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Fluid-Structure Interaction analysis of a Cropped Delta Wing

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Abstract

The fluid-structure interaction behaviour of a cropped delta wing is investigated using a computational fluid dynamics (CFD) based aeroelastic solver in time domain. Here, an N-S based finite volume solver is coupled with a finite element based linear and nonlinear structural solvers to study the nonlinear aeroelastic characteristics of the delta wing over a range of dynamic pressures. The amplitude and frequency obtained from the present coupled solver at various dynamic pressures are compared with the available experimental and computational results.

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Keywords: Fluid structure interaction; Delta wing; Limit cycle oscillations; Geometric nonlinearity, Navier-Stokes equations

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1. Introduction

Static or dynamic deflections of moderate to large amplitudes are generally a serious problem in the design of both military and commercial aircrafts. Limit Cycle Oscillation (LCO) is such a limited-amplitude self-sustaining vibration resulting from the nonlinear interaction between the unsteady aerodynamics and the structural response. The limiting of the oscillation amplitude is due to the presence of nonlinearity in the aeroelastic system i.e. in fluid, structural or in both. This is generally undesirable resulting in a reduction in vehicle performance which leads to airframe limiting structural fatigue and compromises the ability of pilots to perform critical mission related tasks [1]. Most of the LCOs are associated with nonlinear aerodynamic mechanisms such as shock wave motions, shock-induced flow separation and leading edge vortex flows [2]. Predicting and investigating LCO is challenging due to the inherent non-linear nature of the problem. Accurate prediction of LCOs is necessary in expanding the flight

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envelope and mission requirements of the existing aircrafts. Flutter prediction methods based on linear approaches are inadequate in computing the occurrence of LCOs. Even the usage of accurate methods such as flight testing and wind tunnel experiments for finding the LCO occurrence is difficult owing to the fact that it is unable to distinguish LCO from the onset of flutter divergence [1]. Therefore, it is necessary to develop good non-linear prediction techniques for the simulation of LCOs.

Improvements in the computational fluid dynamics (CFD) technologies and its coupling with computational structural dynamics (CSD) solver paved a way for its usage in the areas of many transonic aeroelastic analyses such as transonic flutter simulations, transonic buffet, etc. Also it provides a prediction capability for simulating the complex, nonlinear aerodynamics and structural dynamics associated with LCO phenomena. Nonlinearities in fluid dynamics have been reasonably understood well with the help of CFD in the context of aeroelasticity. But in most of these analyses, the structural methods are assumed to be linear. Investigations using non-linear structural models in computational aeroelasticity have been done by few researchers. Gordnier [2, 3] used von Karman plate theory as the nonlinear structural model for the LCO study of the swept delta wing. Attar et al. [4] used a higher fidelity structural theory based on co-rotational formulation for the prediction of LCO of the delta wing configuration. In all the above works, the structural model is based on two-dimensional finite element with different non-linear structural theories. In the present work, the LCO problem of the cropped delta wing is investigated using three dimensional solid elements.

1.1. Objective

The objective of the present work is to validate the coupled aeroelastic solver for its nonlinear aeroelastic prediction capabilities. The combination of high fidelity models for both aerodynamics and structural dynamics is used to predict the LCO of the delta wing [3]. The aerodynamic model is based on N-S based CFD method and the computational structural dynamics (CSD) model is based on three-dimensional nonlinear finite element method with geometric nonlinearity. Both the solvers are coupled explicitly in time domain to study the nonlinear FSI behavior of the delta wing, particularly its LCO characteristics. The comparison of the present results is also done with the available experimental and computational results.

2. Methodology

The methodology adopted for the present FSI analysis is the partitioned based coupled CFD and CSD solvers in time domain. The governing equations and the solution methodology for each of the solvers are discussed in the following sections.

