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ORIGINAL ARTICLE

Steady boundary layer flow and reactive mass transfer past an exponentially stretching surface in an exponentially moving free stream

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Abstract An analysis is made to study the steady two-dimensional boundary layer flow and reactive mass transfer past an exponentially stretching sheet in an exponentially moving free stream. The reaction rate of solute and the wall concentration distribution are taken variable. The governing equations are transformed and then solved numerically. The study reveals that the momentum boundary layer thickness is considerably smaller than that of stagnation point flow over stretching sheet. Due to increase of Schmidt number and reaction rate parameter the mass transfer considerably enhances. Importantly, for solute distribution, in addition to mass transfer, mass absorption occurs in certain situations.

1. Introduction

The flow due to stretching sheet is a vital problem in classical fluid mechanics due to its vast applications in many manufacturing processes in industry, such as, extraction of polymer sheet, wire drawing, paper production, glass-fiber production, and hot rolling. Crane [1] first investigated the steady boundary layer flow of an incompressible viscous fluid over a linearly stretching plate and gave an exact similarity solution in closed analytical form. Numerous studies [2–10] have been conducted later to extend the pioneering work of Crane [1]. On the other hand, Hiemenz [11] first studied the steady flow in the neighborhood of a stagnation-point. Chiam [12] considered a problem which is a combination of the works of Hiemenz [11] and Crane [1], i.e. the stagnation-point flow towards a stretching sheet taking identical stretching rate of the sheet and strain rate of the stagnation-point flow and he found no boundary layer structure near the sheet. Mahapatra and Gupta [13] reinvestigated the same stagnation-point flow towards a stretching sheet with different stretching and straining rates and found two kinds of boundary layer near the sheet depending on the ratio of the stretching and straining rates. In addition, some very important investigations in this direction can be found in the articles [14–22].

The diffusion of spices with chemical reaction in the boundary layer flow also has many applications in water and air pollutions, fibrous insulation, atmospheric flows and many other

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chemical engineering problems. Chambre and Young [23] considered the diffusion of a chemically reactive species in a laminar boundary layer flow. Later, Andersson et al. [24] investigated the effect of transfer of chemically reactive species in the laminar flow over a stretching sheet. Afify [25] explained the MHD free convective flow of viscous incompressible fluid and mass transfer over a stretching sheet with chemical reaction. Cortell [26] investigated the motion and mass transfer for two classes of viscoelastic fluid over a porous stretching sheet with chemically reactive species. Recently, Bhattacharyya and Layek [27,28] discussed the behavior of chemically reactive solute distribution in MHD boundary layer flow over a permeable stretching sheet and also described the slip effects on the boundary layer flow and mass transfer over a vertical stretching sheet.

Last few decades in almost all investigations on the flow over a stretching sheet, the flow occurs because of linear variation of stretching velocity of the flat sheet along the sheet. So, the boundary layer flow induced by an exponentially stretching sheet is not studied much though it is very important and realistic flow frequently appeared in many engineering processes. Magyari and Keller [29] first considered the boundary layer flow due to an exponentially stretching sheet and they also studied the heat transfer in the flow taking exponentially varied wall temperature. Elbashbeshy [30] numerically examined the flow and heat transfer over an exponentially stretching surface considering wall mass suction. Khan and Sanjayanand [31] investigated the flow of viscoelastic fluid and heat transfer over an exponentially stretching sheet with viscous dissipation effects. Furthermore, the behavior of the flow over an exponentially stretching sheet under different physical aspects was discussed by Partha et al. [32], Sanjayanand and Khan [33], Al-Odat et al. [34] and Sajid and Hayat [35]. Recently, Bhattacharyya [36] studied the boundary layer flow due to an exponentially shrinking sheet.

In the present paper, the steady boundary layer flow and mass transfer with first order chemical reaction over an exponentially stretching sheet in an exponential free stream are studied. Nature of this flow is comparable with the stagnation-point flow over a stretching sheet. Here, the reaction rate of the solute and the wall concentration distribution are also taken in exponentially varying form. The obtained self-similar equations are solved by shooting method. The numerical computations are presented through some figures and the various characteristics of flow and diffusion are discussed.

