Alexandria Engineering Journal (2015) 54, 615–622

HOSTED BY
ELSEVIER

Alexandria University
Alexandria Engineering Journal
www.elsevier.com/locate/aej
www.sciencedirect.com

ORIGINAL ARTICLE

Thermodynamics analysis of hydromagnetic third grade fluid flow through a channel filled with porous medium

Samuel O. Adesanya [a,*], **J.A. Falade** [b]

[a] *Department of Mathematical Sciences, Redeemer's University, Ede, Nigeria*
[b] *Department of Physical Sciences, Redeemer's University, Ede, Nigeria*

Received 23 February 2015; revised 21 May 2015; accepted 24 May 2015
Available online 13 June 2015

KEYWORDS

Entropy generation;
Perturbation method;
Third grade fluid;
Porous medium

Abstract In this work, analysis has been carried out to study the entropy generation rate in the flow and heat transfer of hydromagnetic third grade fluid between horizontal parallel plates saturated with porous materials. The flow is induced by a constant pressure gradient applied in the flow direction and also influenced by a uniform magnetic field that is applied across the flow channel. The equations governing the fluid flow are modeled, non-dimensionalized and solved analytically using regular perturbation method. The effect of various flow parameters on the fluid flow is presented graphically and discussed.

1. Introduction

Appreciable progress has been made on the mechanics of third grade fluid in recent times. This is due to its numerous applications in geology, petro-chemical engineering, pharmaceutical industries, lubricants, confectioneries etc. Since no single constitutive model can adequately describe the complex rheological properties of non-Newtonian fluid. The third grade model belongs to an important class of non-Newtonian fluid with the ability to represent the shear thickening and thinning property of the fluid. In fact, quite a lot has been done on the flow and heat transfer characteristics of the fluid. For instance, Hayat et al. [1] analyzed the flow and heat transfer characteristics in the electrically conducting third grade fluid through parallel plates under different geometries. Hayat et al. [2] examined the two-dimensional and magnetohydrodynamic (MHD) flow of thixotropic fluid over a stretched surface in the presence of thermal radiation with variable thermal conductivity. Shafiq et al. [3] investigated the magnetohydrodynamic axisymmetric flow of a third grade fluid between two porous disks.

Ellahi and Riaz [4] studied the influence of magnetohydrodynamics (MHD) on the pipe flow and heat transfer of a third-grade fluid with variable viscosity. Hayat and Kara [5] studied the time-dependent flow of an incompressible third-grade fluid under the influence of a magnetic field of variable strength between the annular regions in two coaxial cylinders.

Moreover, Ellahi and Afzal [6] studied the effect of temperature-dependent viscosity and viscous dissipation on the non-Newtonian fluid flow through a porous medium using

* Corresponding author.
E-mail address: adesanyaolumide@yahoo.com (S.O. Adesanya).
Peer review under responsibility of Faculty of Engineering, Alexandria University.

http://dx.doi.org/10.1016/j.aej.2015.05.014
1110-0168

modified Darcy's law. In the study, Hayat et al. [7] modeled the flow of a third grade fluid in a porous half space based on modified Darcy's law. The result showed that that modified Darcy's law for unidirectional flow of a third grade fluid yields non-linear expression in terms of velocity whereas it is linear for Newtonian, Oldroyd-B, Maxwell and second grade fluids. Also, Ellahi et al. [8], addressed the heat transfer analysis on the laminar flow of an incompressible third grade fluid through a porous flat channel in which the lower plate is assumed to be at a higher temperature than the upper plate. Hayat et al. [9] examined the slip effects on the flow and heat transfer of a third grade fluid through a porous plate while in [10], Hayat et al. presented the exact solution for a thin film of third grade fluid flow down an inclined plane.

All the above studies neglect the entropy production in the fluid flow. However, from thermodynamics point of view, entropy generation is continuous in moving fluid with temperature difference and can significantly alter the success of the desired goal. This is due to the fact that increased entropy generation rate translates into destruction of the available energy for work between the heat reservoirs. This is one of the indices used in predicting the thermal performance of a system. For instance, Das and Jana [11] studied the combined effect of magnetic field and Navier slip on the entropy generation in a flow of viscous incompressible and electrically conducting fluid between two infinite horizontal parallel plates under a constant pressure gradient. Similarly, Adesanya [12] applied the second law of thermodynamics to analyze the steady flow viscous incompressible liquid with temperature dependent properties. Moreover, Makinde [13] studied the inherent irreversibility in the flow of a variable viscous fluid through a narrow channel with a non-uniform wall temperature. As a result, the present analysis will follow the second law of thermodynamics. This approach has been successfully applied in the literature to investigate the thermal performance of some fluid system under different flow conditions in Refs. [14–22] and lots more.

Motivated by the study [1], in the present paper, attention is focused on the flow of hot moving third grade fluid through a porous medium together with the thermodynamic analysis, which was not accounted for in the previous models. Emphasis is on the isothermal case due to its application in geology. For instance, in a petroleum reservoir containing hydrocarbons with low American Petroleum Institute (API) degree and high viscosity. Intervention wells are drilled to introduce hot steam with temperature T_1 into a porous bed of temperature T_0. So that the flow may be induced by temperature difference and the pressure gradient in the flow direction. A survey of literature shows that little or nothing much has been done in this area of study. This explains why the research is worthwhile. The paper is organized as follows; in Section 2, the problem is formulated. The third section gives the method of solution. Results are presented and discussed in section four while section five concludes the paper.