2.1. Governing equation of fluid dynamics

The equation governing the fluid flow is the integral form of the three dimensional, unsteady, compressible N-S equations written in strong conservation form for the moving/deforming control volume given by:

$$\frac{\partial}{\partial t}\int_V \mathbf{W}dV + \oint\left[\mathbf{F}-\mathbf{G}\right]\cdot d\mathbf{A} = \int_V \mathbf{H}dV \tag{1}$$

where the vectors **W** (vector of conservative variables), **F** (vector of convective fluxes) and **G** (vector of diffusive fluxes) are defined as:

$$\mathbf{W}=\begin{Bmatrix}\rho\\ \rho\mathbf{v}\\ \rho E\end{Bmatrix}\quad \mathbf{F}=\begin{Bmatrix}\rho\left(\mathbf{v}-\mathbf{v}_g\right)\\ \rho\left(\mathbf{v}-\mathbf{v}_g\right)\otimes\mathbf{v}+p\mathbf{I}\\ \rho\left(\mathbf{v}-\mathbf{v}_g\right)H+p\mathbf{v}_g\end{Bmatrix}\quad \mathbf{G}=\begin{Bmatrix}0\\ \mathbf{T}\\ \mathbf{T}\cdot\mathbf{v}+\dot{\mathbf{q}}''\end{Bmatrix} \tag{2}$$

and the vector **H** contains the body forces and energy sources. Here ρ, $\mathbf{v}$, $\mathbf{v}_g$, E and p are the density, velocity vector, grid velocity vector, total energy per unit mass and pressure of the fluid respectively. **T** is the viscous stress tensor and $\dot{\mathbf{q}}''$ is the heat flux. Total energy E and the total enthalpy H is related by $E = H - p/\rho$, where $H = h + |\mathbf{v}|^2/2$ and $h = C_p T$.

The governing equation is then converted to discrete form by applying it to a cell-centered control volume. The discrete form will have spatial terms and temporal terms which are discretized as given in table 1.

Table 1. Spatial and temporal discretization schemes

Terms	Discretization
Temporal term	implicit, second order, 3-point backward Euler
Convective term	Weiss-Smith preconditioned Roe's flux-difference splitting scheme
Diffusive term	Second-order accurate scheme
Interpolation	Second-order upwind scheme
Gradients	Hybrid Gauss-LSQ
Limiter function	Venkatakrishnan limiter

For unsteady flow simulations, dual-time stepping approach is used. The resulting nonlinear set of equations is solved using algebraic multi-grid approach. For the problems involving moving/deforming domains, the geometric conservation law (GCL) needs to be satisfied.

2.2. Governing equation of structural dynamics

The nonlinear differential equations based on the principle of virtual work in combination with finite element method can be written in matrix form as:

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = \{F(t)\} \tag{3}$$

where $[M]$, $[C]$ and $[K]$ are the mass, damping and stiffness matrices respectively and $\{F(t)\}$ is the applied load vector which are due to pressure and shear stress on the wing. $\{u\}$, $\{\dot{u}\}$ and $\{\ddot{u}\}$ are the vector of displacement, velocity and acceleration respectively. Since the geometrically nonlinearities are included in the analysis, the stiffness matrix is a function of the nodal displacements. The above system of nonlinear differential equations is solved using iterative Newton-Raphson procedure at each time instance and the Hilbert-Hughes-Taylor (HHT) time integration algorithm is used to advance the structural solution forward in time.

3. Fluid-Structure Coupling

For the modelling of FSI problems, the structural and aerodynamic models must be coupled. The coupling is due to the fact that the aerodynamic force acting on the structure is computed by the unsteady CFD solver whereas the displacements required to define the new position of the aerodynamic mesh is computed by the nonlinear structural solver. The loosely coupled strategy is adopted in the present work in which the aerodynamic forces are updated on the structure and the aerodynamic mesh are updated using the structural displacements at the end of every physical time step. In the tightly coupled approach, this update is done at every sub-iteration of the unsteady flow solver. Figure 1 explains the co-simulation process used in a bi-directional FSI analysis. This methodology is adopted in the present work to study the nonlinear FSI characteristics of the cropped delta wing.