2. Analysis of problem

Consider the steady two-dimensional flow and mass transfer undergoing first order chemical reaction over an exponentially stretching sheet with velocity $U_w(x)$ in an exponential free stream with velocity U_∞. The sheet coincides with the plane $y = 0$ and the flow confined to $y > 0$. Using boundary layer approximation, equations for the flow and the concentration distribution are written in usual notation as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U_\infty\frac{dU_\infty}{dx}+\upsilon\frac{\partial^2 u}{\partial y^2} \tag{2}$$

and

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-R(C-C_\infty), \tag{3}$$

where u and v are the velocity components in x- and y-directions respectively, $\upsilon(=\mu/\rho)$ is the kinematic fluid viscosity, ρ is the fluid density, μ is the coefficient of fluid viscosity, C is the concentration, D is the diffusion coefficient and C_∞ is the concentration in the free stream. $R(x)$ is the variable reaction rate and is given by $R(x) = R_0\exp(x/L)$, L is the reference length and R_0 is a constant.

The boundary conditions are given by:

$$u = U_w(x), \quad v = 0 \quad \text{at } y = 0; \qquad u \to U_\infty(x) \text{ as } y \to \infty \tag{4}$$

and

$$C = C_w = C_\infty + C_0\exp\left(\frac{\lambda x}{2L}\right) \quad \text{at } y = 0;$$
$$C \to C_\infty \text{ as } y \to \infty, \tag{5}$$

where C_w is the variable concentration of the sheet, C_0 is a constant which measures the rate of concentration increase along the sheet and λ is a parameter which is physically very important in controlling the exponential increment of surface concentration and it may have both positive and negative values. The stretching velocity U_w and free stream velocity U_∞ are respectively given by:

$$U_w(x) = b\exp\left(\frac{x}{L}\right) \text{ and } U_\infty(x) = a\exp\left(\frac{x}{L}\right), \tag{6}$$

where b and a are constants with $b > 0$ and $a > 0$.

This flow is quite similar with the stagnation-point flow over a stretching sheet though here no stagnation point appears. Actually, this type of flow is more generalized.

The following similarity transformations [29,33] are introduced:

$$\psi = \sqrt{2\upsilon Lb}f(\eta)\exp\left(\frac{x}{2L}\right), \; C = C_\infty + (C_w - C_\infty)\phi(\eta) \quad \text{and } \eta = y\sqrt{\frac{b}{2\upsilon L}}\exp\left(\frac{x}{2L}\right), \tag{7}$$

where ψ is the stream function defined in the usual notation as $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$ and η is the similarity variable.

In view of (7), the Eq. (1) is identically satisfied and the Eqs. (2) and (3) reduce to the following self-similar equations:

$$f''' + ff'' - 2f'^2 + 2\varepsilon^2 = 0 \tag{8}$$

and

$$\phi'' + Sc(f\phi' - \lambda f'\phi - \beta\phi) = 0, \tag{9}$$

where $\varepsilon = a/b$ is the velocity ratio parameter and $Sc = \upsilon/D$ is the Schmidt number and $\beta = 2LR_0/b$ is the reaction rate parameter.

The boundary conditions (4) and (5) reduce to the following forms:

$$f(\eta) = 0, \; f'(\eta) = 1 \quad \text{at } \eta = 0; \qquad f'(\eta) \to \varepsilon \text{ as } \eta \to \infty \tag{10}$$

and

$$\phi(\eta) = 1 \quad \text{at } \eta = 0; \qquad \phi(\eta) \to 0 \text{ as } \eta \to \infty. \tag{11}$$

When $\varepsilon = 1$, i.e. $b = a$, the Eq. (8) with boundary condition (10) gives closed form analytical solution $f(\eta) = \eta$ which is similar to the stagnation-point flow over stretching sheet [14].

3. Numerical method for solution

The nonlinear coupled differential Eqs. (8) and (9) along with the boundary conditions (10) and (11) form a two point boundary value problem (BVP) and is solved using shooting method, by converting it into an initial value problem (IVP). In this method, it is necessary to choose a suitable finite value of $\eta \to \infty$, say η_∞. The following first-order system is set:

$$f' = p, \quad p' = q, \quad q' = 2p^2 - fq - 2\varepsilon^2 \tag{12}$$

and

$$\phi' = z, \quad z' = -Sc(fz - \lambda p\phi - \beta\phi) \tag{13}$$

with the boundary conditions

$$f(0) = 0, \quad p(0) = 1, \quad \phi(0) = 1. \tag{14}$$

To solve (12) and (13) with (14) as an IVP we must need the values for $q(0)$, i.e. $f''(0)$ and $z(0)$, i.e. $\phi'(0)$ but no such values are given. The initial guess values for $f''(0)$ and $\phi'(0)$ are chosen and the fourth order Runge–Kutta method is applied to obtain the solution. The calculated values of $f'(\eta)$ and $\phi(\eta)$ at $\eta_\infty(=30)$ are compared with the given boundary conditions $f'(\eta_\infty) = \varepsilon$ and $\phi(\eta_\infty) = 0$ and adjust values of $f''(0)$ and $\phi'(0)$ using Secant method to give better approximation for the solution. The step-size is taken as $\Delta\eta = 0.01$. The process is repeated until we get the results correct up to the desired accuracy of 10^{-6} level.