2. Mathematical formulation

Consider the steady flow of a third grade fluid flow between two infinite porous plates of distance 2 h apart kept at temperature T_0. Due to the introduction of hot steam with temperature T_1 into the channel the fluid temperature T_0 increases and the fluid temperature attains the temperature T_1–T_0 where $T_1 > T_0$. The entropy lost in the hot region is gained in the cold region in accordance to second law of thermodynamics. The equations governing the fluid flow are formulated based on the following assumptions; the flow is acted upon by a constant pressure gradient, the induced magnetic field is negligible and buoyancy forces are neglected.

Under this configuration, the velocity field is given by (see Fig. 1)

$$V = (u', 0, 0), \tag{1}$$

while the conservation of momentum and balanced energy equation for the hydromagnetic fluid are given by

$$\nabla \cdot V = 0, \tag{2}$$

$$\rho \frac{DV}{Dt} = \nabla \cdot S, \tag{3}$$

$$\rho c_p \frac{DT'}{Dt} = k\nabla^2 T' + S \cdot L. \tag{4}$$

The Cauchy stress tensor S is defined as

$$S = -p_1 I + \mu A_1 + \alpha_1 A_2 + \alpha_2 A_1^2 + \beta_1 A_3 + \beta_2 (A_1 A_2 + A_2 A_1) + \beta_3 (tr A_1^2) A_1, \tag{5}$$

where the first three kinematic tensors $A_i (i = 1, 2, 3)$ are defined by

$$A_1 = \nabla V + (\nabla V)^t, \tag{6}$$

$$A_n = \frac{dA_{n-1}}{dt} + A_{n-1}(\nabla V) + (\nabla V)^t A_{n-1}, \; n = 2, 3. \tag{7}$$

where $V = [u(y), 0, 0]$ is the velocity field, $L = \text{grad } V$, ρ is the constant fluid density, J is the current density, B is the total magnetic field, E is the total electric field, μ_m is the magnetic permeability, R is the Darcy's resistance, S is the stress tensor, C_p is the specific heat, k is the thermal conductivity, T' is the dimensional fluid temperature, $\frac{D}{Dt}$ is the material derivative, ∇ is the gradient operator, p is the fluid pressure, μ is the dynamic viscosity of the fluid, $\alpha_i (i = 1, 2)$ and $\beta_j (j = 1, 2, 3)$ are the material constants, and $A_i (i = 1, 2, 3)$ are the Rilvin–Ericksen tensors.

Now, if the motion of the fluid is compatible with the thermodynamics, then the Clausius – Duhem inequality together

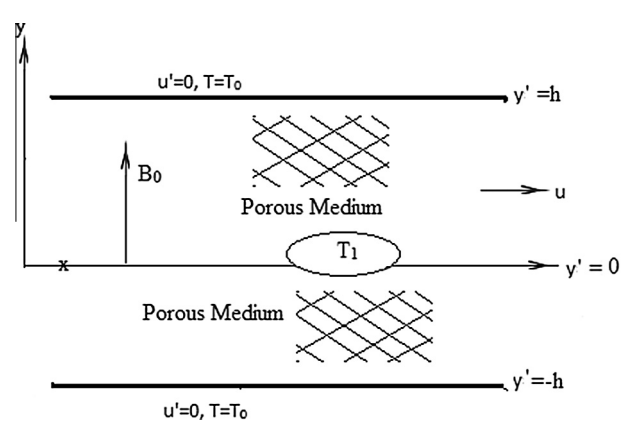

Figure 1 Flow geometry.

with the assumption that the Helmholtz free energy is minimum at equilibrium which implies that

$$\mu \geqslant 0,\ \beta_1 = \beta_2 = 0,\ \beta_3 \geqslant 0,\ \alpha_1 \geqslant 0, |\alpha_1 + \alpha_2| \leqslant \sqrt{24\mu\beta_3}, \tag{8}$$

for thermodynamically compatible third grade fluid, Eq. (5) becomes

$$S = -p_1 I + \mu_{eff} A_1 + \alpha_1 A_2 + \alpha_2 A_1^2. \tag{9}$$

where the effective shear-dependent viscosity is

$$\mu_{eff} = \mu + \beta_3 tr A_1^2. \tag{10}$$

Then in view of Gauss law of magnetism

$$\nabla \cdot B = 0, \tag{11a}$$

the Ampere's law

$$\nabla \times B = \mu_m J, \tag{11b}$$

Faraday's law of induction

$$\nabla \times E = -\frac{\partial B}{\partial t}, \tag{11c}$$

Ohm's law

$$J = \sigma[E + V \times B], \tag{11d}$$

in view of (11), we have

$$\left.\begin{aligned} &\nabla \cdot V = 0, \\ &\rho \tfrac{DV}{Dt} = J \times B + \nabla \cdot S + R, \\ &\rho c_p \tfrac{DT}{Dt} = k\nabla^2 T + J * E + S \cdot L. \end{aligned}\right\} \tag{12}$$

