Simulation of A symmetry Metamaterial Waveguide Absorber(TE&TM)

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Abstract

This communication presents a simulation of metamaterials absorber in lossy waveguide structure for solar cell applications. Both transverse electric (TE) and transverse magnetic (TM) waves propagating in a three layered waveguide structure containing metamaterials as thin film have been studied in infrared and visible regions. The odd symmetry solutions of the Eigen value equation describing lossy – guided modes with complex – valued propagation constants have been computed. They exhibit very strong longitudinal attenuation which increases with the increasing of the mode's number and increases with the decreasing of the film thickness. The longitudinal attenuations are also computed and illustrated versus the wavelength of the incident waves in the visible regions for both TE and TM waves. LHM is better absorber of higher TE modes than that of higher TM modes and the best absorption is attained at shorter wavelengths in the visible region. Moreover, normalized electric profile of TE waves and normalized magnetic profile of TM waves shows more stronger electric field and magnetic field respectively. Moreover, light absorption of the trapped modes in LHM slab are also observed than that in right handed material (RHM) or metal medium which is appropriate potentially for solar-cell application.

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1. Introduction

In the last decade, artificial sub-wavelength materials, called Left Handed Material (LHM), or meta-material whose unit cell is devised to show unnatural electromagnetic (EM) properties applicable to advanced devices. These materials have attracted great interest for research and applications [1-6]. The manipulation of effective parameters for the artificial medium diversifies the application of LHM. One of these applications, the perfect absorption (PA),
which is potentially used for sensing [7] and solar energy [8], has become one of the significant issues related to LHM. The initial PA was demonstrated for the GHz regime by Land et al [9] in 2008. To date, PAs have been developed in every relevant spectral range, from microwave [10], THz [7], near-IR [11] to the near-optical range. The study of light trapping in thin film solar cells is related to the problem of mode coupling into a lossy waveguide structure similar to the field of communications, where loss is something to be minimized and is often neglected. The goal of photovoltaic structures is to absorb as much light as possible within specific layers. It is therefore important for researchers in the field of light trapping to understand physics of wave guidance in lossy dielectric films. Until recently, such problems were normally solved through the use of perturbation theory applied to an ideal lossless model [12,13]. Such methods are useful for describing guided waves in low loss structures, but fail to account for the effects of a high-loss material on the exact field solutions. James et al [12] and [13] examined the problem of lossy waveguide propagation and derives the full-field solution to the problem of wave guidance in a symmetric and an asymmetric three layer slab. They explored loss mode propagation in the context of photovoltaic by modeling a thin film solar cell made of morphous silicon (right handed material (RHM)). S. Zhang et al [14] numerically demonstrated a metamaterial with both negative permittivity and negative permeability overlapping near-infrared wavelength range resulting in a low loss negative-index material and thus a much higher transmission, which will lead to more extensive applications. The negative index material consists of a pair of gold films separated by a dielectric layer with a two dimensional square periodic array of circular holes performing the entire multilayer structure. The negative refractive index was obtained at a wavelength around 2000 nm, the real part is as negative as -2. Furthermore, the proposed structure has a minimum feature size of ~ 100 nm. The goal of this paper is to examine the problem of lossy waveguide absorption when this left handed material (LHM) is implemented as lossy thin film in the waveguide. The model structure is illustrated in Fig.1. It consists of LHM slab with thickness surrounded by cladding and substrate layers which are lossless and symmetric with real valued index $n_c$. LHM film is lossy and has a complex index of refraction $n_h$. The simulations are also performed for another arbitrary LHM which has negative index in the visible region of frequency band at wavelength of value (600, 700 to 1400 nm) [10,15].

2. Theory

The propagation of TE waves through a thin lossy film of left handed material (LHM) with thickness $h$ covered by a lossless cladding is considered. The basic structure of interest for this work is shown in Fig.1. LHM film occupies the region $0 < x < h$. The cladding occupies the region $x > h$.