4. Results and discussion

In the boundary layer flow with exponentially moving free stream over an exponentially stretching sheet, two different kinds of boundary layer structures near the sheet have formed depending upon the ratio of the two constants relating to free stream and stretching velocities, i.e. on the velocity ratio parameter ε, for $\varepsilon > 1$ and $\varepsilon < 1$. Also for $\varepsilon = 1$, no boundary layer is formed near the sheet. The velocity profiles for various values of ε are depicted in Fig. 1 and corresponding concentration profiles are plotted in Fig. 2. The boundary layer thicknesses are physically very important. The viscous and solute boundary layer thicknesses are denoted by δ and δ_C, respectively and are described by the equations $\delta = \eta_\delta\sqrt{\frac{2\upsilon L}{b}}\exp\left(-\frac{x}{2L}\right)$ and $\delta_C = \eta_{\delta C}\sqrt{\frac{2\upsilon L}{b}}\exp\left(-\frac{x}{2L}\right)$. The dimensionless boundary layer thicknesses η_δ and $\eta_{\delta C}$ are respectively defined as the values of η (non-dimensional distance from the surface) at which the difference of dimensionless velocity $f'(\eta)$ and the parameter ε decays to 0.001 and the dimensionless concentration $\phi(\eta)$ reduces to 0.001. The values η_δ and $\eta_{\delta C}$ are given in Table 1 for several values of ε. From the table, it is seen that the viscous boundary layer thickness decreases when ε increases (both for $\varepsilon > 1$ and $\varepsilon < 1$). But, it is worth noting that the viscous boundary layer thickness is considerably thinner than that of the boundary layer of stagnation-point flow over a stretching sheet which is obtained by Mahapatra and Gupta [14]. So, in boundary layer flow over an exponentially stretching sheet with exponential free stream, though the flow dynamics is of similar pattern with that of the stagnation-point flow over a stretching sheet but the viscous boundary layer thickness is smaller and these are physically realistic. It is important to note that the solute boundary layer thickness also decreases with increasing ε.

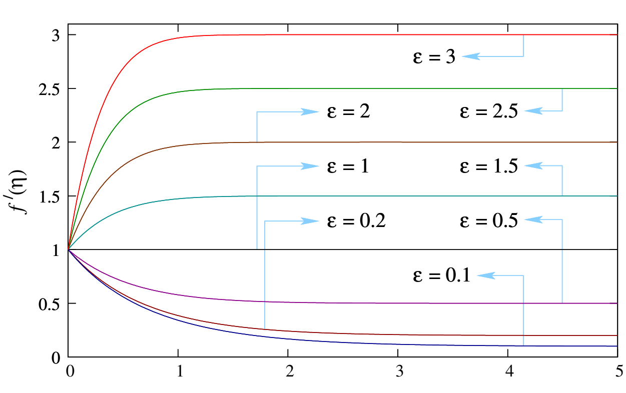

Figure 1 Velocity profiles $f'(\eta)$ for several values of ε.

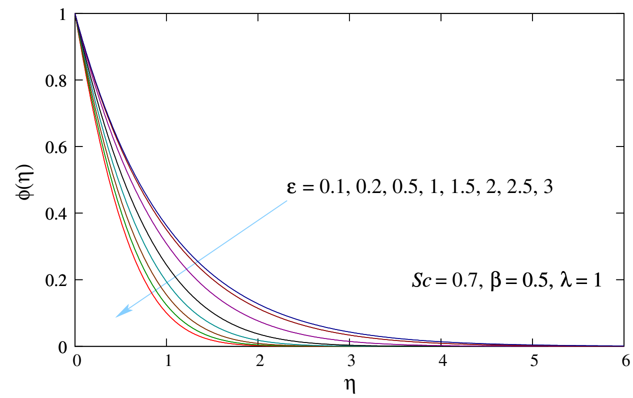

Figure 2 Concentration profiles $\phi(\eta)$ for several values of ε.