Then the fully developed flow is governed by the momentum equation

$$0 = -\frac{dp}{dx} + \mu\frac{d^2u'}{dy'^2} + 6\beta_3\frac{d^2u'}{dy'^2}\left(\frac{du'}{dy'}\right)^2 - \sigma B_0^2 u - \frac{\mu}{K_1}\left(1 + \frac{2\beta_3}{\mu}\left(\frac{du'}{dy'}\right)^2\right)u' = 0 \tag{13}$$

while the energy equation for the thermodynamically developed flow is given by [3]

$$0 = k\frac{d^2T'}{dy'^2} + \mu\left(\frac{du'}{dy'}\right)^2 + 2\beta_3\left(\frac{du'}{dy'}\right)^4 + \sigma B_0^2 u'^2 + \frac{\mu}{K_1}\left(1 + \frac{2\beta_3}{\mu}\left(\frac{du'}{dy'}\right)^2\right)u'^2. \tag{14}$$

Additional terms in (14) are due Ohmic heating of the fluid and the porosity of the channel. The appropriate boundary conditions for the isothermal fluid flow arethe following.

$$\left.\begin{aligned} &u' = 0, T' = T_0 \quad on \quad y = -h, \\ &u' = 0, T' = T_0 \quad on \quad y = h. \end{aligned}\right\} \tag{15}$$

Introducing the following dimensionless variables

$$\begin{aligned} &y = \tfrac{y'}{h}, u = \tfrac{u'}{U},\ \gamma = \tfrac{\beta_3 U^2}{\mu h^2},\ \theta = \tfrac{T - T_0}{T_1 - T_0},\ Br = \tfrac{\mu U^2}{k(T_1 - T_0)}, \\ &Ha^2 = \tfrac{\sigma B_0^2 h^2}{\mu},\ \tfrac{1}{K} = \tfrac{h^2}{K_1},\ N_s = \tfrac{h^2 E_G T_0^2}{k(T_1 - T_0)^2},\ \Omega = \tfrac{T_1 - T_0}{T_0} \end{aligned} \tag{16}$$

we obtain the following dimensionless boundary-valued problems

$$\frac{d^2u}{dy^2} + 6\gamma\frac{d^2u}{dy^2}\left(\frac{du}{dy}\right)^2 - Ha^2 u - \frac{u}{K}\left(1 + 2\gamma\left(\frac{du}{dy}\right)^2\right) + G = 0; u(-1) = 0 = u(1), \tag{17}$$

$$\frac{d^2\theta}{dy^2} + Br\left\{\left(\frac{du}{dy}\right)^2\left(1 + 2\gamma\left(\frac{du}{dy}\right)^2\right) + \left(Ha^2 + \frac{1}{K}\left(1 + 2\gamma\left(\frac{du}{dy}\right)^2\right)\right)u^2\right\} = 0; \theta(-1) = 0 = \theta(1). \tag{18}$$

where u' is the dimensional fluid velocity, T' is the dimensional fluid temperature, γ is the dimensionless third grade material parameter, Br isviscous heating parameter, θ is the dimensionless temperature, u is the dimensionless velocity, G is the dimensionless pressure gradient, Ha^2 is the Hartmann number that measures the magnetic field intensity, B_0 is the electromagnetic induction, σ is the conductivity of the fluid, (K_1, K) are the dimensional and non-dimensional porous permeability parameters, and U is the characteristic velocity.

In order to obtain the solution by regular perturbation method, we assume that $0 < \gamma << 1$ so that we can form solutions in the following form

$$u(y) = u_0 + \gamma u_1 + O(\gamma^2), \tag{19}$$

using (21) in (18) and equating orders, we have

$$\gamma^0: \quad \frac{d^2u_0}{dy^2} - \left(Ha^2 + \frac{1}{K}\right)u_0 = -G;\ u_0(\pm 1) = 0, \tag{20}$$

$$\gamma^1: \quad \frac{d^2u_1}{dy^2} + 6\frac{d^2u_0}{dy^2}\left(\frac{du_0}{dy}\right)^2 - Ha^2 u_1 - \frac{1}{K}\left(u_1 - 2u_0\left(\frac{du_0}{dy}\right)^2\right) = 0;\ u_1(\pm 1) = 0, \tag{21}$$

using DSolve algorithm in a symbolic computer algebra package – MATHEMATICA, the solutions of the boundary valued problems (20) and (21) are obtained and substituted in (19). With the solution (19) known then (18) can be elegantly solved. The huge size of the solution suggests that only graphical solutions can be presented as shown in Figs. 2–8.

3. Entropy generation

Heat transfer by convection process within the channel is irreversible and entropy generation rate within the channel becomes continuous due to exchange of energy and momentum within the fluid particles channel. Therefore, the total entropy generation within the fluid system can be written as

$$E_G = \frac{k}{T_0^2}\left(\frac{dT}{dy'}\right)^2 + \frac{\mu}{T_0}\left[\left(\frac{du'}{dy'}\right)^2\left(1 + \frac{2\beta_3}{\mu}\left(\frac{du'}{dy'}\right)^2\right) + \frac{\sigma B_0^2}{\mu}u'^2 + \frac{1}{K_1}\left(1 + \frac{2\beta_3}{\mu}\left(\frac{du'}{dy'}\right)^2\right)u'^2\right], \tag{22}$$

where the first part is the contribution due to heat transfer in the direction of finite temperature gradient and the second part represents the contribution due to fluid friction irreversibility.

