



Hybrid natural inflation from non-Abelian discrete symmetry

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ABSTRACT

A spontaneously broken global discrete symmetry may have pseudo Goldstone modes associated with the spontaneous breaking of the approximate continuous symmetry of the low dimension terms in the Lagrangian. These provide natural candidates for an inflaton that can generate slow-roll inflation. We show that, in the case of a non-Abelian discrete symmetry, the pseudo Goldstone modes readily couple to further scalar fields in a manner that the end of inflation is determined by these additional scalar fields, generating hybrid inflation. We give a simple parameterisation of the inflationary potential in this case, determine the inflationary parameters resulting, and show that phenomenological successful inflation is possible while keeping the scale of symmetry breaking sub-Planckian. Unlike natural inflation the inflation scale can be very low. We construct two simple hybrid inflation models, one non-supersymmetric and one supersymmetric. In the latter case no parameters need be chosen anomalously small.

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1. Introduction

Field theory inflationary models typically suffer from the “ η ” problem which arises from the need to keep the inflaton very light, lighter than the mass scale set by the Hubble constant during inflation. To solve this problem, including the effects of radiative corrections, it is necessary to have an underlying symmetry which, when exact, forbids the inflaton mass. The mass arises through the (small) terms breaking the symmetry. Radiative corrections to the mass are then under control because, due to the symmetry, they must be proportional to the (small) terms breaking the symmetry. The only symmetries capable of forbidding a scalar mass are a Goldstone symmetry, associated with the spontaneous breaking of a continuous global symmetry, and a supersymmetry in which the scalar mass is related to fermion mass and the latter can be forbidden by a chiral symmetry.

A particularly attractive model based on a Goldstone symmetry is *Natural Inflation* [1]. The Goldstone boson, ϕ , arises due to the spontaneous breaking of a Global symmetry at a scale f . However the symmetry is broken at the quantum level through an anomaly and the would be Goldstone boson is a pseudo Goldstone boson acquiring a mass through the anomaly. The resulting inflationary potential for the inflaton ϕ is given by

$$V = V_0 \left(1 + \cos\left(\frac{\phi}{f}\right) \right). \quad (1)$$

The potential has only two parameters and describes the period at which the density perturbations are produced until the end of inflation at which the potential vanishes. The inflationary parameters resulting from this potential are consistent with observation and the model makes definite predictions for the tensor to scalar perturbations and the running of the spectral index [2].

Given the limited ways available to solve the η problem it is of interest to explore more general forms of the inflationary potential resulting from a pseudo Goldstone inflaton [3]. Here we construct a class of models which give rise to hybrid natural inflation in which a second field is responsible for ending inflation and setting the cosmological constant to zero. The form of the inflationary potential when the density perturbations are produced is given by

$$V = V_0 \left(1 + a \cos\left(\frac{\phi}{f}\right) \right), \quad (2)$$

where a is a constant. As we shall discuss the appearance of the parameter a , and the fact that the end of inflation is set by a second hybrid field, can significantly change the expectations for the inflationary parameters. Moreover it is possible to generate acceptable inflation for a lower symmetry breaking scale than is the case for the original natural inflation. This means one may have $f < M$, where M is the reduced Planck scale, $M = 2.44 \times 10^{18}$ GeV, so that higher dimension corrections to the inflationary potential are under control. In addition the scale of inflation is not determined and can be much lower than the upper bound set by the non-observation of tensor fluctuations.

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The models implementing hybrid natural inflation are based on non-Abelian discrete symmetries.¹ Such discrete symmetries can be equivalent to continuous symmetries for the low dimension terms in the Lagrangian, the approximate continuous symmetry being explicitly broken only by (small) higher dimension terms. If the symmetry is spontaneously broken there will be a light pseudo Goldstone boson that may be identified with the inflaton. The non-Abelian structure of the discrete symmetry is important in constructing the hybrid nature [5] of the model because it allows for a non-trivial coupling between the pseudo Goldstone inflaton field and the field responsible for ending inflation. We are aware of two earlier papers which have explored this idea [6,7] although they do not emphasise the general structure of Eq. (2). A more recent paper [8] explores the possibility that the fields responsible for breaking a non-Abelian discrete family symmetry should be inflaton(s) but does not identify the pseudo Goldstone modes as the candidate inflatons.

