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Reducing θ_{13} to 9°

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ABSTRACT

We propose to consider the possibility that the observed value of θ_{13} is not the result of a correction from an initially vanishing value, but rather the result of a correction from an initially *larger* value. As an explicit example of this approach, we consider analytically and numerically well-known CKM-like charged lepton corrections to a neutrino diagonalization matrix that corresponds to a certain mixing scheme. Usually this results in generating $\theta_{13} = 9°$ from zero. We note here, however, that 9 is not only given by $0 + 9$, but also by $18 - 9$. Hence, the extreme case of an initial value of 18 degrees, *reduced* by charged lepton corrections to 9 degrees, is possible. For some cases under study, new sum rules for the mixing parameters, and correlations with CP phases, are found.

1. Introduction

Remarkable experimental activity in the past decades has established that the phenomenon of neutrino flavor transition is described by neutrino oscillations. Recent measurements of the smallest mixing angle θ_{13} at reactor [1–4] and accelerator [5] neutrino experiments have finally led to an emerging picture where the order of magnitude of all elements of the PMNS matrix is known. Theorists now face the task to understand and/or explain that structure. Most flavor symmetry models [6–8] were constructed when only an upper limit on θ_{13} was known, and therefore aimed at explaining $\theta_{13} = 0$. Corrections to generate a non-zero value are then applied. In the present paper we depart from the historically motivated approaches to generate non-zero θ_{13} from an initially vanishing value, and consider the possibility that initially θ_{13} is already large. Now, the usual corrections to model predictions can reduce the initial value of θ_{13} to its observed value. Of course, the phenomenology will then be different from the standard case. As an explicit example on the consequences that follow, we consider charged lepton corrections.

No matter if neutrinos are Majorana or Dirac particles, the lepton flavor mixing matrix stems from the mismatch between the diagonalization of the charged lepton mass matrix m_ℓ and the neutrino mass matrix m_ν, i.e.

$$U = U_\ell^\dagger U_\nu, \qquad (1)$$

where U_ℓ and U_ν are the unitary matrices diagonalizing m_ℓ and m_ν, respectively. Now, one can apply the following strategy to generate non-zero $\theta_{13} = \arcsin|U_{e3}|$. Assuming that $(U_\nu)_{13} = 0$, as well as $(U_\nu)_{23} = (U_\nu)_{33}$, and that U_ℓ is related to the CKM matrix, i.e. essentially the unit matrix except for $(U_\ell)_{12} = \lambda = \sin\theta_C$, it follows that $|U_{e3}| = \lambda/\sqrt{2}$, or $\theta_{13} = 9° = 0 + 9°$. Numerically, this is basically the observed value of about $\theta_{13} = 9°$, and the fact that this lepton mixing parameter is numerically connected to quark parameters seems to support this argument, but is of course not a proof.[1] Nevertheless, relating the charged lepton diagonalization to the CKM matrix can be arranged in grand unified models, especially based on $SU(5)$, for which $m_\ell = m_d^T$ is a typical outcome. Such a relation has to be viewed as an approximation due to the distinct mass spectra of leptons and quarks, and is modified by higher order corrections or Clebsch–Gordon coefficients. Nevertheless, models predicting $U_{\rm CKM} \simeq U_\ell$ have been constructed, which in addition have $(U_\nu)_{13} = 0$ [9–13]. Hence, the above strategy to generate $|U_{e3}| = \lambda/\sqrt{2}$, where $\lambda \simeq \sin\theta_C \simeq 0.23$, is based on actual model building foundations. We will use for the sake of simplicity and definiteness $U_{\rm CKM} = U_\ell$ in what follows.

While the relation $9° = 0 + 9°$ has its virtues and attraction, one should not ignore the possibility that $9° = 18° - 9°$. Such an extreme value of $18° \equiv \pi/10$ can be obtained in flavor symmetry models. For instance, mixing angles of π divided by integer n are known to be achievable in models based on dihedral groups D_n, such as in Refs. [14,15]. This means that initially U_ν contains a too large value of its 13-element, which is *reduced* to its observed

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[1] The observed value of $|U_{e3}|$ is also close to $\sqrt{m_s/m_b}$, which is presumably only a coincidence.

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value by a sizable charged lepton correction, a CKM-like one in our case. Since the remaining lepton mixing angles are necessarily non-zero both in U and in U_ℓ, the question arises whether θ_{13} should initially be non-zero in the first place. This so far overlooked possibility is what we investigate here, by performing a general analysis of Eq. (1) when U_ℓ is fixed to the CKM matrix. The case of initially vanishing $(U_\nu)_{13} = 0$ has been analyzed countless times, but the cases when $|(U_\nu)_{13}| \simeq |U_{e3}|$, or more interestingly $|(U_\nu)_{13}| > |U_{e3}|$, have never been considered. As a result we find new interesting sum rules, and also note the already mentioned extreme case of reducing θ_{13} from 18 degrees to 9 degrees, where the initial value could be obtained from flavor symmetries, as $18^\circ = \pi/10$ is related to symmetries of geometrical objects.

The remainder of this paper is organized as follows. In Section 2, we present the general formalism and derive the charged lepton corrections to an arbitrary U_ν. Interesting sum rules between neutrino mixing parameters are summarized. In Section 3, a detailed numerical analysis of the model parameters and predictions is performed. Finally, in Section 4, we state our conclusions.

