \]  
(28)

The dimensionless mean flow rate \( \Theta \) in the fixed frame is related to the non-dimensional mean flow rate \( F \) in wave frame by
\[ \Theta = F + 1 + d \]  
(29)

and in which
\[ F = \int_{h_1}^{h_2} \frac{\partial \psi}{\partial y} \, dy \]  
(30)

3. Solution of the problem

Under the assumptions of long wavelength \((\delta << 1)\) and low Reynolds number, neglecting the terms of order \( \delta \) and higher order in Eqs. (23)–(26) can be written as
\[ \frac{\partial p}{\partial x} = \frac{\partial^2 \psi}{\partial y^2} \left( \frac{M^2 + 1}{Da} \right) \left( \frac{\partial \psi}{\partial y} + 1 \right) + Gr \theta \sin x + Gc \phi \sin x \]  
(31)

\[ \frac{\partial p}{\partial y} = 0 \]  
(32)

\[ \left( \frac{1}{Fr} + Rd \right) \frac{\partial^2 \theta}{\partial y^2} + \beta = 0 \]  
(33)

\[ \frac{\partial^2 \phi}{\partial y^2} - Sc \gamma_1 \phi + ScSr \frac{\partial^2 \theta}{\partial y^2} = 0 \]  
(34)

Eliminating the pressure from (31) and (32) by cross differentiation, we have
\[ \frac{\partial^2 \psi}{\partial y^2} - \frac{1}{\gamma^2} \frac{\partial^2 \psi}{\partial y^2} \left( M^2 + \frac{1}{Da} \right) \frac{\partial^2 \psi}{\partial y^2} + Gr \frac{\partial \theta}{\partial y} \sin x + Gc \frac{\partial \phi}{\partial y} \sin x = 0 \]  
(35)

Solving (33) and (34) with the dimensionless boundary conditions (27), the expressions for the temperature and concentration are
\[ \theta = B_1 + B_2y + B_3y^2 \]  
(36)

\[ \phi = C_1 \cosh(N_1y) + C_2 \sinh(N_2y) + C_3 \]  
(37)

Using dimensionless temperature and concentration (36) and (37) in (35) through boundary conditions (27), we obtain the stream function as
\[ \psi = A_1 + A_2y + A_3 \cosh(N_3y) + A_4 \sinh(N_3y) + A_5 \]  
\[ \times \cosh(N_4y) + A_6 \sinh(N_4y) + A_7 + A_8y + A_9y^2 \]  
\[ + A_{10}y^3 + A_{11} \sinh(N_3y) + A_{12} \cosh(N_2y) \]  
(38)

The axial velocity is obtained with the help of Eq. (38) and is given by
\[ u = A_2 + A_3 \sinh(N_3y) + A_4 \cosh(N_3y) + A_5 \]  
\[ \times \cosh(N_4y) + A_6 \sinh(N_4y) + A_8 + 2A_9y \]  
\[ + 3A_{10}y^2 + A_{11} \sinh(N_3y) + A_{12} \cosh(N_2y) \]  
(39)

The pressure gradient obtained from the non-dimensional momentum equation for the axial velocity is given by
\[ \frac{dp}{dx} = D_1 \sinh(N_3y) + D_2 \cosh(N_3y) + D_3 \sinh(N_4y) + D_4 \]  
\[ \times \cosh(N_4y) + D_5 \sinh(N_2y) + D_6 \sinh(N_2y) \]  
\[ + D_7y + D_8y^2 + D_9 \]  
(40)

The dimensionless pressure difference \( \Delta P_x \) is given by
\[ \Delta P_x = \int_0^1 \left( \frac{dp}{dx} \right) dx \]  
(41)

where the involved constants in Eqs. (36)–(41) are given in Appendix.