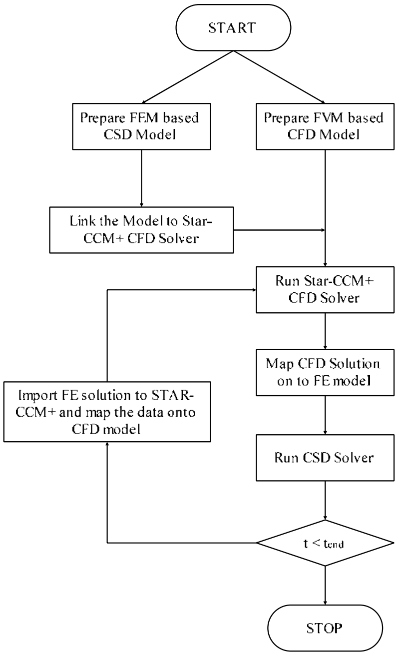


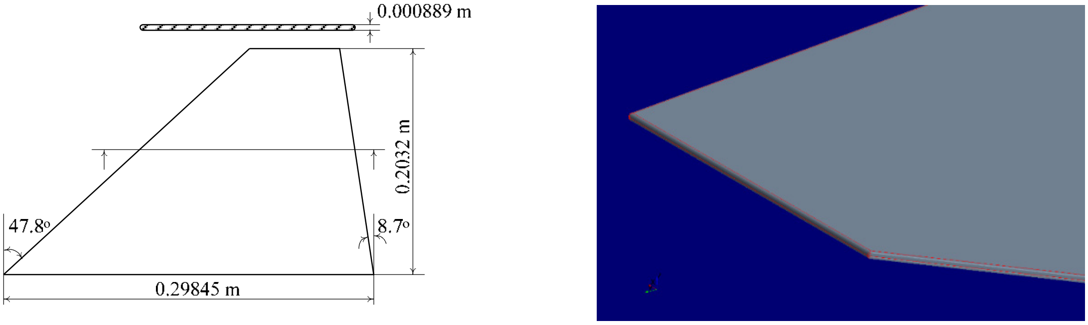
Fig. 1. Co-simulation using CSD and CFD solvers for FSI analysis.

4. Results and Discussion

In this section, the nonlinear FSI characteristics of the cropped delta wing is investigated using the coupled FSI solver in time domain.

4.1. Problem Definition

Figure 2 shows the cropped delta wing model considered in the present study. This model is based on the experimental model of Schairer [5]. The model has a leading edge sweep angle of 47.8° and the trailing edge sweep angle of -8.7°. The thickness (*t*) and semi-span length (*b*) of the model is 0.000889 *m* and 0.2032 m respectively. The leading edge, trailing edge and wingtip of the model are rounded as shown in Fig. 2 (b).



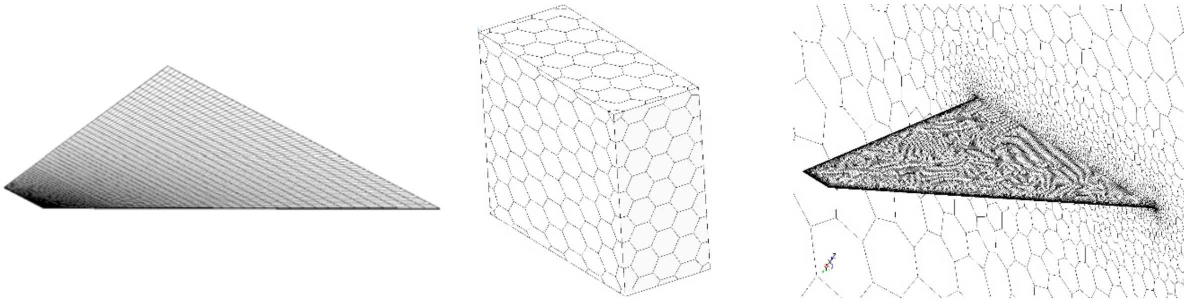
(a) Geometry

(b) Wing tip

Fig. 2. Geometric details of the cropped delta wing

4.2. Computational Domain

The computational domains used for structural dynamics and aerodynamics calculations are shown in Fig. 3. The structural dynamic model is created using solid element with 29219 elements. The fluid dynamics model is created using polyhedral cells. The inlet and the outlet boundaries are located at 6 *m* and 9 *m* respectively from the wing root leading edge. The top and bottom boundaries are located at 6 *m* from the mid plane. The side boundary is located at 6 *m* from the symmetry face. The total number of cells in the fluid domain is 2051096. The boundary layer is modeled with 20 prism layers with first cell height 0.0000017 *m* and total thickness 0.0001 *m*. Further the edges of the wing are refined finely using volumetric control refinement in order to capture the flow gradients accurately.