In Figs. 3 and 4, the concentration profiles for various values of Schmidt number Sc are demonstrated with $\varepsilon = 1.5$ and 0.1 respectively. It is found that the concentration at a point and the solute boundary layer thickness rapidly decreases with increase of Sc for both values of ε. This is caused due to the fact that the mass transfer from the surface to fluid strongly depends on Sc and the mass transfer increases as Sc increases. Actually, a increase of Schmidt number means a decrease in diffusion coefficient D and the mass transfer is inversely proportional to D.

The variations in concentration profiles for different values of reaction rate parameter β are exhibited in Figs. 5 and 6 for $\varepsilon = 1.5$ and 0.1 respectively. For both values of ε, concentration decreases with β. The solute boundary layer thickness slightly reduced for increase of β when $\varepsilon = 1.5$, whereas, for $\varepsilon = 0.1$ that thickness significantly decreases with β. The mass transfer is also affected by β. The mass transfer enhances due to β and for which the solute boundary layer thickness reduces.

The effect of the parameter λ on the reactive concentration distribution is very important in physical point of view. The dimensionless solute profiles $\phi(\eta)$ are presented in Figs. 7 and 8 for various λ with $\varepsilon = 1.5$ and $\varepsilon = 0.1$ respectively. In both the cases the overshoot of concentration for some negative values of λ are observed. When $\varepsilon = 1.5$, the concentration overshoot is started for smaller negative values of λ and on the other hand, the overshoot of concentration is observed for larger negative values of λ for $\varepsilon = 0.1$. In these flow situations

Table 1 Values of η_δ and $\eta_{\delta C}$ for several values of ε with $Sc = 0.7$, $\beta = 0.5$, and $\lambda = 1$.

	ε	0.1	0.2	0.5	1.5	2.0	2.5	3.0
Mahapatra and Gupta [14] (linear stagnation-point flow)	η_δ	6.96	5.91	4.36	–	2.62	–	2.30
Present study	η_δ	5.04	4.17	2.91	1.87	1.79	1.68	1.59
	$\eta_{\delta C}$	6.15	5.51	4.37	2.92	2.59	2.35	2.16

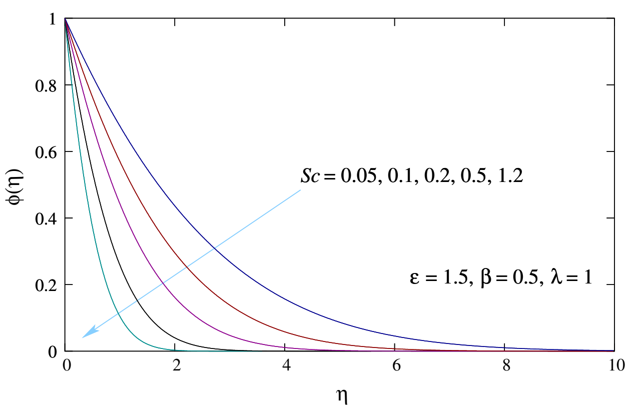

Figure 3 Concentration profiles $\phi(\eta)$ for several values of Sc with $\varepsilon = 1.5$.

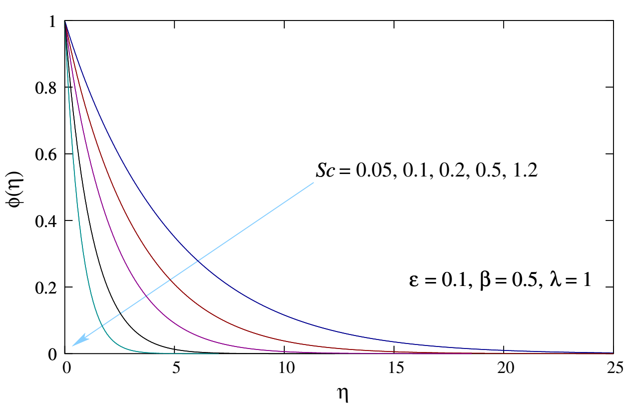

Figure 4 Concentration profiles $\phi(\eta)$ for several values of Sc with $\varepsilon = 0.1$.

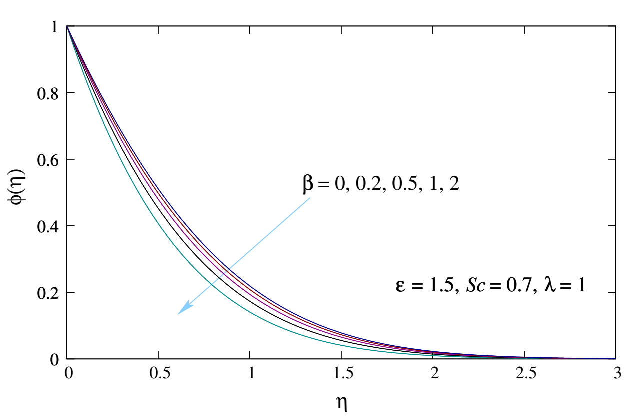

Figure 5 Concentration profiles $\phi(\eta)$ for several values of β with $\varepsilon = 1.5$.