2. Methodology

In the picture of three-flavor neutrino oscillations, the lepton flavor mixing is described by a 3×3 unitary matrix U, which is conventionally parametrized by three mixing angles (θ_{12}, θ_{23} and θ_{13}), and three CP violating phases out of which one is the Dirac phase (δ) and the other two are the Majorana phases (ρ and σ). In the standard parametrization, the lepton mixing matrix is given by

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-\mathrm{i}\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{\mathrm{i}\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{\mathrm{i}\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{\mathrm{i}\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{\mathrm{i}\delta} & c_{23}c_{13} \end{pmatrix} \times \begin{pmatrix} e^{\mathrm{i}\rho} & 0 & 0 \\ 0 & e^{\mathrm{i}\sigma} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (2)$$

where $s_{ij} \equiv \sin\theta_{ij}$ and $c_{ij} \equiv \cos\theta_{ij}$ (for $ij = 12, 23, 13$). In case of Dirac neutrinos the phases ρ and σ will be irrelevant. The results of this paper are independent on the nature of the neutrino. The latest global analysis of current neutrino oscillation data yields [16]

$$\begin{aligned} \sin^2\theta_{12} &= 0.313^{+0.013}_{-0.012}, \\ \sin^2\theta_{23} &= 0.444^{+0.036}_{-0.031}, \\ \sin^2\theta_{13} &= 0.0244^{+0.0020}_{-0.0019}, \end{aligned} \quad (3)$$

where short baseline reactor data with baseline shorter than 100 m are not included. Another recent fit result is obtained in [17], with similar results. There are also non-trivial results on the CP phase δ, with best-fit results around $3\pi/2$, or $\cos\delta \simeq 0$. However, the 1σ ranges are very large, including essentially also the case $\cos\delta \simeq -1$. We note that for some of the cases that we will discuss it is actually crucial whether $\cos\delta$ is 0 or -1, and therefore we use only the obtained ranges of the mixing angles in our fits.

The concrete form of U_ℓ cannot be fixed unless a specific mode is considered. Motivated by the connection between the CKM matrix and U_ℓ in many grand unified models we assume here for definiteness $U_\ell = U_{\rm CKM}$. As for the unitary matrix U_ν diagonalizing the neutrino mass matrix, one can parametrize it in analogy to U by using three rotation angles $\tilde\theta_{12}$, $\tilde\theta_{23}$, and $\tilde\theta_{13}$ together with a phase ϕ. Note that we have ignored the Majorana-like phases in this parametrization, since they are located on the right-hand side of U_ν and hence do not affect our discussions on the mixing angles and Dirac CP phase. Now, the lepton flavor mixing matrix is given by[2]

$$U = U^\dagger_{\rm CKM} P U_\nu(\tilde\theta_{12}, \tilde\theta_{23}, \tilde\theta_{13}, \phi). \quad (4)$$

Here $P = \mathrm{diag}(e^{\mathrm{i}x}, e^{\mathrm{i}y}, 1)$ is a phase matrix stemming from the mismatch between U_e and U_ν [19].

We proceed to expand the mixing matrix U in order to obtain the charged lepton corrections. Different from the lepton sector, the CKM matrix takes a nearly diagonal form, and is typically parametrized by using four parameters (λ, A, ρ and η) in the Wolfenstein parametrization. Since we are mainly interested in the lepton flavor mixing which has not been measured as precisely as $U_{\rm CKM}$, we will keep the Wolfenstein parametrization only up to λ^2, i.e.

$$U_{\rm CKM} \simeq \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & 0 \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ 0 & -A\lambda^2 & 1 \end{pmatrix}. \quad (5)$$

Now, by inserting Eq. (5) into (4) we obtain the matrix elements of U to order λ^2 as[3]

$$U_{e1} = \tilde c_{12}\tilde c_{13} + \left(\tilde s_{12}\tilde c_{23}e^{-\mathrm{i}\varphi} + \tilde s_{23}\tilde c_{12}\tilde s_{13}e^{-\mathrm{i}(\varphi-\phi)}\right)\lambda - \frac{1}{2}\tilde c_{12}\tilde c_{13}\lambda^2, \quad (6)$$

$$U_{e2} = \tilde s_{12}\tilde c_{13} + \left(-\tilde c_{12}\tilde c_{23}e^{-\mathrm{i}\varphi} + \tilde s_{23}\tilde s_{12}\tilde s_{13}e^{-\mathrm{i}(\varphi-\phi)}\right)\lambda - \frac{1}{2}\tilde s_{12}\tilde c_{13}\lambda^2, \quad (7)$$

$$U_{e3} = \tilde s_{13}e^{-\mathrm{i}\phi} - \tilde s_{23}\tilde c_{13}e^{-\mathrm{i}\varphi}\lambda - \frac{1}{2}\tilde s_{13}e^{-\mathrm{i}\phi}\lambda^2, \quad (8)$$

$$U_{\mu3} = \tilde s_{23}\tilde c_{13} + \tilde s_{13}e^{\mathrm{i}(\varphi-\phi)}\lambda - \left(\frac{1}{2}\tilde s_{23}\tilde c_{13} + A\tilde c_{23}\tilde c_{13}e^{-\mathrm{i}y}\right)\lambda^2, \quad (9)$$

where $\varphi = x - y$ has been defined, and the notation $\tilde s_{ij} \equiv \sin\tilde\theta_{ij}$, $\tilde c_{ij} \equiv \cos\tilde\theta_{ij}$ is adopted. Since the charged lepton mixing matrix takes the CKM form, only the 12-rotation plays a role. Consequently, one can rotate away one of the phases, leaving only the difference between two CP phases x and y in the above results.

Comparing with the standard parametrization given in Eq. (2), we find

$$\sin^2\theta_{13} \simeq \tilde s^2_{13} - 2\lambda\tilde s_{13}\tilde c_{13}\tilde s_{23}\cos(\varphi-\phi) + \lambda^2\left(\tilde s^2_{23}\tilde c^2_{13} - \tilde s^2_{13}\right), \quad (10)$$

$$\sin^2\theta_{12} \simeq \tilde s^2_{12} - 2\lambda\frac{1}{\tilde c_{13}}\tilde s_{12}\tilde c_{12}\tilde c_{23}\cos\varphi, \quad (11)$$

$$\sin^2\theta_{23} \simeq \tilde s^2_{23} + 2\lambda\frac{1}{\tilde c_{13}}\tilde s_{23}\tilde s_{13}\tilde c^2_{23}\cos(\varphi-\phi), \quad (12)$$

where the $\mathcal{O}(\lambda^2)$ terms are only kept for $\sin^2\theta_{13}$, since θ_{13} is relatively smaller compared to the other mixing angles. As for the Dirac phase δ, to leading order we have

$$\tan\delta = \frac{\tilde s_{13}s_\phi - \tilde s_{23}\tilde c_{13}\lambda s_\varphi}{\tilde s_{13}c_\phi - \tilde s_{23}\tilde c_{13}\lambda c_\varphi}, \quad (13)$$

where $s_\phi = \sin\phi$, $s_\varphi = \sin\varphi$ and so on. It might also be useful to express the Jarlskog invariant [21,22] in terms of the model pa-

[2] For the case that U_ν is CKM-like, see [18].