Here we present two simple models capable of giving the form of Eq. (2). The first is not supersymmetric and requires that the coefficient of the leading term breaking the approximate continuous symmetry should be anomalously small. It may be that such smallness follows if the non-Abelian discrete symmetry arises from the spontaneous breaking of an underlying continuous symmetry, but we have not been able to demonstrate this yet. The second example implements the non-Abelian discrete symmetry in a supersymmetric theory. The additional constraints of supersymmetry allow for the construction of a viable model with no parameter chosen to be anomalously small.

In Section 2 we determine the implications for the inflationary parameters that result from the hybrid natural inflationary form of Eq. (2). Sections 3 and 4 present the non-supersymmetric and supersymmetric hybrid natural inflation models respectively. Finally in Section 5 we present our conclusions.

2. Inflationary parameters from hybrid natural inflation

It is straightforward to determine the constraints on the parameters of the potential of Eq. (2) following from the requirement of acceptable inflation. This then leads to a determination of the detailed form of the inflation observables produced by this potential. The observables are given in terms of the usual slow-roll parameters [9]

$$\epsilon \equiv \frac{M^2}{2} \left(\frac{V'}{V} \right)^2 \simeq \frac{M^2}{2} \left(\frac{a}{f} \right)^2 \sin^2 \left(\frac{\phi}{f} \right), \quad (3)$$

$$\eta \equiv M^2 \frac{V''}{V} \simeq -M^2 \left(\frac{a}{f^2} \right) \cos \left(\frac{\phi}{f} \right), \quad (4)$$

$$\xi \equiv M^4 \frac{V'V'''}{V^2} \simeq - \left(\frac{M}{f} \right)^4 a^2 \sin^2 \left(\frac{\phi}{f} \right) = -2 \left(\frac{M}{f} \right)^2 \epsilon. \quad (5)$$

In terms of these the tensor to scalar ratio is

$$r = 16\epsilon. \quad (6)$$

The spectral index is given by

$$n_s = 1 + 2\eta - 6\epsilon, \quad (7)$$

and the ‘running’ of the spectral index characterised by $n_r = dn_s/d\ln k$ is

$$n_r = 16\epsilon\eta - 24\epsilon^2 - 2\xi. \quad (8)$$

The density perturbation at wave number k being

$$\delta_H^2(k) = \frac{1}{150\pi^2} \frac{V_H}{\epsilon_H M^4}. \quad (9)$$

The subscript H above signals the scale at which observable inflation starts, some ≈ 60 e-folds before the end of inflation when the scalar density perturbation on the scale of our present Hubble radius² was generated.

What are the predictions for the inflationary parameters following from the form of Eq. (2)?

2.1. Spectral index and tensor to scalar ratio

Let us start with the condition that the slow-roll parameters are small. Since the end of inflation is determined by the hybrid sector there is no reason to expect $\cos(\frac{\phi}{f})$ to be anomalously small. In this case Eq. (4) requires that $a(\frac{M}{f})^2 \ll 1$. If we further require $f < M$ so that the higher order gravitational corrections to the potential are small we need $a \ll 1$ and hence $\epsilon \ll \eta$ corresponding to a small tensor to scalar ratio. The observed small value for $1 - n_s$ for $\cos(\frac{\phi}{f}) \approx O(1)$ can be obtained simply by fixing a

$$a \approx \frac{1}{2} (1 - n_s) \left(\frac{f}{M} \right)^2. \quad (10)$$

From Eqs. (6) and (3) follows an upper bound for the tensor

$$r < 2(1 - n_s)^2 \left(\frac{f}{M} \right)^2. \quad (11)$$

For example, with $f = 0.1M$ and for $\cos(\frac{\phi}{f}) \approx O(1)$, we have $a \approx 2 \times 10^{-4}$ corresponding to $r < 5.6 \times 10^{-5}$, where we have used the value $n_s > 0.947$ [10]. Smaller values of f lead to much smaller values of a . The only way a significant value for r can be obtained is for $f > M$ corresponding to $a = O(1)$ as is the case for natural inflation. However this is in the region where one may expect significant higher order gravitational corrections to the potential. For $n_s > 0.947$ (the lowest bound on n_s given in [10]) and f taking its maximum value $f = M$ the absolute bound on r is 5.6×10^{-3} .