![Fig.1. Waveguide configuration for a lossy LHM waveguide.](image)

We present the Eigen value equation for transverse electric (TE) waves propagating in the $z$ direction with a
propagation wave constant in the form \( \exp \left[ i \left( k_z z - 2\pi f t \right) \right] \), \( f \) is the operating frequency. The electric and magnetic field vectors for TE waves propagating along z-axis with angular frequency \( \omega \) and wave number \( k_z \) are defined as:

\[
E = [0, E_y(\omega, z), 0] \exp \left( i \left( k_z z - \omega t \right) \right) \\
H = [H_y(\omega, z), 0, H_z(\omega, z)] \exp \left( i \left( k_z z - \omega t \right) \right)
\]  

(1)

The wave equation in each media is obtained from Maxwell's equations:

2.1. In lossy LH film, \( 0 < x < h \)

The wave equation can be found easily from the Maxwell's equations as:

\[
\frac{\partial^2 E_y}{\partial x^2} - k_h^2 E_y + k_h^2 E_y = 0
\]

(2)

where \( k_h = \sqrt{k_0 n_h} \) and \( n_h = \sqrt{-\mu_h e_h + ik} \) is the refractive index of LH film. \( e_h, \mu_h \) is the electric permittivity and magnetic permeability of LHM respectively. 

\( k_z = \beta_z + i\alpha_z \), where \( \beta_z \) is the longitudinal phase constant, \( \alpha_z \) is the mode attenuation coefficient along z-axis, \( k_0 = 2\pi / \lambda \) is the wave propagation length in free space and \( \lambda \) is the free-space wavelength for the model.

The symmetric(odd) solution of Eq(2) has the form [12]:

\[
E_y = E_0 e^{ik_{\parallel}z} \sin(k_z x),
\]

(3)

\[
H_z = (1/ i \omega \mu_0 \mu_h) E_0 k_z e^{ik_{\parallel}z} \cos(k_z x)
\]

(4)

If this solution is substituted into Eq.(2) the resulted relation is

\[
k_h^2 = k_z^2 + k_{\parallel}^2
\]

(5)

2.2. In lossless cladding, \( x > h \)

The wave equation is:

\[
\frac{\partial^2 E_y}{\partial x^2} + \left( k_c^2 - k_z^2 \right) E_y = 0
\]

(6)

Where, \( k_c = k_0 n_c \) is the wave number of the cladding region.

The symmetric(odd) solution for Eq.(6) is given by [1]

\[
E_y = c E_0 e^{ik_{\parallel}z} e^{i\gamma(x-h)},
\]

(7)

\[
H_z = (\gamma / \omega \mu_0) E_0 c e^{ik_{\parallel}z} e^{i\gamma(x-h)}
\]

(8)

c is a constant determined by boundary conditions and \( \gamma \) is the complex propagation constant, the real component of \( \gamma \) causes the phase oscillation with respect to x-axis. It satisfies the relation

\[
k_{\parallel}^2 = k_z^2 + \gamma^2.
\]

(9)

The continuity of \( E_y \) and \( H_z \) at the boundary \( x = h \) leads to the following equations:

\[
\sin(k_z h) = c
\]

(10)
\[ -i k_z \cos k_x h = c \gamma \mu_h \]

By Eq. (5) and Eq. (9),

\[ \gamma = i \sqrt{k_i^2 - k_c^2 - k_y^2} \]

By dividing Eq. (11) by Eq. (10) the eigen value equation of the waves is then obtained as:

\[ -k_x \cot(k_x h) = \mu_h \sqrt{k_i^2 - k_c^2 - k_x^2} \]  

(12)

In similar way, the eigen value equation of the TM waves is obtained as:

\[ -k_z \cot(k_z h) = \varepsilon_h \sqrt{k_i^2 - k_c^2 - k_z^2} \]  

(13)

Equation (12) determines the allowed values for the TE complex wave number of odd modes and Eq. (13) determines the allowed values for the TM complex wave number of odd modes. Since the Eigen value equation are transcendental, they can only solved through iterative methods. We used Steepest descent method with linear line search[12].

3. Numerical results and discussions

The parameters were used in carrying out the numerical calculations at near infrared frequencies, for example for the operating frequency 160 THz (\( \lambda = 1900 \text{nm} \)), LHM has effective refractive index of \( n_h = -3.74 + i2 \) [14]. The film thickness \( h = 500 \text{nm} \) and the cladding refractive index \( n_c = 1 \). For arbitrary LHM, the frequency-dependent permittivity is described by the Drude medium model as [15].

\[ \varepsilon = \varepsilon_{lattice} - \frac{\omega_p^2}{\omega^2 + i \omega \gamma} \]  

(14)