[3] Ignoring CP phases, expressions for the PMNS mixing angles in case of CKM-like corrections to U_ν, with angles in U_ν all larger than the ones in U_ℓ can be found in [20].

rameters, i.e.

$$\begin{aligned} J_{\rm CP} &= \tilde{J}_{\rm CP} + \lambda \tilde{c}_{13}\tilde{c}_{23}\big\{\tilde{s}_{12}\tilde{c}_{12}\big(\tilde{c}_{23}^2 - \tilde{c}_{13}^2\big)\sin\varphi \\ &\quad + \tilde{s}_{13}\tilde{s}_{23}\big[\tilde{c}_{23}\big(\tilde{c}_{12}^2 - \tilde{s}_{12}^2\big)\sin(\varphi-\phi) \\ &\quad - \tilde{s}_{12}\tilde{s}_{23}\tilde{s}_{13}\tilde{c}_{12}\sin(\varphi - 2\phi)\big]\big\} \\ &\simeq \tilde{J}_{\rm CP} + \lambda\tilde{s}_{12}\tilde{c}_{12}\tilde{c}_{13}\tilde{c}_{23}\big(\tilde{c}_{23}^2 - \tilde{c}_{13}^2\big)\sin\varphi \end{aligned} \tag{14}$$

where, as usual, $\tilde{J}_{\rm CP}$ is defined as

$$\tilde{J}_{\rm CP} = \tilde{s}_{12}\tilde{s}_{23}\tilde{s}_{13}\tilde{c}_{12}\tilde{c}_{23}\tilde{c}_{13}^2 \sin\phi. \tag{15}$$

Of course, even if $\tilde{\theta}_{13} = 0$ is assumed, CP violation can still be induced by the λ correction, when $\sin\varphi = \sin(x-y) \neq 0$.

Both θ_{13} and θ_{23} are independent of $\tilde{\theta}_{12}$ at leading order. The leading corrections to θ_{13} and θ_{23} are proportional to $\lambda\tilde{s}_{13}$, whereas the leading correction to θ_{12} is proportional to λ. This indicates that a larger deviation of $\tilde{\theta}_{12}$ from θ_{12} than for the other mixing angles is allowed. However, there are terms including cosines of phases in the expressions, which can suppress the corrections. Note that the same combination of phases appears in the expressions for $\sin^2\theta_{23}$ and $\sin^2\theta_{13}$, which implies a correlation between both observables, if the second order term in $\sin^2\theta_{13}$ can be ignored. It reads

$$\sin^2\theta_{23} - \sin^2\tilde{\theta}_{23} = -\frac{\cos^2\tilde{\theta}_{23}}{\cos^2\tilde{\theta}_{13}}\big(\sin^2\theta_{13} - \sin^2\tilde{\theta}_{13}\big). \tag{16}$$

In case $\phi = 0$, there is a correlation between the 23- and 12-sectors:

$$\begin{aligned} &\sin^2\theta_{23} - \sin^2\tilde{\theta}_{23} \\ &\quad = -\frac{\cos\tilde{\theta}_{23}\sin\tilde{\theta}_{23}\sin\tilde{\theta}_{13}}{\cos\tilde{\theta}_{12}\sin\tilde{\theta}_{12}}\big(\sin^2\theta_{12} - \sin^2\tilde{\theta}_{12}\big). \end{aligned} \tag{17}$$

However, the general case is complicated and depends on many parameters. The obvious extreme cases are $\tilde{\theta}_{13} = 0$, $\tilde{\theta}_{13} > \theta_{13}$ and $\tilde{\theta}_{13} \simeq \theta_{13}$. We will in the following discuss these cases analytically, before performing a general numerical analysis.

2.1. The case of $\tilde{\theta}_{13} = 0$

We will start from the most simple case with $\tilde{\theta}_{13} = 0$, though there is nothing new too add to existing knowledge (see e.g. [19,20,23–36]). In the limit under study, the expressions for the mixing angles reduce to leading order to

$$\begin{aligned} &\sin\theta_{13} \simeq \lambda\sin\theta_{23}, \\ &\delta \simeq \varphi + \pi, \\ &\sin^2\theta_{12} \simeq \tilde{s}_{12}^2 - 2\lambda\tilde{s}_{12}\tilde{c}_{12}\tilde{c}_{23}\cos\varphi. \end{aligned} \tag{18}$$

From the relation $\sin\theta_{13} \simeq \lambda\sin\tilde{\theta}_{23}$ one obtains for $\tilde{\theta}_{23} = \pi/4$ the value $\sin^2\theta_{13} \simeq 0.0255$, in very good agreement with the measured value. In the tri-bimaximal mixing case, we have

$$\begin{aligned} &\sin\theta_{23} = \frac{1}{\sqrt{2}}, \\ &\sin\theta_{13} = \frac{1}{\sqrt{2}}\lambda, \\ &\delta = \varphi + \pi, \\ &\sin^2\theta_{12} = \frac{1}{3} + \frac{2\sqrt{2}}{3}\sin\theta_{13}\cos\delta, \end{aligned}$$

whereas for the bimaximal mixing case we obtain

$$\begin{aligned} &\sin\theta_{23} = \frac{1}{\sqrt{2}}, \\ &\sin\theta_{13} = \frac{1}{\sqrt{2}}\lambda, \\ &\delta = \varphi + \pi, \\ &\sin^2\theta_{12} = \frac{1}{2} + \sin\theta_{13}\cos\delta. \end{aligned}$$

In the tri-bimaximal based case, δ has to be close to $\pi/2$ (or $3\pi/2$) in order to suppress the θ_{13} correction to $\sin^2\theta_{12} = 1/3$. The situation is however different in the bimaximal case, in which a sizable and negative θ_{13}-correction is required in order to reduce the maximal mixing value $\sin^2\tilde{\theta}_{12} = 1/2$. Hence, $\delta \simeq \pi$ or 2π has to be fulfilled. This interplay of the mixing scheme (bimaximal/tri-bimaximal) in U_ν and the Dirac phase in neutrino oscillations has first been noticed in [31]. Recall that the fit results from Refs. [16,17] include at 1σ essentially both cases, $\delta \simeq 2\pi$ and $\delta \simeq 3\pi/2$, where the latter value is close to the best-fit one.