(a) CSD mesh (b) CFD domain (c) CFD grid on wing surface

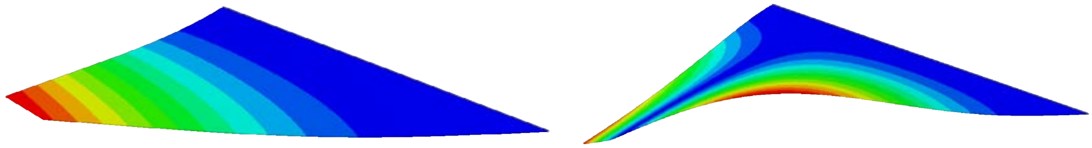
Fig. 3. CSD and CFD Models

4.3. Free Vibration Analysis

The free vibration analysis is carried out in order to verify the dynamic characteristics of the delta wing with the available experimental results [5]. The wing is made of cold rolled steel plate whose material properties are Young's modulus (E_s) = 206.842 *GPa*, density (ρ_s) = 7833.41 kg/m^3 and Poisson's ratio (v) = 0.25. The wing is rigidly constrained at its root section in all the degrees of freedom. The free vibration analysis is carried out and the obtained natural frequencies are compared with the experimental results [5]. Table 2 shows that the present computed natural frequencies are in good agreement with those available in the literature. The mode shapes corresponding to the first four natural frequencies are also shown in Fig. 4.

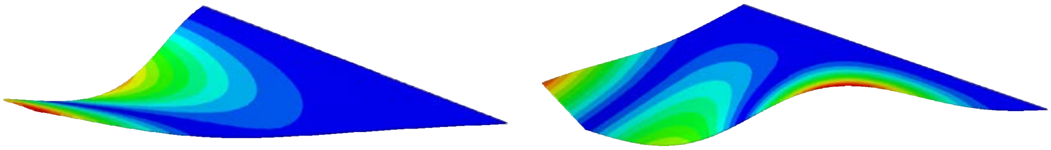
Table 2. Comparison of natural frequencies.

Mode Shape	Present Computation (Hz)	Experiment [5] (Hz)
First Bending	26.814	26.7
First Torsion	89.749	88.2
Second Bending	134.16	131.8
Second Torsion	206.59	-



(a) First Bending (26.814 Hz)

(b) First Torsion (89.749 Hz)



(c) Second Bending (132.3 Hz)

(a) Second Torsion (206.59 Hz)

Fig. 4. First four mode shapes of the delta wing

4.4. Fluid Structure Interaction Analysis

In the previous section, the dynamic characteristics of the delta wing are compared with the experimental results. In this section, the validation of the coupled CFD-CSD solver is carried out by computing the nonlinear aeroelastic characteristics of the delta wing. The parameters taken for the comparison are the LCO amplitude and the frequency. The conditions chosen for the FSI analysis are free stream Mach number (M_∞) of around 0.87 and free stream Reynolds number (Re_∞) varying between 2.7 x 10^6 and 3.7 x 10^6 [2, 3]. At these conditions, the dynamic pressure is varied ranging from 17.788 *kPa* to 23.787 *kPa* and the corresponding responses at wing tip leading and trailing edges are observed. Table 3 summarizes the flow conditions and structural parameters used in the present work.

Table 3. Flow conditions and structural parameters.

Dynamic pressure (kPa)	λ	μ_s	Mach number	Reynolds number, x 10^6
17.788	73.21	0.0216	0.879	2.700765
19.167	78.89	0.0235	0.878	2.931988
20.546	84.56	0.0253	0.874	3.154261
23.787	97.9	0.0304	0.860	3.722870

Here μ_s and λ are the non-dimensional freestream dynamic pressure and mass ratio respectively and are given by $\lambda = 12\left(1-\nu^2\right)\rho_\infty u_\infty^2 c^3 / \left(E_s h^3\right)$ and $\mu_s = \rho_\infty c / \left(\rho_s h\right)$, where ρ_∞ is the freestream density, u_∞ is the freestream velocity and *h* is the non-dimensional plate thickness.