2.2. Spectral index running

The running of the spectral index is dominated by the third term in Eq. (8) and is given by

$$n_r \approx \frac{1}{4} \left(\frac{M}{f} \right)^2 r, \quad (12)$$

and from Eq. (11) it follows that

$$n_r < \frac{1}{2} (1 - n_s)^2. \quad (13)$$

The inequality above $n_r < 1.4 \times 10^{-3}$, is stronger than the current observational constraint $n_r < 0.019$ as given by [10,11].

2.3. Scale of inflation

The scale of inflation is set by the requirement that the density perturbations have the observed magnitude. From Eqs. (9) and (4) we have

$$V_H \approx V_0 \simeq M^4 A_H^2 \left(\frac{M}{f} \right)^2 a^2 \sin^2 \left(\frac{\phi_H}{f} \right) = \frac{1}{8} M^4 A_H^2 r, \quad (14)$$

² Numerically this is $H_0^{-1} \simeq 3000h^{-1}$ Mpc, where $h \equiv H_0/100$ km s⁻¹ Mpc⁻¹ ~ 0.7 is the Hubble parameter. The density perturbation can be measured down to ~ 1 Mpc, a spatial range corresponding to just 8 e-folds of inflation.

¹ Such symmetries often arise in string compactification [4].

Table 1

We show expectations for the spectral indices coming from WMAP5 and WMAP5 + BAO + SN measurements and compare them with similar results from *Natural Inflation* and *Hybrid Natural Inflation* models. To obtain the numerical bounds the lowest reported value of the scalar spectral index $n_s \sim 0.947$ was used. M is the reduced Planck mass $M = 2.44 \times 10^{18}$ GeV and N_e is the number of observable e-folds.

	WMAP5	WMAP5 + BAO + SN	Natural Inflation	Hybrid Natural Inflation
n_s	$0.963^{+0.014}_{-0.015}$	0.960 ± 0.013	$\approx \begin{cases} 1 - (\frac{M}{f})^2, & f \leq 4M \\ 1 - \frac{2}{N_e}, & f \geq 10M \end{cases}$	$\approx 1 - 2a(\frac{M}{f})^2$
r	< 0.43 (95% CL)	< 0.22 (95% CL)	$< \frac{8}{N_e} < 0.2$	$< 2(1 - n_s)^2 (\frac{f}{M})^2 < 5.6 \times 10^{-3}$
n_r	$-0.090 < n_r < 0.019$	$-0.068 < n_r < 0.012$	$> -\frac{8}{(1+2N_e)^2} > -10^{-3}$	$< \frac{1}{2}(1 - n_s)^2 < 1.4 \times 10^{-3}$
Δ	–	–	$\sim 5.5 \times 10^{15} (\frac{f}{M})^{1/2}$ GeV	$< 9 \times 10^{15} (\frac{f}{M})^{1/2}$ GeV

Table 2

The table shows the discrete transformations of the field components under the Z_2 and Z'_2 groups. The semi-direct product of Z_2 with Z'_2 results in the non-Abelian dihedral group D_4 and the breaking of this symmetry leads to the natural hybrid inflation models discussed below.

	Z_2	Z'_2
φ_1	φ_2	φ_1
φ_2	φ_1	$-\varphi_2$
χ_1	$-\chi_1$	χ_1
χ_2	$-\chi_2$	$-\chi_2$

where $A_H \equiv \sqrt{75}\pi\delta_H \approx \sqrt{75}\pi(1.91 \times 10^{-5})$. From Eq. (11) we get a bound for the inflationary scale

$$\Delta < \sqrt{\frac{A_H}{2}}(1 - n_s)^{1/2} \left(\frac{f}{M}\right)^{1/2} M. \quad (15)$$

For $n_s > 0.947$ we get $\Delta < 9 \times 10^{15} (\frac{f}{M})^{1/2}$ GeV. This can nearly saturate the bound $V_0 < (10^{16} \text{ GeV})^4$ [16] but typically is much less and can be very low. We shall illustrate this when we discuss two specific models of hybrid natural inflation.