2.2. The case of $\tilde{\theta}_{13} > \theta_{13}$

If $\tilde{\theta}_{13}$ is larger than the observed value of θ_{13}, the term proportional to λ^2 term in Eq. (10) can be neglected, leaving us with a set of novel sum rules. Appealing values of the initial value are e.g. $\tilde{\theta}_{13} = \pi/10$ or $\tilde{\theta}_{13} = \pi/12$. Assuming $\tilde{\theta}_{13} = \pi/10$ (or $\tilde{\theta}_{13} = 18^\circ$) and for simplicity also $\tilde{\theta}_{23} = \pi/4$, the following sum rules can be deduced:

$$\sin^2\theta_{13} \simeq \frac{3-\sqrt{5}}{8} - \frac{(\sqrt{5}-1)\sqrt{5+\sqrt{5}}}{8}\lambda\cos(\varphi - \phi), \tag{19}$$

$$\sin^2\theta_{23} \simeq \frac{1}{2} - \frac{4}{5+\sqrt{5}}\left(\sin^2\theta_{13} - \frac{3-\sqrt{5}}{8}\right), \tag{20}$$

$$\sin^2\theta_{12} \simeq \sin^2\tilde{\theta}_{12} - \frac{2}{\sqrt{5+\sqrt{5}}}\lambda\sin 2\tilde{\theta}_{12}\cos\varphi. \tag{21}$$

Thus, using the measured value $\theta_{13} \simeq 9^\circ$ and Eq. (20), one predicts $\theta_{23} \simeq 47.3^\circ$. Another interesting example is $\tilde{\theta}_{13} = \pi/12$ (or $\tilde{\theta}_{13} = 15^\circ$), which leads to the following rum rules,

$$\sin^2\theta_{13} \simeq \frac{2-\sqrt{3}}{4} - \frac{\sqrt{2}}{4}\lambda\cos(\varphi - \phi), \tag{22}$$

$$\sin^2\theta_{23} \simeq \frac{1}{2} - 2(2-\sqrt{3})\left(\sin^2\theta_{13} - \frac{2-\sqrt{3}}{4}\right), \tag{23}$$

$$\sin^2\theta_{12} \simeq \sin^2\tilde{\theta}_{12} - (\sqrt{3}-1)\lambda\sin 2\tilde{\theta}_{12}\cos\varphi. \tag{24}$$

By inserting $\theta_{13} = 9^\circ$ into Eq. (23) we obtain the prediction $\theta_{23} \simeq 46.3^\circ$. As in the previous example, we find θ_{23} in the second octant.

It is obvious from Eq. (13) or from (6)–(9) that in case $(\tilde{U}_\nu)_{e3} > U_{e3}$ at leading order $\delta \simeq \phi$ holds. In addition, from (22) it is clear that the first and second terms should cancel to a large extent in order to reduce to the observed value of $|U_{e3}|^2$. To this end, the cosine in (22) should be close to 1, which gives

$$\delta \simeq \phi \simeq \varphi. \tag{25}$$

Similar to the discussion in the previous subsection, if $\sin^2\tilde{\theta}_{12} = 1/3$ holds, $\delta \simeq \pi/2$ (or $3\pi/2$) is required to suppress its corrections to θ_{12}. In contrast, for $\sin^2\tilde{\theta}_{12} = 1/2$, $\delta \simeq \pi$ is expected in order to avoid a too large solar mixing angle. Amusingly, the correlation between $\sin^2\tilde{\theta}_{12}$ and CP violation is identical to the one for vanishing $\tilde{\theta}_{13}$. Both cases can in principle be distinguished by their prediction for θ_{23}, see the blue and red points in the lower left plot in Fig. 6.

2.3. The case of $\tilde{\theta}_{13} \simeq \theta_{13}$, or $\sin\tilde{\theta}_{13} \simeq \sin\tilde{\theta}_{23}\lambda$

This is obviously the most complicated case, and does not allow much analytical results. The Dirac CP phase is determined by

$$\delta = -\mathrm{Arg}\left(\tilde{s}_{13}e^{-\mathrm{i}\phi} - \tilde{s}_{23}\tilde{c}_{13}e^{-\mathrm{i}\varphi}\lambda\right), \tag{26}$$

or by Eq. (13). In principle, any value for δ is possible. As an interesting example, we look at the scenario with $\tilde{\theta}_{13} = 9^\circ$ (or $\tilde{\theta}_{13} = \pi/20$). In this special case, the sum of the first and third term of Eq. (10) is about 0.05, the same size as the second term if the cosine would not be there. Since the measured θ_{13} is also very close to 9°, one would naturally expect that the phase difference between ϕ and φ is around $\pm\pi/3$. Concretely, we have the following relation

$$\delta \simeq \phi \pm \pi/3 \simeq \varphi \pm 2\pi/3. \tag{27}$$

Note also that corrections to θ_{12} are not sensitive to $\tilde{\theta}_{13}$ as shown in the general formula (11), which implies that the CP phase δ is restricted to be close to $\pm\pi/6$ and $\pm\pi/3$ for $\tilde{s}^2_{12} = 1/3$ and $\tilde{s}^2_{12} = 1/2$, respectively.

3. Numerics

In this section we fit the five parameters ($\tilde{\theta}_{12}$, $\tilde{\theta}_{23}$, $\tilde{\theta}_{13}$, ϕ and φ) to the experimental data using the exact form of Eq. (4). To figure out the allowed parameter spaces of the model parameters, we compare the latest global-fit data with a χ^2-function defined as

$$\chi^2_{ij} = \sum_{i<j} \frac{(\sin^2\theta_{ij} - \sin^2\theta^0_{ij})^2}{\sigma^2_{ij}}, \tag{28}$$

where θ^0_{ij} represents the experimental data given in Eq. (3), σ_{ij} denote the corresponding 1σ absolute errors, and θ_{ij} are the predictions of the model and can be expressed in terms of the model parameters.

