Out-of-equilibrium behavior of coarsening ferromagnets

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Abstract
The out-of-equilibrium properties of ferromagnets quenched below the critical point are discussed in the dynamical scaling limit. The autoresponse and the autocorrelation are studied for both perfect and disordered ferromagnets. The scaling functions of the autoresponse are found to be in agreement with the predictions coming from the theory of local scale-invariance. Large finite time corrections to the scaling behavior of the autocorrelation are revealed for the disordered magnets.

Keywords: aging, out-of-equilibrium, disordered magnets

1. Introduction
Ferromagnets undergoing phase ordering when quenched inside the ordered phase are studied since many years, see [1] for a review. Recently, a series of studies has addressed the out-of-equilibrium properties of these systems in the dynamical scaling or aging regime. This renewed interest was triggered by the observation that the ferromagnets share many properties with complex systems displaying slow dynamics (glasses, spin glasses), even though they are conceptually much simpler. Therefore, new insights into the non-equilibrium properties of these complex systems can be expected from a thorough study of the out-of-equilibrium properties of magnetic systems.

In this contribution I summarize the recent investigation of two-time quantities (like the autoresponse and the autocorrelation functions) in various ferromagnetic systems with phase-ordering kinetics [2, 3, 4, 5, 6]. I thereby discuss both perfect and disordered systems.

2. The model
The model under study is the two-dimensional Ising model with only nearest neighbor ferromagnetic interactions, defined by the Hamiltonian

$$\mathcal{H} = -\sum_{\langle i,j \rangle} J_{ij} S_i S_j$$

(1)

where the spin variable $S_i$ can take on the values $\pm 1$. Note that we allow for bond randomness as long as the couplings remain ferromagnetic, i.e. $J_{ij} \geq 0$. For $J_{ij} = J > 0$ we recover the perfect ferromagnetic Ising model. In addition to the perfect model I shall also discuss the random-bond case where the couplings are drawn with equal probability from the square distribution $[J(1-\varepsilon/2), J(1+\varepsilon/2)]$. The width of the distribution is thereby controlled by the parameter $\varepsilon$ with $0 \leq \varepsilon \leq 2$. In all cases we impose on the system non-conserved dynamics through the heat-bath rule. We prepare the system in a fully disordered initial state (corresponding to infinite temperature) and bring it in contact at
time \( t = 0 \) with a heat bath at a temperature \( T \) smaller than the critical temperature. After the quench coarsening sets in, which leads to the formation and growth of ordered domains. The typical length \( L(t) \) of the domains increases in time following a simple power-law, i.e. \( L(t) \sim t^{1/z} \). The exponent \( z \) is called the dynamical exponent. Whereas it takes on the value 2 for the perfect Ising model, much larger values of \( z \) can be achieved in the random-bond model. Indeed it has been proposed [7] (and this agrees with our data [5, 6]) that in the disordered model the dynamical exponent is given by

\[
z = 2 + \varepsilon/T. \tag{2}
\]

For this reason the random-bond Ising model is a perfect system for verifying the predictions coming from a recent theoretical approach, the theory of local scale-invariance [2, 8], which exploits the presence of special space-time symmetries in non-equilibrium systems. In the past, checks of this theory have mainly been restricted to values of \( z \) close to 2.

3. Two-time quantities

Non-equilibrium properties can for example be studied through two-time quantities. Examples include the autoreponse and autocorrelation functions. Consider the time dependent order parameter (magnetization) \( \phi(t) \). The autoreponse measures the reaction of the system to an external field \( h(t) \) that has been switched on at time \( s \). It is given by the expression

\[
R(t, s) = \frac{\delta(\phi(t))}{\delta h(s)} \quad (t > s) \tag{3}
\]

where the average is an average over the noise. The times \( s \) and \( t \) are usually called waiting time and observation time, respectively. The autocorrelation, on its side, measures the correlation between configurations at times \( t \) and \( s \) and is defined by

\[
C(t, s) = \langle \phi(t)\phi(s) \rangle. \tag{4}
\]