2.4. Constraints from cosmological observations

Putting all this together, in Table 1 we compare the predictions of *Hybrid Natural Inflation* with those of *Natural Inflation* and with recent observational bounds coming from the five years data of the WMAP team [10,11] and those from BAO [12] and SN surveys [13–15].

3. A simple model of hybrid natural inflation

We first consider a simple (non-supersymmetric) model which has a non-Abelian discrete symmetry, the semi-direct product group D_4 with generators given by the semi-direct product group elements of $Z_2 \times Z'_2$. This will illustrate the mechanism leading to hybrid natural inflation but will require a parameter to be chosen very small. In the next section we will construct a supersymmetric version of the model which avoids the need to have such a small parameter.

D_4 has 5 irreducible representations 1^{++} , 1^{+-} , 1^{-+} , 1^{--} , and 2. The fields in the model consist of a doublet of real scalar fields $\varphi = (\varphi_1, \varphi_2)$ and two real singlet fields $\chi_{1,2}$ transforming as 1^{++} and 1^{--} respectively. Under the $Z_2 \times Z'_2$ group the fields φ_i and $\chi_{1,2}$ transform as shown in Table 2. The leading order independent terms in the scalar potential consistent with this symmetry are given by

$$V(\varphi, \chi) = c - m_\varphi^2 \varphi^2 + \lambda_1 (\varphi^2)^2 + \sum_{i=1}^2 m_{\chi_i}^2 \chi_i^2 + \lambda_2 \sum_{i=1}^2 \varphi_i^4 - \lambda_3 \varphi_1 \varphi_2 \chi_1 \chi_2 + O(\chi^4), \quad (16)$$

where c is a constant and $\varphi^2 = \varphi_1^2 + \varphi_2^2$. The first four terms are invariant under a larger $SO(2)$ ($U(1)$) symmetry, but this is broken to D_4 by the fifth and sixth terms. For clarity, we have not included the terms $\phi^2 \chi_{1,2}^2$ as these do not break the $U(1)$ symmetry and do not affect the nature of the inflation produced by the potential. After symmetry breaking φ^2 acquires a vacuum expectation value (v_{ev}) v^2 . Treating the fifth (and sixth) term as a perturbation it is convenient to use the following parameterisation

$$\varphi = (\rho + v) \begin{pmatrix} \cos(\frac{\phi}{v}) \\ \sin(\frac{\phi}{v}) \end{pmatrix}. \quad (17)$$

Here ϕ is the pseudo Goldstone boson associated with the spontaneous symmetry breaking of the approximate $U(1)$ symmetry. It will be identified with the inflaton. The field ρ is massive and does not play a role in the inflationary era. The potential for the ϕ and χ fields has the form

$$V(\phi, \chi) = c' + 64\lambda_2 f^4 \cos\left(\frac{\phi}{f}\right) + \sum_{i=1}^2 m_{\chi_i}^2 \chi_i^2 - 8\lambda_3 f^2 \sin\left(\frac{\phi}{2f}\right) \chi_1 \chi_2 + O(\chi^4), \quad (18)$$

where $v = 4f$ and $c' \equiv c - 16m_\varphi^2 f^2 + 256\lambda_1 f^4 + 192\lambda_2 f^4$ is a constant. One may see that,³ starting from $\phi \simeq 0$ with $8\lambda_3 f^2 \sin(\frac{\phi}{2f}) < m_\chi^2$, ϕ will “roll” until the term quadratic in the χ fields drives χ_i to acquire v_{ev} s. If the stabilising terms, quartic in χ , have $O(1)$ coefficients the minimum will be at $\chi_i \simeq m_\chi$ and so the change in potential energy following from the χ roll is of $O(m_\chi^4)$. In order to have zero (or very small) cosmological constant after inflation it is necessary to tune the parameters such that $V(\phi_{min}, \chi_{min}) \simeq 0$ (in the absence of a solution to the cosmological constant problem such fine tuning is an ingredient of all inflationary models). In this case such tuning requires the choice $c' = O(m_\chi^4)$. With this the form of the inflationary potential during inflation near the scale $\phi = \phi_H$ at which the density perturbations are produced is given by