3.1. $\tilde{\theta}_{12}$–$\tilde{\theta}_{13}$ plane

We start from projecting the parameter space to the $\tilde{\theta}_{12}$–$\tilde{\theta}_{13}$ plane. The parameter ranges for $\tilde{\theta}_{12}$ and $\tilde{\theta}_{13}$ are shown in Fig. 1 using contour lines for the most general case. We also consider the case of maximal $\tilde{\theta}_{23}$ using colored contours, and make assumptions about the CP phases.

From Fig. 1 we see that $\tilde{\theta}_{13}$ can be as large as 19.2°, which inspires us with mixing patterns such as $\sin^2(\pi/10) = (3-\sqrt{5})/8$ and $\sin^2(\pi/12) = (2-\sqrt{3})/4$. Such values of π divided by n can be obtained in flavor symmetry models such as in Refs. [14,15]. The range of $\tilde{\theta}_{12}$ is wide and a maximal $\tilde{\theta}_{12}$ can be accommodated. If $\tilde{\theta}_{23}$ is fixed to $\pi/4$, the parameter space shrinks only slightly, which is a consequence of the suppressed (by both λ and $\tilde{\theta}_{13}$) correction terms to $\tilde{\theta}_{23}$, see Eq. (12). In the limit $\phi = 0$, for which the 12- and 13-sectors are correlated, see Eq. (17), a sizable $\tilde{\theta}_{13}$ demands a relatively large value of $\cos\varphi$ in order to suppress its contribution to θ_{13}. This in turn requires $\tilde{\theta}_{12}$ to be close to maximal. In contrast, if $\tilde{\theta}_{13}$ is tiny, the constraint on $\tilde{\theta}_{12}$ becomes less stringent, which can be seen clearly from our analytical results Eq. (11). Explicitly, for a vanishing $\tilde{\theta}_{13}$, one has the approximate relation $\sin\theta_{13} \simeq \lambda\sin\theta_{23}$. In such a case, the leading order correction to $\tilde{\theta}_{12}$ is flexible since it is proportional to $\cos\varphi$. If all phases are zero, a significant and negative correction to $\tilde{\theta}_{12}$ is expected, and consequently only the nearly maximal value $\tilde{\theta}_{12} \simeq \pi/4$ can be accommodated.

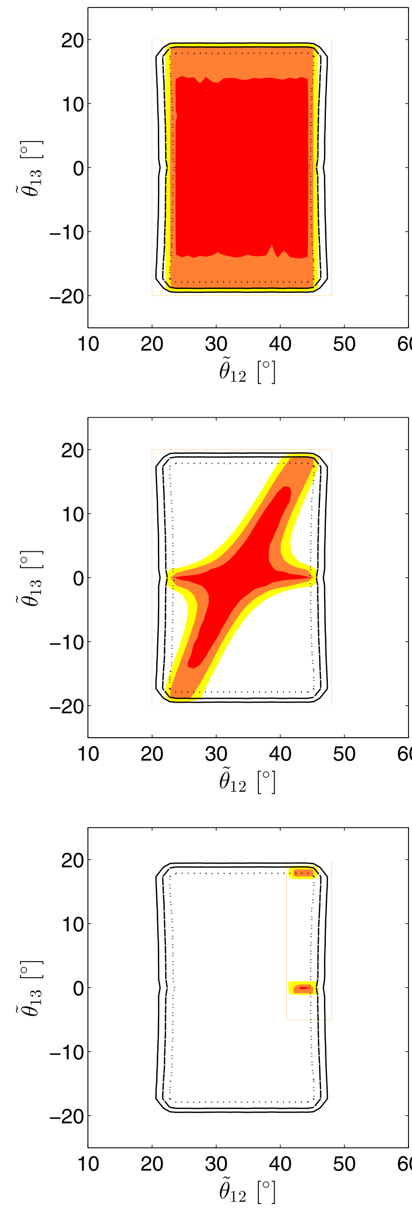

Fig. 1. Parameter ranges of $\tilde{\theta}_{12}$ and $\tilde{\theta}_{13}$ at 1, 2 and 3σ. For the color contours, we have fixed $\tilde{\theta}_{23} = 45^\circ$. In the upper panel, we allow all phases to freely vary between 0 and 2π. In the middle panel, we switch off ϕ but not φ, whereas in the lower panel, all CP phases are set to zero.

3.2. $\tilde{\theta}_{12}$–$\tilde{\theta}_{23}$ plane

The allowed parameter space in the $\tilde{\theta}_{12}$–$\tilde{\theta}_{23}$ plane is shown in Fig. 2. As special cases, we choose $\tilde{\theta}_{13} = 0$ and $\tilde{\theta}_{13} = \pi/10$, both for the general case and for all phases being set to zero.

As expected from the suppressed corrections to $\tilde{\theta}_{23}$, the parameter range of $\tilde{\theta}_{23}$ is similar to that of θ_{23}. If we neglect the CP phases, $\tilde{\theta}_{13} = 0$ leads to a large negative correction to $\tilde{\theta}_{12}$, and a relatively larger $\tilde{\theta}_{12}$ is favored. In case of large $\tilde{\theta}_{13}$, $\tilde{\theta}_{23}$ is driven towards smaller values, see Eq. (16).

3.3. $\tilde{\theta}_{13}$–$\tilde{\theta}_{23}$ plane

The allowed parameter space in the $\tilde{\theta}_{13}$–$\tilde{\theta}_{23}$ plane is shown in Fig. 3. As special cases we choose $\sin^2\tilde{\theta}_{12} = 1/3$ and $\sin^2\tilde{\theta}_{12} = 1/2$.