The time step used for the simulation is 0.0001 *s* with 30 inner iterations per physical time step. The turbulence in the flow is modelled using *k-omega* SST turbulence model. The steady flow analysis is carried out at very small angles of attack (0.1°) that serve as the initial condition for the FSI analysis. In the present computations, it is also assumed that there is no structural damping present in the system. In the next section, the influence of the nonlinear aerodynamic and linear/nonlinear structural models on the LCO amplitudes of the delta wing is presented.

4.5. Linear Structural Dynamics Model

Initially, the FSI analysis of the delta wing is carried out by considering the geometrically linear structure dynamics model at dynamic pressure, 19.167 *kPa*, and the obtained time histories of wing tip displacements and lift coefficient are shown in Fig. 5. From the figures it can be observed that at q_∞ = 19.167 *kPa*, the amplitude and frequency of the wing tip trailing edge response are 0.098 *m* and 43.29 *Hz* respectively. It also shows an out of phase oscillations between wing tip trailing edge motion and the lift coefficient.

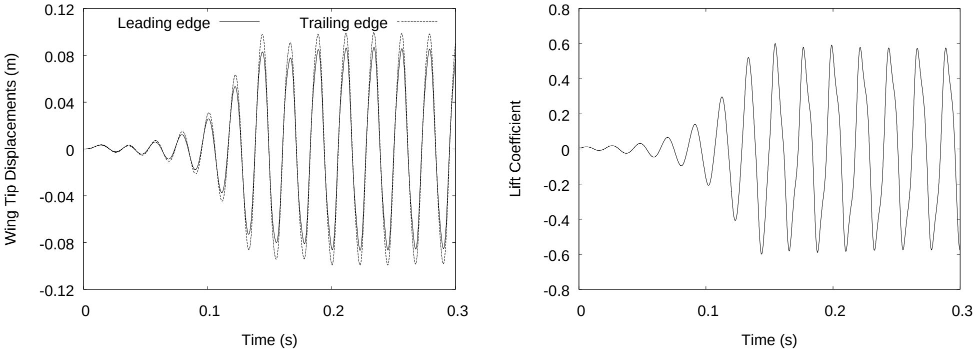


Fig. 5. Time history of wing tip displacements and lift coefficient at $M_\infty = 0.878$, $Re_\infty = 2.932 \times 10^6$ and $q_\infty = 19.167$ *kPa* using linear structural model

The above LCO is due to the presence of the nonlinear mechanism in the aerodynamics since the structural model is linear. The nonlinear mechanism is the creation of the large wing tip leading edge vortices due to the large deformation of the linear structural model. The sustaining LCO oscillation is due to the bending-torsion motion of the wing. As the wing deforms, a nonzero local angle of attack is created by the torsional component of the aerodynamic loads. The increase in this angle of attack results in the production of a leading edge vortex. This vortex creates normal force which is 180° out of phase with the motion of the wing [4]. The vortex acts like an aerodynamic spring limiting the wing motion and cause the LCOs [6]. Thus, the generation of LCO in the present case is due to the nonlinearity in the aerodynamic sources.

4.6. Nonlinear structural dynamics model

The FSI analysis using nonlinear structural dynamics model is then carried out at dynamic pressure, 19.167 *kPa*, and the respective LCO amplitudes and frequencies are observed. Figure 6 gives the time history of the wing tip displacements and lift coefficient at $q_\infty = 19.167$ *kPa*. It can be observed that the nonlinear structure model shows a large reduction in the LCO amplitudes compared to linear structural model. Since the structural model is nonlinear, the wing tip displacement is found to be small and the formation of wing tip vortices is very weak. Thus, the constant amplitude oscillations of LCO are primarily caused by the nonlinearity present in the structure.