$$V(\phi, 0) = V_0 \left(1 + a \cos\left(\frac{\phi}{f}\right)\right), \quad (19)$$

where $V_0 \equiv c' = O(m_\chi^4)$ and $a \equiv (\frac{64\lambda_2 f^4}{V_0}) = O(\frac{64\lambda_2 f^4}{m_\chi^4})$. The end of inflation occurs when $\phi = \phi_e$ where

$$8\lambda_3 f^2 \sin\left(\frac{\phi_e}{2f}\right) \simeq m_\chi^2. \quad (20)$$

The freedom in choosing the free parameters λ_2 , λ_3 and m_χ means one can adjust V_0 , ϕ_e and a at will for any choice of f . Given this

³ To simplify the presentation we take $m_{\chi_{1,2}} = m_\chi$. Note that the hybrid mechanism applies also to the case the masses are not equal.

it is reasonable to treat ϕ_e as a parameter in place of some combination of m_χ and λ_3 . Further, since ϕ_H is directly determined by ϕ_e and the number of e-folds of inflation, N , between ϕ_H and ϕ_e , it is even more convenient to treat ϕ_H as the independent parameter. In this case the hybrid natural inflation is completely parameterised by the potential Eq. (19) for $\phi \simeq \phi_H$, with N subsequent e-folds of inflation. This is the form of the hybrid natural inflationary potential that was analysed above. Note that the scale of inflation is given by $V_0 = O(m_\chi^4)$ and, since m_χ is a free parameter, it can be made arbitrarily small.⁴ This represents a significant difference compared to the case of natural inflation in which the scale is constrained to be high [2].

It remains to check whether acceptable inflation occurs for reasonable values of the parameters. It is easy to generate a sufficient number of e-folds between ϕ_H and ϕ_e , consistent with our assumption that $\phi \ll f$. This follows because $N \propto e^{Ht}$ while $\phi_e = \phi_H e^{\frac{m_\phi^2}{3H} t}$ and $m_\phi \ll H$ due to its pseudo Goldstone nature. Thus during the last 50–60 e-folds of inflation ϕ rolls only a small amount and so both ϕ_H/f and ϕ_e/f can be small. The condition that the χ field ends inflation, $8\lambda_3 f^2 \sin(\frac{\phi}{2f}) > m_\chi^2$, is readily satisfied even for the largest value of m_χ , $m_\chi \simeq 10^{16}$ GeV, consistent with the upper bound on V_0 . To see this note that using Eq. (10) and Eq. (14) together with $\phi_e \approx \phi_H$ Eq. (20) is satisfied provided

$$\lambda_3 \simeq \frac{A_H}{4} \left(\frac{1-n_s}{2} \right) \frac{M}{f}, \quad (21)$$

independent of m_χ . Finally, the scale of inflation is set by Eq. (14). Using this together with $V_0 \simeq m_\chi^4$, Eq. (20) and $\phi_H \simeq \phi_e$ one finds

$$\lambda_2 \simeq \frac{A_H^2}{64} \left(\frac{1-n_s}{2} \right)^3 \tan^2 \left(\frac{\phi_H}{f} \right). \quad (22)$$

From this, since inflation is generated for large values of $\cos(\frac{\phi_H}{f})$, we see that it is necessary to choose λ_2 to be anomalously small, $\lambda_2 \ll 10^{-14}$, certainly a defect of this model. From Eq. (22) we can give an estimate for the inflaton mass $m_\phi^2 \simeq 64\lambda_2 f^2 \simeq A_H^2 (\frac{1-n_s}{2})^3 f^2 \sim 2 \times 10^{-12} f^2$. From Eq. (21) we see that for $\lambda_3 \leq 1$, the scale of symmetry breaking f should be $f \geq 3 \times 10^{-6} M$. Thus the inflationary scale Eq. (14) satisfies $V_0 \geq (6 \times 10^{-6} M \tan^{1/2}(\frac{\phi_H}{f}))^4 \simeq (10^{13} \text{ GeV} \tan^{1/2}(\frac{\phi_H}{f}))^4$. However since $\phi_H \ll f$ there is no lower bound for the inflationary scale and it can be much lower than 10^{13} GeV. To eliminate the need for the λ_2 fine tuning we turn now to the construction of a supersymmetric model.