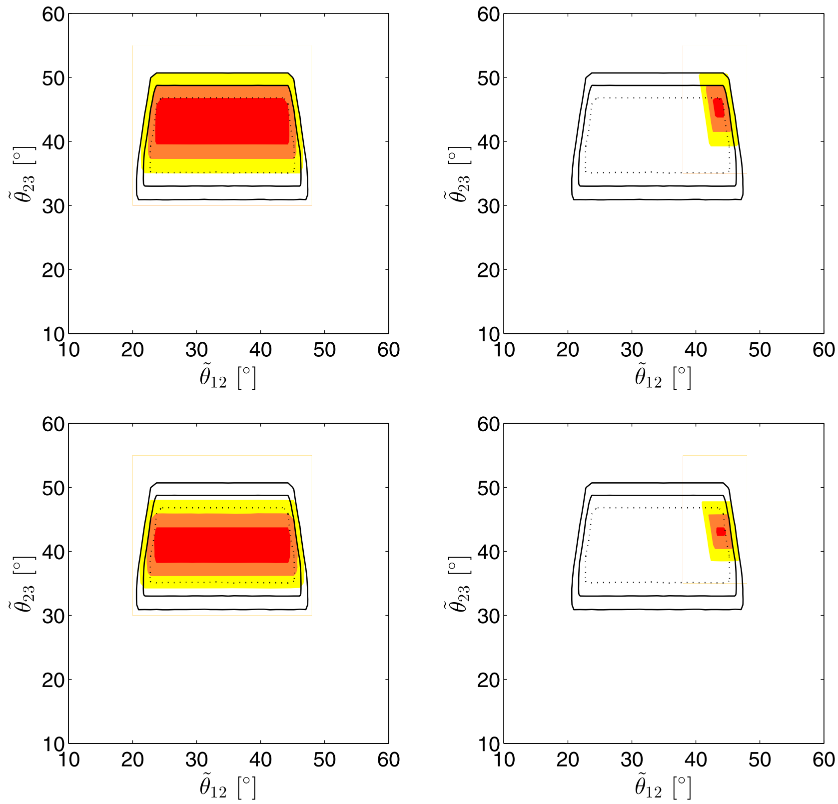

Fig. 2. The parameter ranges of $\tilde{\theta}_{12}$ and $\tilde{\theta}_{23}$ at 1, 2 and 3σ. For the color contours $\tilde{\theta}_{13} = 0$ (upper row) or $\tilde{\theta}_{13} = \pi/10$ (lower row) is fixed, but the choices of the phases are different. In the left column, we allow all phases to freely vary between 0 and 2π, whereas in the right column, all phases are set to zero.

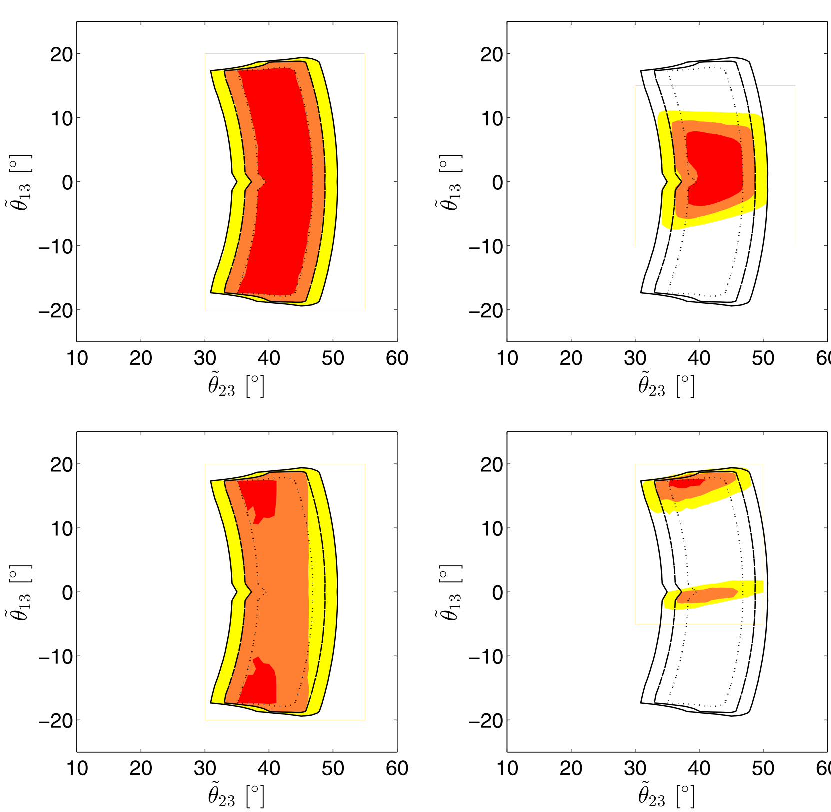

Fig. 3. The parameter range of $\tilde{\theta}_{13}$ and $\tilde{\theta}_{23}$ at 1, 2 and 3σ. For the color contours, we fix $\sin^2\tilde{\theta}_{12} = 1/3$ in the upper row and $\sin^2\tilde{\theta}_{12} = 1/2$ in the lower row. In the left column, we allow all phases to freely vary between 0 and 2π, whereas in the right column $\phi = 0$ is fixed.

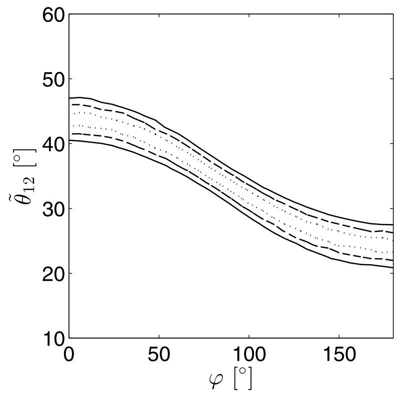

Fig. 4. The parameter range of φ and $\tilde\theta_{12}$ at 1, 2 and 3σ. All other model parameters are marginalized.

As the figure shows, $\tilde\theta_{13}$ and $\tilde\theta_{23}$ are not sensitive to the choice of $\tilde\theta_{12}$, which has already been shown in the analytical part above, cf. Eqs. (10), (12). They are however very sensitive to the CP phases, i.e. $\phi = 0$ restricts the range of $\tilde\theta_{13}$ down to $-10^\circ \lesssim \tilde\theta_{13} \lesssim 10^\circ$ in the case of $\sin^2\tilde\theta_{12} = 1/3$, and in two distinct regions around 0 and 18° in the case of $\sin^2\tilde\theta_{12} = 1/2$, with $\tilde\theta_{13} \sim 9^\circ$ being excluded. It is worth noting that, when all the phases are set to zero, there is no parameter space for $\sin^2\tilde\theta_{12} = 1/3$, since the derived θ_{12} is too small.