4. Results and discussion

In this section, the effects of embedded dynamical parameters on the quantities of interest are examined through graphical plots. Plots for the axial velocity, temperature, concentration, pressure gradient and pressure rise are obtained and studied through Figs. 2–24. In the present study following default fixed parameter values are adopted for numerical computations:
\[ x = 1, a = 0.7, b = 0.5, d = 1, Da = 0.5, \gamma = 5, \phi = \frac{\pi}{3}, Gr = 0.1, \]  
\[ \zeta = \frac{\pi}{3} \]  
\[ L = 0, M = 1, Sr = 1.5, Sc = 1.5, \Theta = -1, \beta = 0.2, \gamma_1 = 1, \]  
\[ Pr = 1, Rd = 0.1, Gc = 0.5 \]  

4.1. Velocity profile

Figs. 2–9 are plotted for axial velocity \( u \) to the effects of Hartmann number \( M \), Darcy number \( Da \), chemical reaction parameter \( \gamma_1 \), couple stress parameter \( \gamma \), heat generation parameter \( \beta \), local temperature Grashof number \( Gr \), local species Grashof number \( Gc \), inclination parameter \( x \) and partial slip parameter \( L \). It is seen that from these figures it
Figure 3  Effect of $\gamma_1$ on velocity profile.

Figure 4  Effect of $\gamma$ on velocity profile.

Figure 5  Effect of Gr on velocity profile.

Figure 6  Effect of $G_c$ on velocity profile.

Figure 7  Effect of $\beta$ on velocity profile.

Figure 8  Effect of $z$ on velocity profile.
Figure 9  Effect of $L$ on velocity profile.

Figure 10  Effect of $Pr$ and $Rd$ on temperature profile.

Figure 11  Effect of $\beta$ on temperature profile.

Figure 12  Effect of $Sc$ and $\gamma_1$ on concentration profile.

Figure 13  Effect of $Sc$ and $Sr$ on concentration profile.

Figure 14  Effect of $M$ and $Da$ on pressure gradient.
Figure 15 Effect of $\gamma_1$ on pressure gradient.

Figure 16 Effect of $\gamma$ on pressure gradient.

Figure 17 Effect of $\beta$ on pressure gradient.

Figure 18 Effect of $Gr$ on pressure gradient.

Figure 19 Effect of $\phi$ on pressure gradient.

Figure 20 Effect of $M$ and $Da$ on pressure rise.
is witnessed that the plots for velocity are concaved downward. These plots show a parabolic trajectory with maximum value occurring near the center of channel. Fig. 2 displays the effects of Hartmann number and Darcy number on the fluid velocity. It shows from this figure that an increase in the value of Hartmann number results in a diminished of the velocity near the center of the channel while the opposite trend exists for the Darcy number. Hartman number and Darcy number have opposite effects on the flow velocity. This identification agrees with the physical interpretation that, because the effect of increasing magnetic field strength dampens the velocity due to the Lorentz force an increasing $Da$ means it reduces the drag force and hence causes the flow velocity to increase. Fig. 3 analyzes the influence of chemical reaction parameter $c_1$ on velocity. It can be observed that the axial velocity increases at the left wall while a reverse trend is seen at the right wall when the chemical reaction parameter $c_1$ increases but the similar trend follows for the couple stress parameter $c$, local temperature Grashof number $Gr$ and local species Grashof number $Gc$ on the velocity profile from Figs. 4–6. Fig. 7 illustrates that the axial velocity increases as the increasing values of heat generation parameter $b$ near the center of the channel. Fig. 8 depicts the influence of inclination angle on the axial velocity and found that the velocity has the similar trend observed that of the chemical reaction parameter. Fig. 9 depicts the variation in axial velocity for different values of the slip parameter $L$. It is noticed that increasing values of slip parameter enhance the axial velocity near the channel walls, while it is decreasing function at the center of the channel.