4. A supersymmetric version of hybrid natural inflation

It is straightforward to build an $N = 1$ supersymmetric generalisation of the model. This model has the advantage that it has no anomalously small parameters due to the additional symmetry forcing the higher order terms, which break the approximate continuous symmetry, to be small. In addition the mass of the hybrid field is protected against radiative corrections larger than the Hubble scale during inflation and can be chosen to have any value above this scale. The field content consists of chiral supermultiplets of the $N = 1$ supersymmetry which transform non-trivially under the D_4 non-Abelian discrete symmetry. The supermultiplets consist of a D_4 doublet $\varphi = \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix}$ and three singlet representations

$\chi_{1,2}$ and A transforming as 1^{-+} , 1^{--} and 1^{++} respectively. The interactions are determined by the superpotential given by

$$P = A \left(\Delta^2 - \lambda_3 \frac{1}{M^2} \varphi_1 \varphi_2 \chi_1 \chi_2 \right), \quad (23)$$

where Δ is a constant with units of mass. These are the leading order terms if we have an additional R -symmetry under which the fields A, φ, χ have charges 2, -2 and 2 respectively. It is this R -symmetry that allows us to construct a model with no anomalously small couplings. Including the effect of SUSY breaking terms the potential has the form

$$V(\varphi, \chi) = \left| \Delta^2 - \lambda_3 \frac{1}{M^2} \varphi_1 \varphi_2 \chi_1 \chi_2 \right|^2 + m_\varphi^2 |\varphi|^2 + \sum_{i=1}^2 m_{\chi_i}^2 |\chi_i|^2 + \lambda_2 \frac{m^2}{M^2} \sum_{i=1}^2 |\varphi_i|^4 + O\left(\frac{m^2}{M^2} |\chi|^4\right), \quad (24)$$

where we have used the same symbol for the scalar components of the chiral supermultiplets as for the supermultiplets themselves. Here $m_\varphi^2, m_{\chi_i}^2$ and m^2 are supersymmetry breaking masses which, during inflation, are of the order of the Hubble parameter, $H \simeq \frac{\Delta^2}{M}$. The last term is a D -term which only arise when supersymmetry is broken and hence is proportional to the SUSY breaking scale, m^2 . It can come from radiative corrections with a messenger field of mass M_M in the loop.

Including radiative corrections m_φ^2 should be interpreted as a running mass squared, $m_\varphi^2 = m_\varphi^2(\varphi)$ and these radiative corrections may readily drive it negative at some scale Λ below the Planck scale. In this case φ will develop a vev of order Λ . As before we introduce a parameterisation for φ which is appropriate if the dominant terms are the second and third terms of the potential that are invariant under a larger continuous $SU(2)$ symmetry acting on the complex doublet of fields φ . We treat the term proportional to λ_2 which breaks $SU(2)$ to D_4 as a perturbation. We write φ in the form

$$\varphi = e^{i\phi\sigma} \begin{pmatrix} 0 \\ \rho + v \end{pmatrix} = \frac{\rho + v}{\phi} \begin{pmatrix} (\phi_2 + i\phi_1) \sin(\frac{\phi}{v}) \\ \phi \cos(\frac{\phi}{v}) - i\phi_3 \sin(\frac{\phi}{v}) \end{pmatrix}, \quad (25)$$

where σ_i are the Pauli spin matrices, ϕ_i are the pseudo Goldstone fields left massless by the second and third terms of the potential and acquiring mass from the fifth term⁵ and finally $\phi^2 = \phi_1^2 + \phi_2^2 + \phi_3^2$. The field ρ acquires a mass of $O(m_\varphi)$ and plays no role in the inflationary era. The potential for the pseudo Goldstone fields ϕ_i alone is given by

$$V(\phi) = \frac{m^2}{M_M^2} v^4 \lambda_2 \left[\left[\left(\frac{\phi_1}{\phi} \right)^2 + \left(\frac{\phi_2}{\phi} \right)^2 \right]^2 \sin^4 \left(\frac{\phi}{v} \right) + \left[\cos^2 \left(\frac{\phi}{v} \right) + \left(\frac{\phi_3}{\phi} \right)^2 \sin^2 \left(\frac{\phi}{v} \right) \right]^2 \right]. \quad (26)$$