Using the dimensionless parameters and variables (16), we get the equation for the dimensionless entropy generation rate as follows

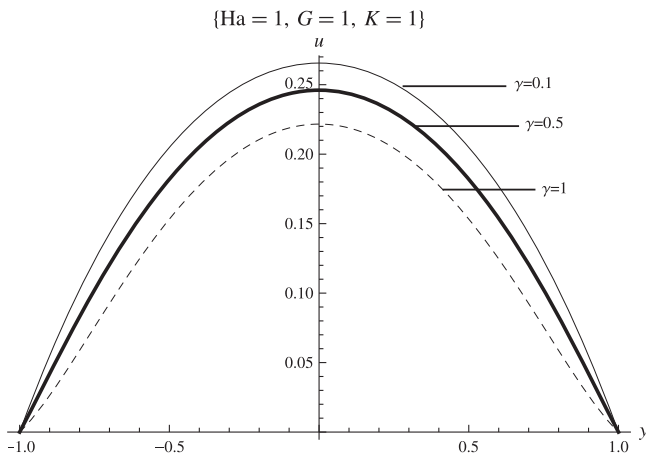

Figure 2 Plot of velocity profile for various values of third grade material effect.

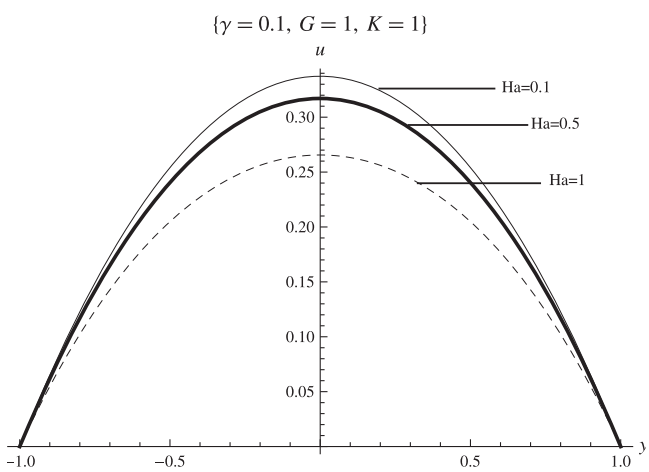

Figure 3 Plot of velocity profile for various values of Hartmann's number.

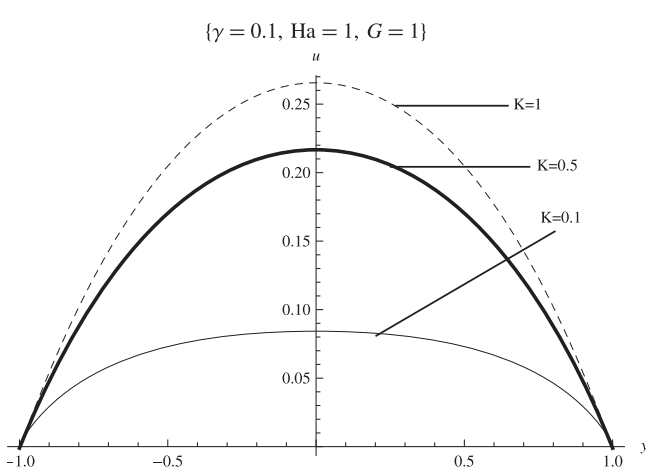

Figure 4 Plot of velocity profile for various values of porous permeability parameter.

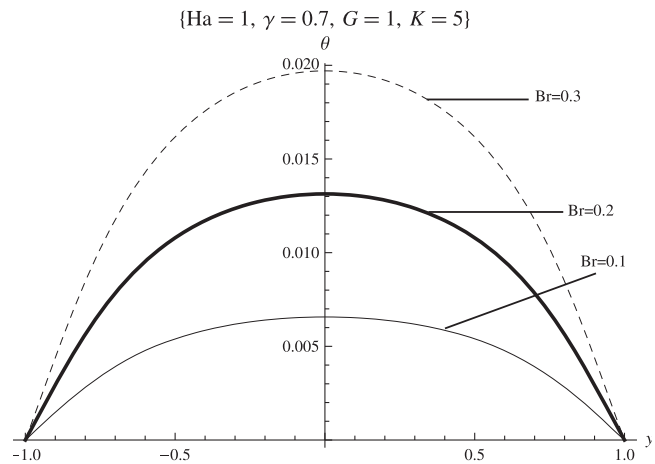

Figure 5 Plot of temperature profile for various values of Brinkman's number.