4.2. Heat transfer characteristics

In this subsection, the influence of the effect of Prandtl number $Pr$ and thermal radiation parameter $Rd$ and heat generation parameter $\beta$ for the Temperature profile are illustrated through Figs. 10 and 11. From Fig. 10, it can be shows that the temperature profile enhances with an increase in Prandtl number where as temperature diminishing function to with the larger values of thermal radiation parameter. Temperature is found to increase considerably with increase in the value of heat generation parameter $\beta$ which is evident from Fig. 11.
4.3. Mass transfer characteristics

In this subsection, the concentration distribution $\phi$ for different values of Schmidt number $Sc$, chemical reaction parameter $\gamma_1$, and Soret number $Sr$ is shown in Figs. 12 and 13. The Schmidt number $Sc$ defines the ratio of the momentum diffusivity to the mass (species) diffusivity. It is evident from Fig. 12, that the concentration field decreases with an increase in $Sc$, which is due to increase in Schmidt number; the mass diffusion decreases and concentration distribution decreases; and the same behavior is for the chemical reaction parameter $\gamma_1$. We found from Fig. 13, that the concentration field decreases with the increasing $Sr$.

4.4. Pumping characteristics

The results are presented to the influence of different emerging parameters on the pressure gradient and are shown in Figs. 14–19. It is understood that from these figures, the pressure gradient is proportionately small in the ample part of the channel $x \in [0, 0.3]$ and $x \in [0.7, 1]$. That is, the flow can easily move without imposition of a larger pressure gradient. On the other hand, in the narrow part of the channel $x \in [0.3, 0.7]$, particularly near $x = 0.41$, the pressure gradient is required to maintain the same flux to pass it. Fig 14 depicts to examine the effect of Hartmann number and Darcy number on the pressure gradient. This figure shows that the value of pressure gradient increases by a small amount when Hartmann parameter enhances in the occluded part of channel while the reverse effect for the Darcy number. Figs. 15–18 illustrate that the pressure gradient decreases with an increase in the values of $\gamma_1$, $\gamma$, $\beta$ whereas the opposite effect is for thermal Grashof number $Gr$. Fig. 19 displays that the pressure gradient decreases in the narrow part of the channel and increases in the wider part of the channel, the narrow region is shifted to the left side with an increase in $\phi$ and lesser amount of pressure gradient is required to pass the flow in ample part in an asymmetric channel to the values of increase in $\phi$.

The pressure rise is an important physical measure in the peristaltic mechanism. The results are prepared and discussed for different physical parameters of interest through Figs. 20–24 and are plotted for dimensionless pressure rise $\Delta P_z$ versus the dimensionless flow rate $\Theta$ to the effects of Hartmann number $M$ with Darcy number $Da$, chemical reaction parameter $\gamma_1$, couple stress parameter $\gamma$, heat generation parameter $\beta$ and partial slip parameter $L$. The pumping regions are peristaltic pumping ($\Theta > 0, \Delta P_z > 0$), augment pumping ($\Theta < 0, \Delta P_z < 0$), retrograde pumping ($\Theta < 0, \Delta P_z > 0$), co pumping ($\Theta > 0, \Delta P_z < 0$) and free pumping ($\Theta = 0, \Delta P_z = 0$). Fig. 20 shows that the pressure rise $\Delta P_z$ decreases with an increase of Hartmann number $M$ in the both peristaltic pumping region and free pumping region while the reverse behavior of pressure rise as the larger values of $Da$. Fig. 21 depicts the dimensionless pressure rise per wavelength $\Delta P_z$ against the dimensionless average flux $\Theta$ to the influence of chemical reaction parameter $\gamma_1$. It is observed that the pressure rise decreases when the chemical reaction parameter $\gamma_1$ increases. The influence of couple stress fluid parameter $\gamma$ on the pressure rise decreases and is elucidated from Fig. 22. Fig. 23 depicts the quite opposite nature to that of Fig. 22 for the effect of the heat generation parameter $\beta$. The impact of slip parameter $L$ on dimensionless pressure rise per wavelength $\Delta P_z$ against the dimensionless average flux $\Theta$ is shown in Fig. 24. We observed that the pressure rise decreases for small values of $\Theta$ with the increase in $L$; however, pressure rise increases for large values of $\Theta$ with the increase in $L$.