Where \( \omega \) is the angular frequency, \( \varepsilon_{lattice} \) is the lattice permittivity, \( \omega_p \) is the effective plasma frequency and \( \gamma \) is the electric damping factor \( \omega_p = 1.2 \times 10^{16} \text{rad/s}, \gamma = 1.2 \times 10^{14} \text{rad/s} \)  \( \varepsilon_{lattice} = 9.1 \). The calculations are also performed for electromagnetic radiations in the visible regions at wavelength 600, 700 to 1400 nm. In this frequency band the real part of refractive indices of LHM according to (14) are -2.338, -3.274 to -8.349 while the permeability of LHM is assumed to be -1. The dispersion equation (12) has been solved to compute the complex wave numbers of the modes. Figure 2 describes the variation ofmode attenuation along z-axis with mode's order for different film’s thicknesses. High order modes are generally more lossy than low-order modes since the mode attenuation along z-axis \( \alpha_z \) increases to the values of (8.1, 12.2 to 51.29) with the mode's number of (0, 1 to 7) as noticed by curve (3). Besides that, the value of the mode attenuation increases sharply with film’s thickness decrease as noticed by curve (1) where the film thickness is \( h = 100 \text{nm} \) which means more absorption length and more absorption is realized with thinner LHM film. Figure 3a displays the corresponding electric field profile (normalized to unit amplitude) for the mode's order of lossy (LHM) waveguide i.e. (M=0, M=1 and M=7) in the infrared region of \( \lambda = 1900 \text{nm} \). In Fig.2a, for M=0, \( k_x = 7.45 - i 1.15 \), \( k_z = 10.99 + i 8.1 \) \( \mu m^{-1} \), for M=1 \( k_x = 14.67 - i 1.7 \), \( k_z = 8.49 + i 12.2 \) \( \mu m^{-1} \) and for M=7 \( k_x = 52.19 - i 4 \), \( k_z = 5.7 + i 51.29 \) \( \mu m^{-1} \), we see that the electric field increases with mode's order increasing. For M=7, it attains three times its value for the mode M=0. This is because of increasing values of the imaginary part of the propagation number along x-axis \( k_x \). In particular, high absorption of the wave is achieved in LHM film as well as a dramatic evanescent decay in the cladding region.
Fig. 2. Longitudinal attenuation coefficient ($\alpha_z$) of TE waves versus mode number of LHM model, for:

1) $h = 100 \text{ nm}$; (2) $h = 300 \text{ nm}$; (3) $h = 500 \text{ nm}$; (4) $h = 700 \text{ nm}$; $\lambda = 1900 \text{ nm}$; $n_z = 1$

In Fig. 3b the simulation is performed as in Fig. 3a for the incident waves in the visible region of $\lambda = 600 \text{ nm}$ where the refractive index of LHM is calculated according to (14) of $n_h = -2.338 + i2$. It has been shown that most of the waves are confined in LHM film so the visible electric waves are more significant for solar cell applications. Figure 4a displays a comparison of normalized electric field profile of $M=2$ TE mode of LHM with that of RHM (amorphous silicon) and with that of a metal in the infrared region of $\lambda = 1900 \text{ nm}$, $n_h = -3.74 + i2$ [14].

Figure 4b displays the same simulation in the visible region of $\lambda = 600 \text{ nm}$ and $n_h = -2.338 + i2$ at $h = 500 \text{ nm}$. The refractive index of amorphous silicon film [RHM][12] is $n_j = 4.9 + i0.3$ and that's of metal film is $n_m = -100 + i0.3$. In Fig. 4a the computed wave numbers are ($k_z = 20.79 - i2.4$, $k_z = 6.9 + i9$) $\mu \text{m}^{-1}$ for LHM, and ($k_z = 15.78 + i0.72$, $k_z = 0.788 + i4.522$) $\mu \text{m}^{-1}$ for RHM, ($k_z = 18.73 - i338 \times 10^{-4}$, $k_z = 330.2 - i0.993$) $\mu \text{m}^{-1}$ for metal. The increasing values of the longitudinal attenuation $\alpha_z$ (imaginary part of $k_z$) is important to the field of light trapping in thin films, as it represents the absorption length of a guided mode in the structure. Negative $\alpha_z$ means loss of wave power from the structure while positive $\alpha_z$ means the absorption of wave power by the structure. It is shown that the longitudinal attenuation $\alpha_z$ of (LHM) is larger than that's for RHM or metal. Figure 4b displays that the most of waves are trapped in the film in the visible region.
Fig. 3. Electric field profile (normalized to unit amplitude) of the $M=0; M=1; M=7$ modes of TE waves for LHM model, for (a) infrared region $n_e = -3.74 + i 2; \lambda = 1900 \text{nm}$; (b) visible region $n_e = -2.338 + i 2; n_c = 1; h = 500 \text{nm}$
Fig. 4. Electric field profile of the M=2 TE mode at $h = 500\,\text{nm}$; $n_e = 1$; $n_f = 4.9 + i0.3$ for RHM, and for metal $n_w = -100 + i0.3$ for (a) (infrared region) $\lambda = 1900\,\text{nm}$; $n_x = -3.74 + i2$; (b) (visible region) $\lambda = 600\,\text{nm}$; $n_x = -2.338 + i2$ for LHM model.
Fig. 5. Magnetic field profile of the $M=0; 2; 4$ TM mode at $h=500\text{nm}$ $n_e=1$; $n_n=-2.338+i2$ for LHM model in the visible region of $\lambda=600\text{nm}$.