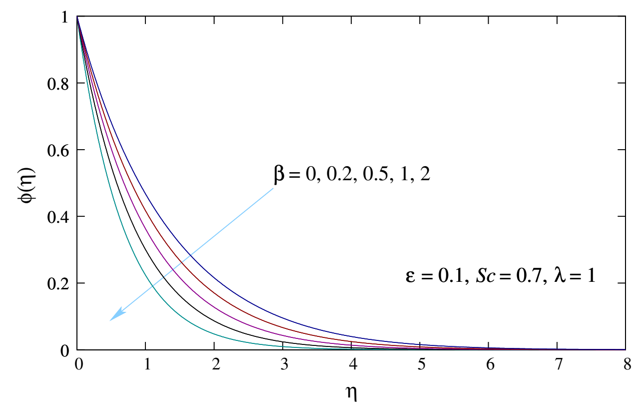

Figure 6 Concentration profiles $\phi(\eta)$ for several values of β with $\varepsilon = 0.1$.

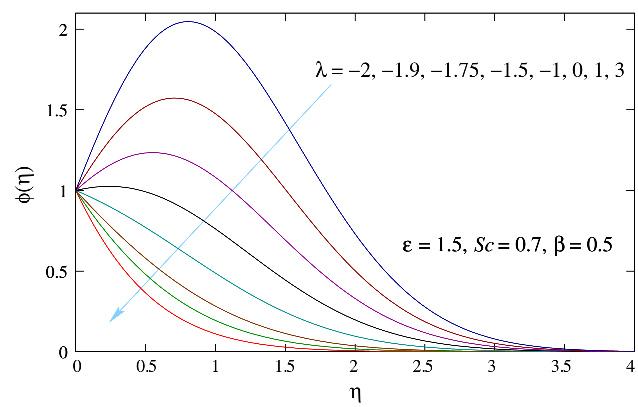

Figure 7 Concentration profiles $\phi(\eta)$ for several values of λ with $\varepsilon = 1.5$.

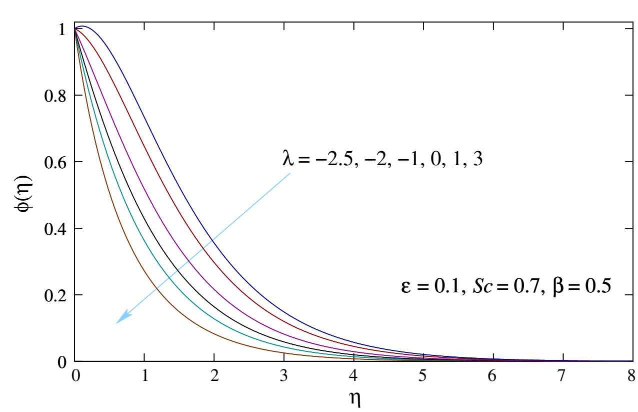

Figure 8 Concentration profiles $\phi(\eta)$ for several values of λ with $\varepsilon = 0.1$.

(concentration overshoot) mass absorption occur at the sheet. Also, it can be concluded that the larger velocity of stretching prevents the overshoot of concentration, i.e. the mass absorption. For $\lambda > 0$, no overshoot is observed for both values of ε. In addition, the solute boundary layer thickness decreases with λ.

5. Conclusions

The effects of various physical parameters on the flow and mass transfer characteristics in the boundary layer over an exponentially stretching sheet with an exponential free stream have been analyzed. The transformed nonlinear self-similar ordinary different equations are solved by shooting technique using Runge–Kutta method. The findings of the graphical analysis of the results of this investigation can be summarized as follows:

(i) The viscous and solute boundary layer thicknesses decrease with increase of velocity ratio parameter ε. Importantly the thickness of the viscous boundary layer in this type of flow is significantly thinner than the linear stagnation point flow over a linearly stretching sheet.
(ii) The mass transfer from the sheet enhances for the increase of velocity ratio parameter, Schmidt number and reaction rate parameter.
(iii) For larger negative value of λ, the mass transfers from the fluid to the surface, i.e. mass absorption occurs and it increases with ε.

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