In Table 1, the numerical results for the pressure rise are prepared by using computational software Mathematica. A comparison of obtained results in limiting case of the present study with the ones from the open literature for particular values of the governing parameters. In the absence of the Hartmann number $M$ ($M = 0$) and $x = \pi/2$ (not inclined channel), it is found from Table 1 that with the increase in couple stress parameter, the pressure rise decreases. These observations are not only in accordance with the physical expectations of pertinent parameter but also have good agreement with the existing literature [42]. The present attempt will be beneficial in innumerable clinical applications. This analysis gives a better perception for the speed of injection and the fluid flow characteristics within the syringe.

5. Conclusion

In the present paper, we have presented the peristaltic motion of a couple stress fluid over an inclined asymmetric channel filled with a porous medium by taking into account the effects of thermal radiation, partial slip and chemical reaction parameter. The two-dimensional governing equations have been modeled and then simplified using the long wavelength approximation and then analytical solution for stream function, velocity, temperature, concentration and pressure gradient is obtained. The results are discussed with the help of graphs. We have concluded the following key observations:

1. Velocity rises in the left wall of the channel with enhancing chemical reaction parameter while the opposite behavior is at the right wall of the channel.
2. Axial velocity increases with increase in slip parameter $L$ and Hartmann number $M$ near the channel walls while velocity decreases at the center of the channel and the opposite effect for the Darcy number $Da$.
3. The pressure gradient decreases as the larger values of chemical reaction parameter.
4. Prandtl number and thermal radiation parameter have opposite effects on the temperature.
5. Our results are in good agreement with Ref. [42] when $L = 0$, $\gamma_1 = 0$ and $Rd = 0$.
6. Concentration decreasing function to the higher values of $Sc$ and $\gamma_1$.

<table>
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<th>$\gamma = 5$</th>
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</tr>
</tbody>
</table>
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Appendix A

\[ N_1 = \left( \frac{M^2 + 1}{Da} \right), \quad N_3 = \sqrt{\frac{\gamma^2 + \gamma^2 - \frac{4N_1^2}{2}}{2}}, \]

\[ N_4 = \sqrt{\frac{\gamma^2 - \gamma^2 - \frac{4N_1^2}{2}}{2}}, \]

\[ A_7 = \frac{Gr y^2 \sin zB_2}{N_1^2}, \quad A_8 = \frac{2Gr y^2 \sin zB_2}{N_1^2}, \quad A_9 = \frac{Gr y^2 \sin zB_2}{2N_1^4}, \]

\[ A_{10} = \frac{Gr y^2 \sin zB_2}{3N_1^2}, \]

\[ A_{11} = \frac{Gr y^2 \sin \alpha C_1}{N_1^2}, \quad A_{12} = \frac{Gr y^2 \sin \alpha C_2}{N_1^2}, \quad A_{13} = \cosh(N_1 h_1) - \cosh(N_1 h_2), \]

\[ A_{14} = \sinh(N_1 h_1) - \sinh(N_1 h_2), \quad A_{15} = \cosh(N_1 h_1) - \cosh(N_1 h_2), \]

\[ A_{16} = \sinh(N_4 h_1) - \sinh(N_4 h_2), \]

\[ A_{21} = \cosh(N_2 h_1) - \cosh(N_2 h_2), \]

\[ A_{22} = A_{23} + A_{24} + A_{25} + A_{26} + A_{27} = \frac{(\alpha - A_{22})}{A_{17}}, \]

\[ A_{28} = N_5 \sinh(N_1 h_1) + LN_5 \cosh(N_1 h_1), \]

\[ A_{29} = N_5 \sinh(N_1 h_1) + LN_5 \cosh(N_1 h_1), \]

\[ A_{30} = N_5 \sinh(N_1 h_1) + LN_5 \cosh(N_1 h_1), \]