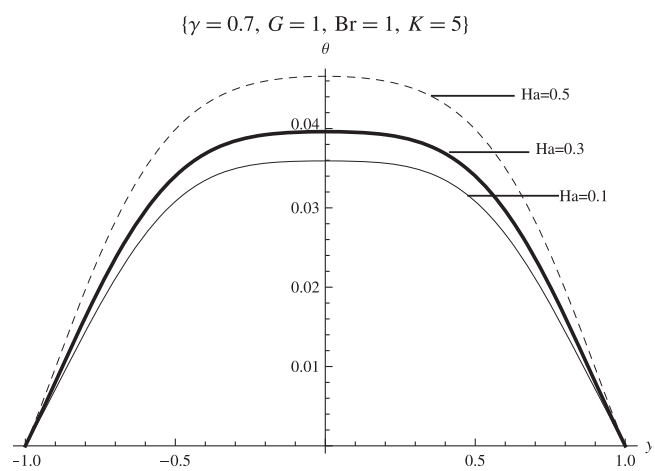

Figure 6 Plot of temperature profile for various values of Hartmann's number.

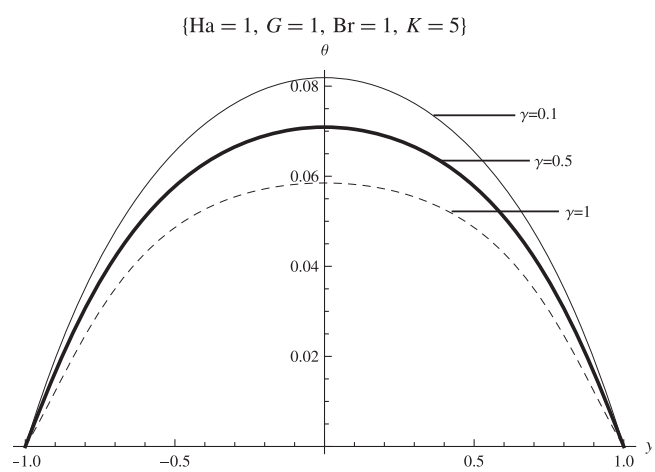

Figure 7 Plot of temperature profile for various values of third grade material effect.

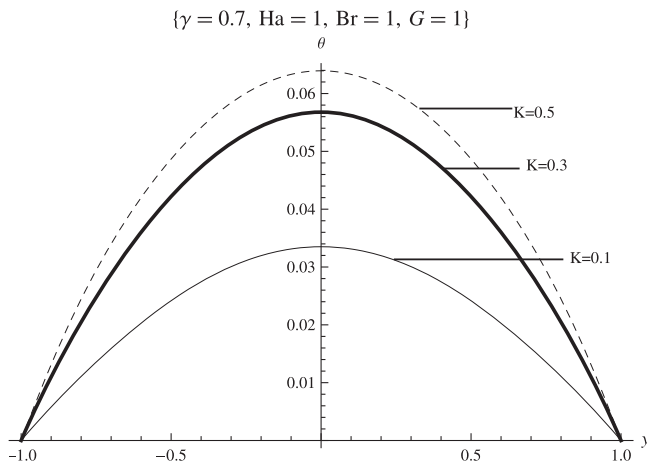

Figure 8 Plot of temperature profile for various values of porous permeability parameter.

$$N_S = \left(\frac{d\theta}{dy}\right)^2 + \frac{Br}{\Omega}\left[\left(\frac{du}{dy}\right)^2\left(1 + 2\gamma\left(\frac{du}{dy}\right)^2\right) + \left(Ha^2 + \frac{1}{K}\left(1 + 2\gamma\left(\frac{du}{dy}\right)^2\right)\right)u^2\right] \tag{23}$$

substituting the results obtained in (19) and (18) in (23) the entropy generation rate within the flow channel can be seen as shown in Figs. 9–12.

The irreversibility ratio within the channel can then be computed using (23) so as to determine the influence of heat transfer and fluid friction on the heat flow within the channel.Let us take N_1, and N_2 as the heat transfer and viscous dissipation respectively then

$$N_1 = \left(\frac{d\theta}{dy}\right)^2, N_2 = \frac{Br}{\Omega}\left[\left(\frac{du}{dy}\right)^2\left(1 + 2\gamma\left(\frac{du}{dy}\right)^2\right) + \left(Ha^2 + \frac{1}{K}\left(1 + 2\gamma\left(\frac{du}{dy}\right)^2\right)\right)u^2\right] \tag{24}$$

such that the irreversibility ratio denoted by Bejan number (Be) can be written as

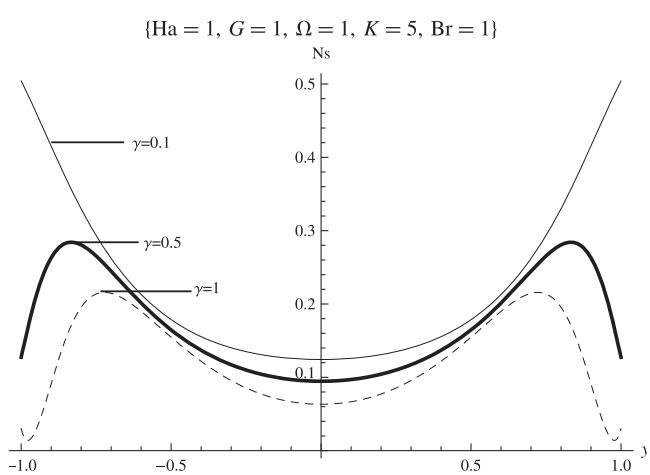

Figure 10 Plot of entropy generation rate for various values of Hartmann's number.