3.4. φ–$\tilde\theta_{12}$ plane

As pointed out in the analytical section, the phase difference $\varphi = x - y$ is very crucial for certain mixing patterns, in particular for $\tilde\theta_{12}$. Thus, we illustrate the relation between φ and $\tilde\theta_{12}$ in Fig. 4.

The correlation between small phases for $\sin^2\tilde\theta_{12} = 1/2$ and phases around π for $\sin^2\tilde\theta_{12} = 1/3$ is reproduced. Note that this feature is present for all values of $\tilde\theta_{13}$.

3.5. J_{CP}–$\tilde\theta_{12}$ plane

Since the choice of $\tilde\theta_{12}$ can be sensitive to the CP phases, we further illustrate in Fig. 5 the 3σ ranges of the Jarlskog invariant with respect to $\tilde\theta_{12}$. As one can read from the plot, J_{CP} is not sensitive to $\tilde\theta_{12}$ in the most general case. However, once $\tilde\theta_{13}$ is fixed, a connection between J_{CP} and $\tilde\theta_{12}$ can be expected. As we have mentioned in Section 2.1, in the case of vanishing $\tilde\theta_{13}$, maximal CP violation ($J_{CP} \simeq \pm 0.04$) is achieved for $\tilde s_{12}^2 \simeq 1/3$ since δ is close to $\pi/2$ (or $3\pi/2$). In contrast, $\tilde s_{12}^2 \simeq 1/2$ leads to a suppressed J_{CP} as can be seen from the upper right plot. For the case of $\tilde\theta_{13} \simeq \theta_{13} \simeq 9^\circ$, our analytical results given in Eq. (27) appear as reasonably good approximations. For instance, the tri-bimaximal value $\tilde s_{12}^2 \simeq 1/3$ suggests $|\sin\delta| \sim 1/2$, corresponding to $J_{CP} \sim J_{CP}^{\max}/2$, which is reflected in the lower left plot. Furthermore, $\tilde s_{12}^2 \simeq 1/2$ results in $|\sin\delta| \sim 0.87$, indicating nearly maximal CP violation. As mentioned above, the situation for the large $\tilde\theta_{13}$ case is similar to the small $\tilde\theta_{13}$ case, as shown in the discussion after Eq. (25).

3.6. Lepton mixing parameters

Finally, the correlations among the leptonic mixing parameters are shown in Fig. 6. We choose four benchmark neutrino mixing matrices U_ν:

a) tri-bimaximal pattern with $\tilde\theta_{13} = 0$, $\sin^2\tilde\theta_{12} = 1/3$ and $\sin^2\tilde\theta_{23} = 1/2$ (red points);

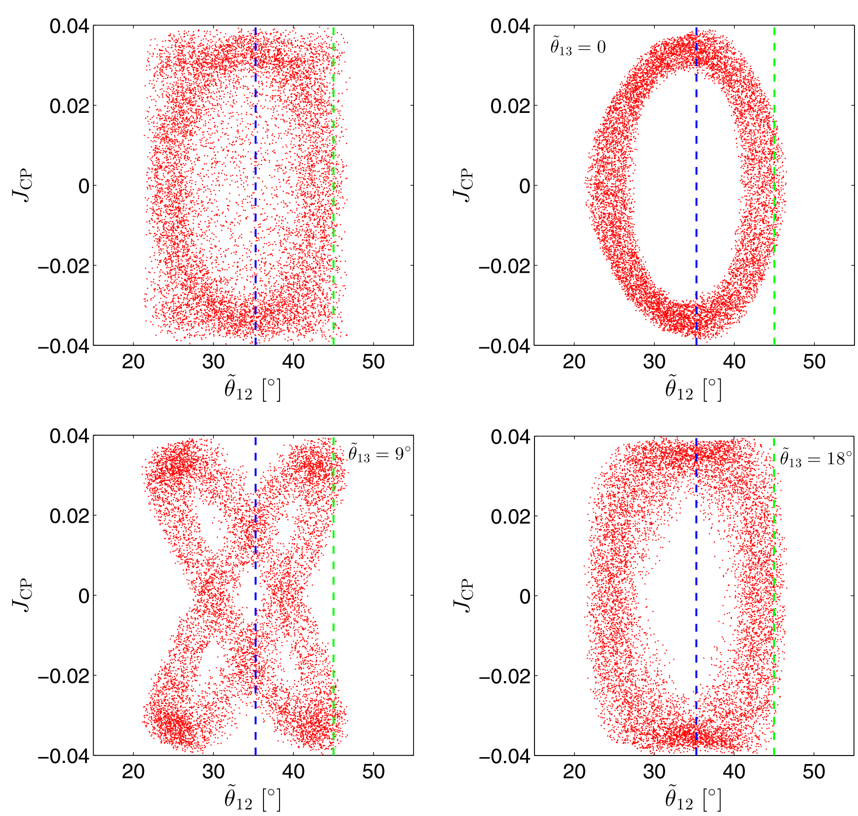

Fig. 5. Scatter plots for the parameter range of J_{CP} and $\tilde\theta_{12}$ at 3σ. Here we marginalize all the model parameters for the upper left plot, and fix $\tilde\theta_{13} = 0$, $\tilde\theta_{13} = 9^\circ$ and $\tilde\theta_{13} = 18^\circ$ in the other plots, respectively. The blue and green dashed lines correspond to $\sin^2\tilde\theta_{12} = 1/3$ and $\sin^2\tilde\theta_{12} = 1/2$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