\[ A_{31} = N_6 \cosh(N_2 h_1) + LN_6 \cosh(N_2 h_1), \]

\[ A_{32} = -1 - (A_8 + 2LA_9 + (2A_8 + 6LA_{10} h_1 + 3A_{10} h_1^2 + N_2 \times \cosh(N_2 h_1)(A_{11} + A_{12} LN_2) + N_2 \sinh(N_2 h_1)(A_{11} + A_{12} LN_2), \]

\[ A_{33} = N_5 \sinh(N_3 h_1) - LN_5 \cosh(N_3 h_2), \]

\[ A_{34} = N_5 \sinh(N_3 h_2) - LN_5 \cosh(N_3 h_2), \]

\[ A_{35} = N_4 \sinh(N_2 h_2) - LN_4 \cosh(N_2 h_2), \]

\[ A_{36} = N_4 \cosh(N_2 h_1) - LN_4 \cosh(N_2 h_2), \]

\[ A_{37} = -1 - (A_8 + 2LA_9) + (2A_8 + 6LA_{10} h_2 + 3A_{10} h_2^2 + N_2 \times \cosh(N_2 h_2)(A_{11} + A_{12} LN_2) + N_2 \sinh(N_2 h_2)(A_{11} + A_{12} LN_2), \]

\[ A_{38} = A_{23} - A_{28}, A_{39} = A_{24} - A_{29}, A_{40} = A_{25} - A_{30}, \]

\[ A_{41} = A_{26} - A_{31}, A_{42} = A_{27} - A_{32}, \]

\[ A_{43} = A_{23} - A_{33}, A_{44} = A_{24} - A_{34}, A_{45} = A_{25} - A_{35}, \]

\[ A_{46} = A_{26} - A_{36}, A_{47} = A_{27} - A_{37}, \]

\[ A_{48} = N_3^2 \sinh(N_3 h_1), A_{49} = N_3^2 \cosh(N_3 h_1), \]

\[ A_{50} = N_3^2 \sinh(N_3 h_1), A_{51} = N_3^2 \cosh(N_3 h_1), \]

\[ A_{52} = - (6A_{10} + A_{11} N_3^2 \cosh(N_2 h_1) + A_{12} N_3^2 \sinh(N_2 h_1)), \]

\[ A_{53} = N_3^2 \sinh(N_3 h_2), \]

\[ A_{54} = N_3^2 \cosh(N_3 h_2), A_{55} = N_3^2 \sinh(N_3 h_2), \]

\[ A_{56} = N_3^2 \cosh(N_3 h_2), \]

\[ A_{57} = - (6A_{10} + A_{11} N_3^2 \cosh(N_2 h_2) + A_{12} N_3^2 \sinh(N_2 h_2)), \]

\[ A_{58} = \frac{A_{39}}{A_{38}} - \frac{A_{44}}{A_{43}} - \frac{A_{59}}{A_{58}} - \frac{A_{40}}{A_{43}} - \frac{A_{45}}{A_{46}}, \]

\[ A_{59} = \frac{A_{40}}{A_{43}}, A_{60} = \frac{A_{44}}{A_{43}}, \]

\[ A_{61} = \frac{A_{42}}{A_{43}}, A_{62} = \frac{A_{44}}{A_{43}} - \frac{A_{49}}{A_{48}}, \]

\[ A_{63} = \frac{A_{45}}{A_{46}}, A_{64} = \frac{A_{46}}{A_{48}}, \]

\[ A_{65} = \frac{A_{47}}{A_{48}}, A_{66} = \frac{A_{49}}{A_{48}}, \]

\[ A_{67} = \frac{A_{50}}{A_{48}} - \frac{A_{55}}{A_{53}}, \]

\[ A_{68} = \frac{A_{53}}{A_{48}} - \frac{A_{56}}{A_{48}}, A_{69} = \frac{A_{52}}{A_{48}} - \frac{A_{57}}{A_{53}}, \]