Fig. 6. Magnetic field profile of the $M=2$ TM mode at $h=500\text{nm}$ in the infrared region; $\lambda=1900\text{nm}$; $n_f=1; n_n=-3.74+i2$ for LHM model, $n_f=4.9+i0.3$ for RHM, and for metal, $n_m=-100+i0.3$. 
Fig. 7. The attenuation of both TE and TM of the (a) M=0; (b) M=2; (c) M=4 mode as a function of wavelength for arbitrary LHM model, in the visible region $h = 500 \text{ nm}$
It is worth to notice that LHM is the best absorber in the visible region leading to more extensive applications. Figure 5 illustrates the magnetic field profile of TM waves (normalized to unit amplitude) for the mode's order of lossy (LHM) waveguide i.e. \((M=0, M=2\) and \(M=4\)) in the visible region of wavelength of \(\lambda = 600 \text{ nm}\). For \(M=2\), \((k_x = 7.18 - i 11.66, k_z = 27 + i 22) \mu \text{m}^{-1}\) and \((k_x = 26.57 - i 10.5, k_z = 24.6 + i 32.2) \mu \text{m}^{-1}\) for \(M=4\). For \(M=4\), \(\alpha_z\) is 32.2 but for \(M=2\) its 22 which means that mode 4 has larger absorption length than mode 2. Figure 6 also shows a comparison of normalized magnetic field profile of \(M=2\) TM mode of LHM with that of RHM (amorphous silicon) and with that of a metal at the same parameters of Fig.(4a). The computed wave numbers are \((k_x = 5.93 - i 4.268, k_z = 128 + i 8.24) \mu \text{m}^{-1}\) for LHM and \((k_x = 16 + i 1.69, k_z = 3.86 - i 2.88) \mu \text{m}^{-1}\) for RH, \((k_x = 24.68 - i 0.15, k_z = 329.7 - i 0.98) \mu \text{m}^{-1}\) for metal. We see that the magnetic field of TM mode of LHM is higher than that of metal or RHM. \(\alpha_z\) of LHM is positive while that of RHM and metal is negative which means that LHM is the best absorber of TM modes. As shown by Fig.7 the longitudinal attenuations are computed and illustrated versus the wavelength of the incident waves in the visible regions (600-1400 nm) for both TE and TM waves of modes \(M=0, 2\). It is noticed that the attenuation in this frequency band increases with the decreasing of the wavelength where best absorption is attained at shorter wavelengths. With respect to mode 0, 2 small difference is observed in the attenuation values of TM and TE waves while a big difference occurs for mode 4. We see that in the region (600-1100 nm) the TE attenuation decreases from 35 to 21 \(\mu \text{m}^{-1}\) but the TM attenuation decreases from 32 to -1 \(\mu \text{m}^{-1}\) where the higher order modes of TE waves such as mode 4 are more attenuated than of TM modes in the visible region (600-1100 nm) of LHM film which leads that LHM is better absorber of higher TE modes than of TM modes. Such results may have useful implications for light trapping applications where the ultimate goal is to maximize the light absorption in a finite film.

4. Conclusions

We presented the simulation of metamaterial absorber in a lossy thin LH film surrounded by lossless dielectric cladding. The propagation characteristics of both TE and TM waves are studied in both infrared region and visible region. In infrared region, the mode attenuation is calculated versus the film thickness and mode's number. It has been shown that the implementation of LH film in the THz range of electromagnetic wave and of minimum 100 nm thickness heightens the waves intensity in the film where maximum light absorption is achieved for the higher mode's number and stimulates evanescent decay of waves in the cladding region. LHM is better absorber of higher TE modes than that of TM modes and the best absorption is attained at shorter wavelengths in the visible region of wavelength (600, 700 to 1400 nm). This study will make promising foundation for future works and provides some insights into the potential applications of LHM, in particular solar cell.

References