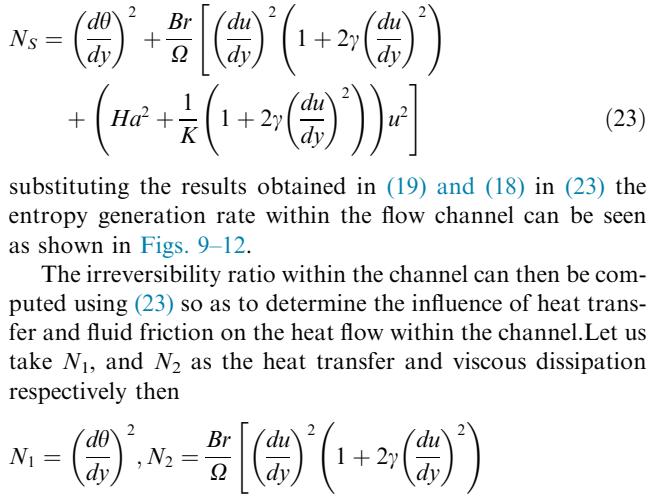

Figure 11 Plot of entropy generation rate for various values of Brinkman's number.

Figure 9 Plot of entropy generation rate for various values of third grade material effect.

Figure 12 Plot of entropy generation rate for various values of porous permeability parameter.

$$Be = \frac{N_1}{N_S} = \frac{N_1}{N_1 + N_2} = \frac{1}{1 + \Phi}, \quad \Phi = \frac{N_2}{N_1} \tag{25}$$

and shown graphically as Figs. 13–16.

Ω is the temperature difference parameter, (E_G, N_S) are the dimensional and non-dimensional entropy generation rates.

4. Results and discussion

In this section, numerical results are presented to show the effect of each fluid parameter on the flow. Fig. 2 depicts the variations in the third grade material effect on the fluid velocity, from the graph it is evident that increase in the third grade material is to decrease the flow velocity due to fluid thickening. This is physically true since the API decreases with specific gravity as well as rise in the viscosity of the fluid. The retarding effect of the Lorentz forces applied in x direction to the flow channel shown in Fig. 3 is to decrease the fluid velocity. This is one of the control measures in handling hot moving hydromagnetic fluid under extremely high temperature. The response of the fluid velocity to the porous permeability parameter is shown in Fig. 4 from the graph it is evident that as the permeability of the medium increases there is corresponding rise in the fluid velocity.

This shows that the homogeneity of the flow channel will be attained as the porous permeability increases. Fig. 5 shows the effect of Brinkman number on the temperature profile. As shown in the graph, an increase in the Brinkman number leads to an increase in the temperature distribution within the flow channel. This is due to the fact that heat energy is stored within the moving fluid as a result of the viscous heating. Hence, as a heat source it enhances the temperature at any point in the fluid. Fig. 6 represents the effect of magnetic field on the fluid flow. From the plot it is observed that an increase in the magnetic field intensity leads to an increase in the temperature of the fluid due to the Ohmic heating. Fig. 7 depicts the effect of third grade material on the fluid flow. It is observed that an increase in the third grade material effect leads to a decrease in the fluid temperature. This is physically true due to the thickening property of the fluid resulting from decreased API degree of the fluid. On the other hand, effect of the porous permeability parameter on the fluid temperature distribution is shown in Fig. 8. The result shows that as the porous

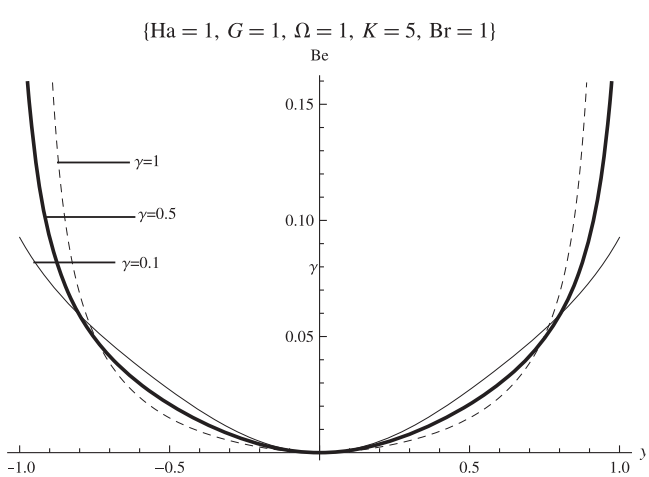

Figure 13 Plot of irreversibility ratio for various values of third grade material effect.