We shall demonstrate that inflation occurs for small $\frac{\phi_i}{v}$. In this region, for positive λ_2 , the field ϕ_3 has a positive mass squared while $\phi_{1,2}$ have negative mass squared. Thus ϕ_3 does not develop a vev. The full potential for the fields acquiring vevs then has the form

⁴ However radiative corrections will typically drive m_χ to be close to the Planck scale—the usual “hierarchy problem”. In the next section we construct a supersymmetric model that avoids this problem.

⁵ For clarity, we have not included the terms $(m^2/M_M^2)|\phi|^2|\chi_{1,2}|^2$ as these do not break the $SU(2)$ symmetry and do not affect the nature of the inflation produced by the potential.

$$\begin{aligned}
V(\phi, \chi) = & \Delta'^4 + 64\lambda_2 \frac{m^2}{M_M^2} f^4 \cos\left(\frac{\phi}{f}\right) + \sum_{i=1}^2 m_{\chi_i}^2 |\chi_i|^2 \\
& - 8\lambda_3 e^{i\alpha} \Delta^2 \frac{f^2}{M^2} \sin\left(\frac{\phi}{2f}\right) \chi_1 \chi_2 + \text{h.c.} \\
& + O\left(\frac{m^2}{M^2} \chi^4\right), \tag{27}
\end{aligned}$$

where $\phi^2 = \phi_1^2 + \phi_2^2$, $\Delta'^4 = \Delta^4 + 16m_\phi^2 f^2 + 192\lambda_2 \frac{m^2}{M_M^2} f^4$ and $\alpha = \tan^{-1}(\frac{\phi_1}{\phi_2})$. Note that since the soft SUSY breaking masses are of $O(\Delta'^2/M)$ the last 3 terms are small for $f < M$ and $\Delta' \simeq \Delta$.

The condition that χ_i should end inflation is that in Eq. (27) the fourth term dominates over the third, i.e.

$$8\lambda_3 \Delta^2 \frac{f^2}{M^2} \sin\left(\frac{\phi}{2f}\right) > m_\chi^2. \tag{28}$$

Provided $\frac{\phi}{f}$ is initially small the χ_i fields will initially have zero vevs but once Eq. (28) is satisfied they will roll to their final minimum to end inflation. As may be seen from Eq. (24) the final minimum sets the first term to zero leaving an $O(m^2 f^2)$ contribution (small relative to Δ^4) that must be cancelled by an additional constant in order to set the cosmological constant to zero. For $f < M$ this term is small and the potential during inflation is dominated by the Δ^4 term.

Thus the first two terms are of the form of the potential given in Eq. (2) with $V_0 \simeq \Delta^4$ and

$$a = 64\lambda_2 \frac{m^2}{M_M^2} \frac{f^4}{\Delta'^4} \simeq 64\lambda_2 \left(\frac{f}{M}\right)^4 \left(\frac{M}{M_M}\right)^2. \tag{29}$$

We can now determine the parameter range that leads to acceptable inflation. Note that, compared to the non-supersymmetric case, the scale of the potential during inflation is still proportional to a^{-1} but the coefficient is suppressed by the factor m^2/M_M^2 . As a result the bound of Eq. (22) is weakened by the same factor. As we will discuss this makes it easy to get satisfactory inflation without fine tuning.