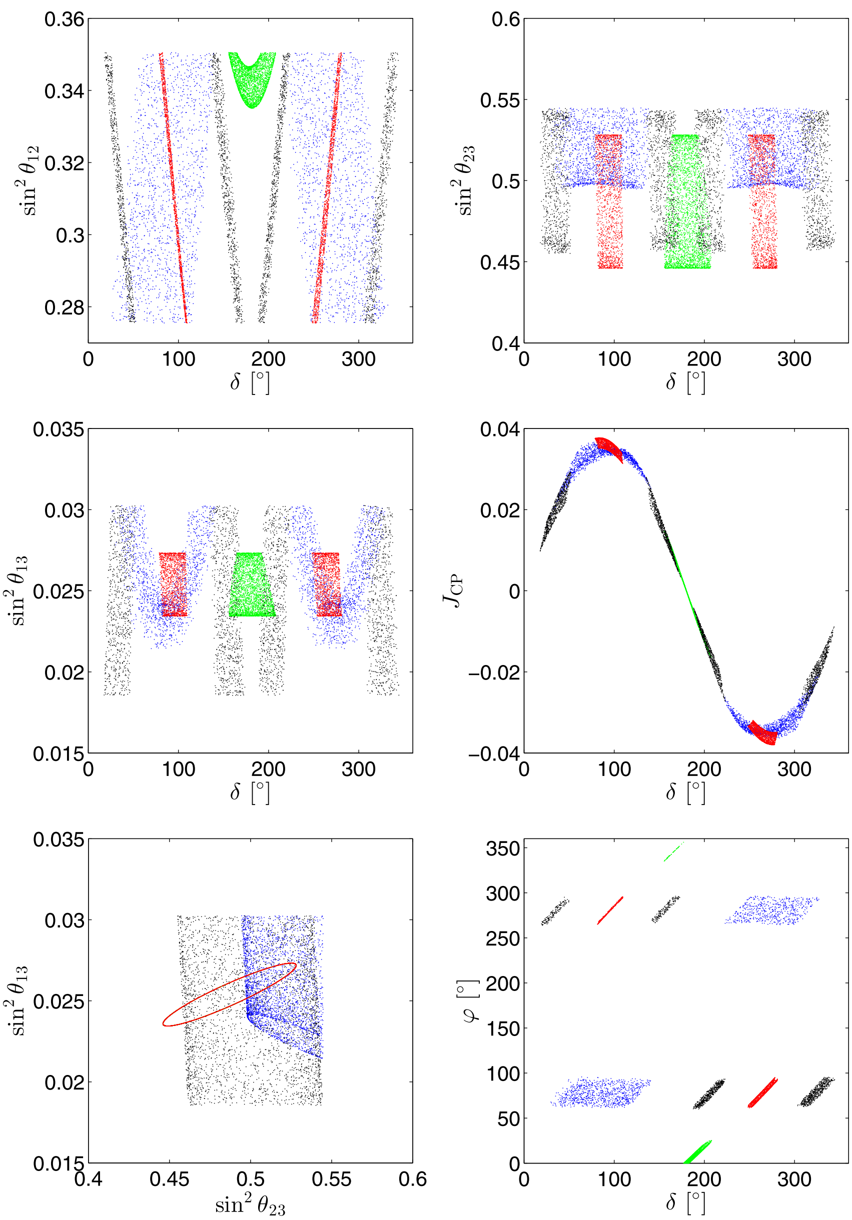

Fig. 6. The allowed 3σ range of the lepton mixing parameters and the Jarlskog invariant. Green (red) points are for bimaximal (tri-bimaximal) mixing in $\tilde{U}_\nu$. The other cases are for $\sin^2\tilde{\theta}_{12} = 1/3$, $\sin^2\tilde{\theta}_{23} = 1/2$ and $\tilde{\theta}_{13} = \pi/10$ (blue) or $\tilde{\theta}_{13} = \pi/20$ (black). Since θ_{13} and θ_{23} are related in the same way for cases a) and b), the red and green points are overlapping in the left bottom plot. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

b) bimaximal pattern with $\tilde{\theta}_{13} = 0$, $\sin^2\tilde{\theta}_{12} = 1/2$ and $\sin^2\tilde{\theta}_{23} = 1/2$ (green points);
c) large $\tilde{\theta}_{13}$ case with $\tilde{\theta}_{13} = \pi/10$, $\sin^2\tilde{\theta}_{12} = 1/3$ and $\sin^2\tilde{\theta}_{23} = 1/2$ (blue points);
d) medium $\tilde{\theta}_{13}$ case with $\tilde{\theta}_{13} = \pi/20$, $\sin^2\tilde{\theta}_{12} = 1/3$ and $\sin^2\tilde{\theta}_{23} = 1/2$ (black points);

Our analytical results from the previous Sections are confirmed, e.g., the tri-bimaximal (bimaximal) pattern leads to $\delta \simeq \pi/2$ ($\delta \simeq \pi$). When $\tilde{\theta}_{13}$ is sizable, the Dirac CP phase depends on ϕ and φ, and therefore is not fixed. However, the choice of φ is restricted from θ_{12}, which in turn sets constraints on δ.

4. Conclusions

Since for a long time only an upper limit on θ_{13} existed, most neutrino models were constructed to generate zero θ_{13}. The recent finding of a sizable value, $\theta_{13} = 9°$, have led to many studies on generating that value from an initially zero value. We have noted here that this approach may be misleading, and that in fact θ_{13} could have initially been larger. The routinely applied corrections in models will then *reduce* θ_{13} to the observed value, a possibility usually not taken into account. We illustrated the consequences of this approach in an explicit example based on charged lepton corrections.[4]

An extreme case is that initially θ_{13} corresponds to 18°, or $\pi/10$. It is then corrected by $\sin\theta_C/\sqrt{2}$ to the observed value of 9°. Hence, here we do not have $0 + 9 = 9$, but rather of $18 - 9 = 9$. An analytical and numerical study of the general case was performed, revealing new correlations and sum rules, different from

[4] Another approach could be to study radiative corrections to reduce the value of θ_{13}, or corrections from vacuum misalignment in flavor symmetry models.

the usually considered charged lepton corrections, that are based on initially vanishing θ_{13}. We find that the correlation of maximal CP violation ($\delta = \pi/2$) for initial tri-bimaximal mixing and CP conservation ($\delta = \pi$) for initial bimaximal mixing is present for both extreme cases, initial $\theta_{13} = 18^\circ$ and $\theta_{13} = 0$.

We conclude that the possibility of a more complex mixing pattern than usually considered should not be ignored. The simple framework studied here is one example where a departure from the usual approaches results in interesting and novel phenomenology.

Acknowledgements

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