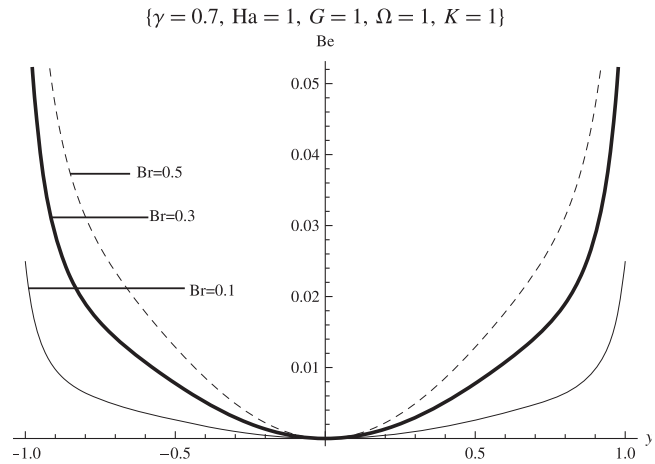

Figure 14 Plot of irreversibility ratio for various values of Brinkman's number.

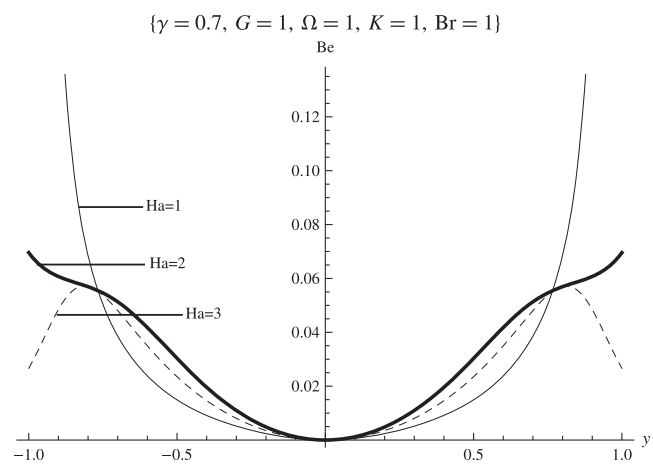

Figure 15 Plot of irreversibility ratio for various values of Hartmann's number.

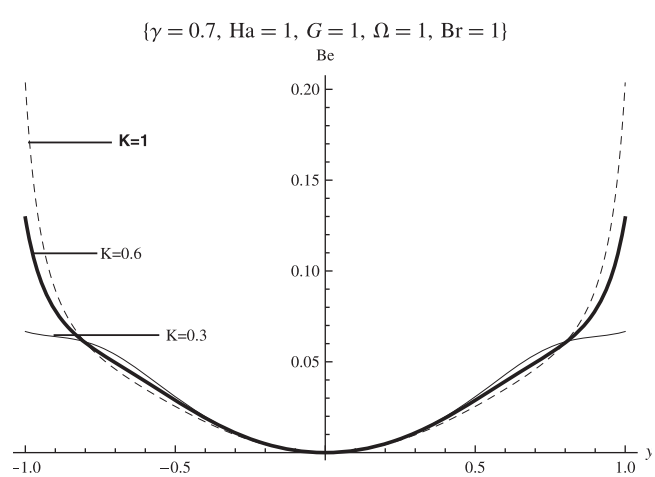

Figure 16 Plot of irreversibility ratio for various values of porous permeability parameter.

permeability parameter increases there is a corresponding increase in the fluid temperature due to increased diffusion of heat within the flow channel.

Fig. 9 shows the effect of the third grade material effect on the fluid flow. As shown in Fig. 7, temperature of the fluid decreases with increasing values of the third grade material effect. Hence it is expected to lower the entropy generation rate within the flow channel. Fig. 10 shows the effect of variations of Hartmann's number on the entropy generation rate within the channel. From the plot it is observed that an increase in the magnetic field intensity increases the entropy generation rate toward the channel walls and oscillated through the channel center line. This is due to the imbalance in the Lorentz forces which decreases the flow velocity and the Ohmic heating of the fluid that enhances the fluid temperature. Moreover, an increase in the Brinkman parameter is observed in Fig. 11 to encourage entropy generation rate. This is due to the excessive production of heat within the channel as the viscous heating increases as shown in Fig. 5. Moreover, as shown in Fig. 12 an increase in the porous permeability parameter increases the entropy production at the fluid layers close to the wall while it decreases at the centerline of the channel.

Figs. 13–16 display the irreversibility ratio for various flow parameters. In Fig. 13 it is observed that as the third grade material effect increases fluid friction dominates over the heat transfer irreversibility in the centerline of the channel while heat transfer dominates over viscous dissipation at the walls. Fig. 14 represents the effect of Brinkman number on the irreversibility ratio of heat within the channel. As observed from the graph, at the centerline of the channel fluid friction dominates over heat transfer irreversibility while heat transfer dominates over fluid friction at the walls. This is because as Brinkman number increases the viscosity of the fluid increases while the thermal conductivity of the fluid decreases. Fig. 15 shows the effect of Hartmann number on the irreversibility ratio within the channel. As observed from the graph, fluid friction dominates over heat transfer at the centerline of the channel and at the walls but the dominance is more pronounced at the centerline than at the walls. Finally, as shown in Fig. 16 fluid friction dominates over heat transfer at the centerline of the channel as the porous permeability parameter increases while heat transfer dominates at the walls.