As before during the last 40–50 e-folds of inflation ϕ rolls only a small amount and so both ϕ_H/f and ϕ_e/f can be small. The spectral index is determined by the value of a which, cf. Eq. (29), can be adjusted for arbitrary f by choosing the appropriate mediator mass scale M_M . From Eqs. (14), (29) and the relation between a and n_s we find

$$\begin{aligned}
\lambda_2 = & \frac{1}{64} \left(\frac{1-n_s}{2}\right) \left(\frac{M_M}{f}\right)^2, \\
\lambda_3 = & \frac{A_H}{8} \left(\frac{1-n_s}{2}\right) \left(\frac{m_\chi}{m}\right)^2 \left(\frac{M}{f}\right). \tag{30}
\end{aligned}$$

From this one may see that $\lambda_{2,3}$ need not be chosen unnaturally small provided the mediator scale is significantly below the Planck scale. Given the smallness of the first two factors in Eq. (30) a small f is favoured, $f < 10^{-4}$, so that higher order supergravity corrections are under control. The mass scale M_M needs to be chosen one or two orders of magnitude larger, quite reasonable given the interpretation of M_M as a mediator mass. From Eq. (30) we can give an estimate for the inflaton mass $m_\phi^2 \simeq 64\lambda_2 f^2 (\frac{m}{M_M})^2 \simeq (\frac{1-n_s}{2}) m^2 \sim 2 \times 10^{-2} m^2$. The scale of inflation is set by Δ and there is no constraint on it coming from Eq. (30). As a result this model of hybrid natural inflation provides another example of inflationary models capable of generating very low scales of inflation [17]. However in this case there is no constraint on

the spectral index compared to the models of [17] which required $n_s < 0.95$. An effect of lowering the inflationary scale is a decrease of the reheat temperature due to the decays of the inflaton field ϕ . The observed baryon asymmetry of the universe may in principle be generated after reheating through anomalous electroweak ($B - L$)-violating processes and/or the Affleck–Dine mechanism, even for reheating temperatures as low as ~ 1 GeV [18]. In this Letter we have concentrated on the inflationary era. However viable models must also have an acceptable reheat phase capable of producing the observed baryon excess. Although a detailed discussion of this is beyond the scope of this Letter it is important to point out that it is the decay of the hybrid field and not the inflaton that is responsible for reheating. This avoids the difficulties associated with the small coupling of a pseudo Goldstone boson and the reheating due to the hybrid field can be very efficient [19]. Furthermore, even if the scale of inflation is very low there are many suggested mechanisms capable of acceptable baryogenesis [20].

5. Summary and conclusions

We have demonstrated a generalisation of *Natural Inflation* which preserves the attractive pseudo Goldstone origin of the inflaton capable of solving the η problem while implementing a hybrid end to inflation through the rapid roll of a second scalar field. The coupling of the hybrid field to the inflaton is possible in models which have a spontaneously broken non-Abelian discrete symmetry and the resulting form of the inflationary potential at the time the density perturbations are produced has a very simple general form. We have analysed the inflationary parameters resulting from hybrid natural inflation and shown that an acceptable value for the spectral index is easy to achieve for a range of inflationary scales from the upper Lyth bound down to arbitrarily low scales. Over most of the allowed range of parameters the tensor to scalar ratio and the running of the spectral index are unobservably small. Moreover there is no need for the symmetry breaking scale to be super Planckian so it is easy to avoid large gravitational corrections.

Using a simple D_4 non-Abelian discrete symmetry we constructed two very simple hybrid natural inflationary models, one non-supersymmetric and one supersymmetric. In the non-supersymmetric case it was necessary to choose the coefficient of the leading term breaking the approximate continuous symmetry to be anomalously small. The second example implements the non-Abelian discrete symmetry in a supersymmetric theory. The additional constraints of supersymmetry allow for the construction of a viable model with no parameter chosen to be anomalously small and the scale of inflation to be arbitrary.

The D_4 based examples provide simple realisations of hybrid natural inflation but there are many other possible realisations following from alternative non-Abelian discrete symmetries. That such symmetries may occur in nature is perhaps made more plausible by the fact that they readily occur in compactified string theories. Irrespective of the details of the discrete group we expect the form of the inflationary potential at the time the density perturbations are produced to have the universal form presented above and so the expectations for the inflationary parameters will have the structure discussed.

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