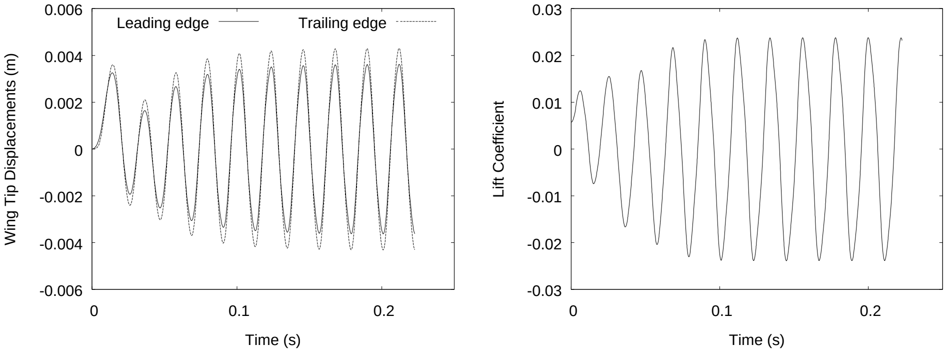


Fig. 6. Time history of wing tip displacements and lift coefficient at $M_\infty = 0.878$, $Re_\infty = 2.932 \times 10^6$ and $q_\infty = 19.167$ *kPa* using nonlinear structural model

Figure 7 shows the comparison of the LCO amplitude and frequency at different dynamic pressures obtained using geometrically linear and nonlinear structural models with the available computational and experimental results. It can be seen that the present coupled FSI solver based on linear structural model predicts very high LCO amplitudes compared to experiments. The LCO amplitudes predicted by the present coupled FSI solver based on nonlinear structural model are slightly lower than the experimental results and very close to the computational results given by Attar et al. [4]. The computational results given in the literature [4] are based on Euler based CFD solver coupled with nonlinear structural model which also indicates that viscosity has negligible effect on the predicted LCO amplitudes at low dynamic pressure [2]. The frequencies computed by the present FSI solver are in close agreement with the experimental results as shown in the figure.

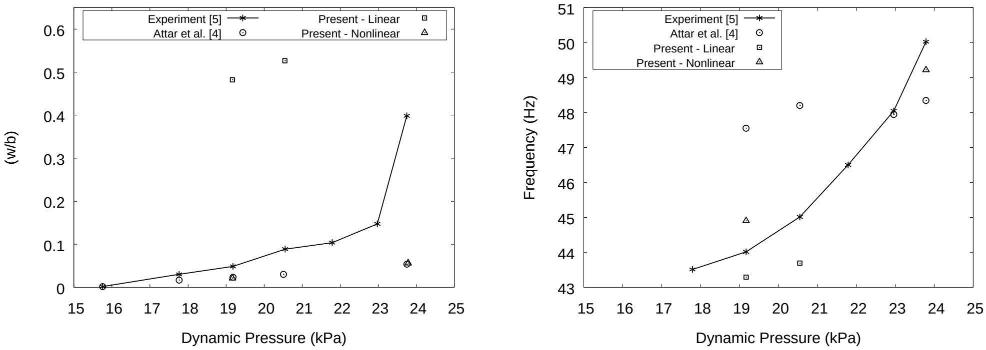


Fig. 7. Comparison of LCO amplitudes and frequencies at different dynamic pressures

5. Conclusions

In this work, a high fidelity computational aeroelastic solver based on coupled CFD and CSD approach is used to study the nonlinear aeroelastic characteristics, namely LCO, of the cropped delta wing. Here, the nonlinear aerodynamic model based on N-S equations are solved by coupling geometrically linear/nonlinear structural dynamic equations to predict LCO of the cropped delta wing. The present results are also compared with the available computational and experimental results in the literature. It is observed that the LCO amplitudes predicted by linear structural model are very high compared to nonlinear structural model. It is also observed that the LCO amplitudes based on nonlinear structural model are slightly lower than the experimental results and match well with the available computational results in the literature. The LCO frequencies computed by the present FSI solver are in better agreement with the experimental results. The present results can be further improved by proper modelling of flow physics in terms of mesh refinement and turbulence.

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