5. Conclusion

In this paper, an analytical study of flow and heat transfer for hydromagnetic third grade fluid through a channel filled with porous medium is investigated. Solutions are obtained using regular perturbation method implemented on a computer symbolic algebra package-mathematica. The solutions are used to compute the entropy generation rate and the irreversibility ratio within the flow channel. The result of the computation shows that

i. An increase in the third grade material effect and magnetic field intensity is observed to decrease the fluid flow velocity while an increase in the porous permeability parameter is observed to enhance the third grade fluid flow velocity within the channel.
ii. An increase in the porous permeability parameter, Brinkmann number and magnetic field intensity is observed to increase the temperature distribution within the channel.
iii. At the centerline of the channel, entropy generation rate increases with an increase in Brinkman number and magnetic field strength intensity while it decreases with an increase in third grade material effect and the porous permeability parameters.
iv. Irreversibility due to fluid friction dominates over the heat transfer in the centerline of the channel while irreversibility due to heat transfer dominates over fluid friction at the walls.

References

[1] T. Hayat, R. Ellahi, F.M. Mahomed, The analytical solutions for magnetohydrodynamic flow of a of third order fluid in a porous medium, Zeitschrift Fur Naturforschung A 64 (a) (2009) 531–539.
[2] T. Hayat, S.A. Shehzad, S. Asghar, MHD low, thixotropic fluid, thermal radiation, variable thermal conductivity and thermal radiation, Walailak J. Sci. Technol. 10 (1) (2013) 29–42.
[3] A. Shafiq, M. Nawaz, T. Hayat, A. Alsaedi, Magnetohydrodynamic axisymmetric flow of a third-grade fluid between two porous disks, Brazilian J. Chem. Eng. 30 (2013) 599–609.
[4] R. Ellahi, A. Riaz, Analytical solution for MHD flow in a third grade fluid with variable viscosity, Math. Comput. Model. 52 (2010) 1783–1793.
[5] T. Hayat, A.H. Kara, Couette flow of a third-grade fluid with variable magnetic field, Math. Comput. Model. 43 (2006) 132–137.
[6] R. Ellahi, S. Afzal, Effects of variable viscosity in a third grade fluid with porous medium: an analytical solution, Commun. Nonlinear Sci. Numer. Simul. 14 (5) (2009) 2056–2072.
[7] T. Hayat, F. Shahzad, M. Ayub, Analytical solution for the steady flow of the third grade fluid in a porous half space, Appl. Math. Model. 31 (2007) 2424–2432.
[8] R. Ellahi, P.D. Ariel, T. Hayat, S. Asghar, Effect of heat transfer on a third grade fluid in a flat channel, Int. J. Numer. Method Fluids 63 (7) (2010) 847–859.
[9] T. Hayat, M. Asif Farooq, T. Javed, M. Sajid, Partial slip effects on the flow and heat transfer characteristics in a third grade fluid, Nonlinear Anal.: Real World Appl. 10 (2009) 745–755.
[10] T. Hayat, R. Ellahi, F.M. Mahomed, Exact solutions for thin film flow of a third grade fluid down an inclined plane, Chaos, Solitons Fractals 38 (5) (2008) 1336–1341.
[11] S. Das, R.N. Jana, Entropy generation due to MHD flow in a porous channel with Navier slip, Ain Shams Eng. J. 5 (2014) 575–584.
[12] S.O. Adesanya, Second law analysis for third-grade fluid with variable properties, J. Thermodyn. vol. 2014, Article ID 452168, 8 pages http://dx.doi.org/10.1155/2014/452168.
[13] O.D. Makinde, Irreversibility analysis of variable viscosity channel flow with convective cooling at the walls, Can. J. Phys. 86 (2008) 383–389.
[14] S.O. Adesanya, O.D. Makinde, Thermodynamic analysis for a third grade fluid through a vertical channel with internal heat generation, J. Hydrodyn. 27 (2) (2015) 264–272.
[15] S.O. Adesanya, O.D. Makinde, Entropy generation in couple stress fluid flow through porous channel with fluid slippage, Int. J. Exergy 15 (3) (2014) 344–362.

[16] S.O. Adesanya, Oluwole D. Makinde, Effects of couple stresses on entropy generation rate in a porous channel with convective heating, Comput. Appl. Math. 34 (2015) 293–307.
[17] A. Bejan, Second law analysis in heat transfer and thermal design, Adv. Heat Trans. 15 (1982) 1–58.
[18] K. Hooman, F. Hooman, S.R. Mohebpour, Entropy generation for forced convection in a porous channel with isoflux or isothermal walls, Int. J. Exergy 51 (2008) 78–96.
[19] S. Chen, Z. Liu, J. Liu, J. Li, L. Wang, C. Zheng, Analysis of entropy generation in hydrogen enriched ultra-lean counter-flow methane–air non-premixed combustion, Int. J. Hydrogen Energy 35 (2010) 12491–12501.
[20] S. Mahmud, R.A. Fraser, Mixed convection-radiation interaction in a vertical porous channel: entropy generation, Energy 28 (2003) 1557–1577.
[21] S. Chen, R. Du, Entropy generation of turbulent double-diffusive natural convection in a rectangle cavity, Energy 36 (2011) 1721–1734.
[22] S. Chen, C. Zheng, Entropy generation in impinging flow confined by planar opposing jets, Int. J. Therm. Sci. 49 (2010) 2067–2075.