\[ A_{70} = \frac{A_{59}}{A_{58}} - \frac{A_{63}}{A_{58}}, \quad A_{71} = \frac{A_{59}}{A_{58}} - \frac{A_{64}}{A_{62}}, \]

\[ A_{72} = \frac{A_{61}}{A_{58}} - \frac{A_{65}}{A_{58}}, A_{73} = \frac{A_{63}}{A_{58}} - \frac{A_{67}}{A_{62}}, \]

\[ A_{74} = \frac{A_{64}}{A_{62}} - \frac{A_{68}}{A_{66}}, A_{75} = \frac{A_{65}}{A_{62}} - \frac{A_{69}}{A_{66}}, A_{76} = \frac{A_{71}}{A_{70}} - \frac{A_{74}}{A_{73}}, \]

\[ A_{77} = \frac{A_{72}}{A_{70}} - \frac{A_{75}}{A_{73}} - \frac{A_{76}}{A_{76}}, \]

\[ A_{5} = \frac{1}{A_{73}}(A_{75} - A_{74} A_{6}), A_{4} = \frac{1}{A_{66}}(A_{69} - A_{65} A_{5} - A_{6} A_{6}), \]

\[ A_{3} = \frac{1}{A_{48}}(A_{52} - A_{49} A_{4} - A_{50} A_{5} - A_{6} A_{5}), \]

\[ A_{2} = \frac{1}{A_{17}}(F - A_{13} A_{3} - A_{4} A_{14} - A_{5} A_{15} - A_{6} A_{16} - A_{22}), \]

\[ A_{1} = \frac{F}{2} - A_{12} h_1 - A_3 \cosh(N_1 h_1) - A_4 \sinh(N_1 h_1) - A_5 \cosh(N_1 h_1) - A_6 \sinh(N_1 h_1) - A_7 - A_8 h_1 - A_9 h_2 \]

\[ - A_9 h_1^2 - A_10 h_2^2 - A_11 \sinh(N_2 h_1) - A_12 \cosh(N_2 h_1) \]

\[ B_{1} = -[B_2 h_1 + B_3 h_2], \quad B_{2} = \frac{1 - B_2 h_1^2}{h_2 - h_1}, \]

\[ B_{3} = \frac{-\beta Pr}{2(1 + Pr R)} h_2^2, \]

\[ N_2 = \sqrt{Sc_1}, \]

\[ C_1 = \frac{\sinh(N_2 h_1) - C_2 \sinh(N_2 h_1) - \sinh(N_2 h_2)}{\sinh(N_2 h_1)}, \]

\[ C_2 = \frac{-C_1 + C_2 \cosh(N_2 h_1)}{\sinh(N_2 h_1)}, \]
\[ C_3 = \frac{2ScSrB_3}{N_2}, \quad D_1 = A_3N_1^3 - A_3N_1^2 - A_3N_1, \]
\[ D_2 = A_4N_1^3 - \frac{A_4N_1^2}{\gamma} - A_4N_1^2, \]
\[ D_3 = A_5N_1^4 - \frac{A_5N_1^3}{\gamma} - A_5N_1^2, \]
\[ D_4 = A_6N_1^3 - \frac{A_6N_1^2}{\gamma} - A_6N_1, \]
\[ D_5 = A_{11}N_1^2 - A_{11}N_1^2 + A_{11}N_1^2 + GcC_1 \sin z, \]
\[ D_6 = A_{12}N_1^2 - A_{12}N_1^2 + A_{12}N_1^2 + GcC_2 \sin z, \]
\[ D_7 = Gr \sin \alpha B_3 + 2A_9, \]
\[ D_8 = Gr \sin \alpha B_3 + 3A_{10}, \]
\[ D_9 = Gr \sin \alpha B_3 + 6A_{10} + GcC_3 \sin z - N_1^2(1 + A_2) + A_8. \]

References