For systems undergoing phase ordering, both quantities should display in the aging or dynamical scaling regime (where \( t \), \( s \), and \( t - s \) are large compared to any microscopic time scale) the following simple scaling behavior:

\[
R(t, s) = s^{-a}f_R(t/s), \quad C(t, s) = f_C(t/s) \tag{5}
\]

where the non-equilibrium exponent takes on the value \( a = 1/z \) for systems undergoing phase ordering [10]. \( f_R(y) \) and \( f_C(y) \) are scaling functions that display a simple power-law behavior for large arguments:

\[
f_R(y) \sim y^{+\lambda_R/z}, \quad f_C \sim y^{-\lambda_C/z} \tag{6}
\]

with the autoreponse and autocorrelation exponents \( \lambda_R \) and \( \lambda_C \).

The recently developed theory of local scale-invariance permits to derive explicit expressions for scaling functions of two-time quantities. For the autoreponse function, we obtain [2, 8]

\[
f_R(y) = f_0 y^{1+a-\lambda_R/z}(y - 1)^{-1-a}. \tag{7}
\]

Interestingly, only the values of the non-equilibrium exponents \( a \) and \( \lambda_R/z \) enter into this expression.

In figure 1 I show numerically determined scaling functions for both the perfect and the disordered Ising models. These data have been produced using the standard thermoremanent magnetization protocol familiar to many experimentalists. In this protocol the system is quenched to low temperatures in presence of the magnetic field with strength \( h_0 \). This field is cut after the waiting time \( s \), and the decay of the magnetization is then monitored as a function of time. In fact, the thermoremanent magnetization \( M(T) \) obtained in this way is an integrated response function, related to \( R(t, s) \) by

\[
M(t, s) = h_0 \int_0^s R(t, u) \, du. \tag{8}
\]
As argued in [9, 10], the scaling form (5) for \(R\) can not be used close to the lower integration bound, thus giving rise to an additional correction-to-scaling term. The more complete scaling form of the thermoremanent magnetization then reads [10]

\[
M(t, s) = s^{-\alpha} f_M(t/s) + s^{-\lambda_R/z} g_M(t/s).
\]  

(9)

The numerical scaling functions shown in figure 1 result from subtracting off the correction term from the data obtained in the simulations [3, 5]. Clearly, the data obtained for different waiting times display a nice data collapse.

![Figure 1: Comparison of the numerically determined scaling functions \(f_M(t/s)\) of the two-dimensional Ising model with the prediction coming from the theory of local scale-invariance (full lines) for (a) \(\varepsilon = 0\) (perfect model) and \(T = 1.5\), (b) \(\varepsilon = 1\) and \(T = 1\) and (c) \(\varepsilon = 2\) and \(T = 0.6\).]

Also included in figure 1 is the theoretical prediction coming from the theory of local scale-invariance where the values of \(z\) (see equation (2)) and of \(\lambda_R/z\) (obtained from the power-law decay at long times) have been plugged in. For all the studied cases a perfect agreement between theory and numerics is obtained. This leads us to conclude that the space-time symmetries underlying the theory are indeed realized in systems undergoing phase ordering. It is worth noting that the same conclusions have been reached in a study of the phase ordering in the \(q\)-state Potts model [11].

In figure 2 I briefly discuss the autocorrelation function for the random-bond model with \(\varepsilon = 2\) and \(T = 1\) (in units of \(J/k_B\)) [6]. At first sight a plot of \(C\) versus \(t/s\) (figure 2a) seems hardly compatible with the simple scaling form \(C(t, s) = f_C(t/s)\), expected for systems undergoing phase ordering, as no clear data collapse is observed. The same observation in the closely related random-site Ising model was recently discussed within an unconventional superaging scenario [12]. However, the data obtained in our study can be consistently described by assuming the existence of a finite-time correction to the asymptotic scaling behavior. Assuming the extended scaling form

\[
C(t, s) = f_C(t/s) - s^{-\nu} g_C(t/s),
\]  

(10)

the correction term can be determined from the numerical data, see figure 2b. After subtracting off this term, a perfect standard scaling behavior is recovered for the random-bond Ising model, as shown in figure 2c, similar to what is observed in the perfect model.

4. Conclusion

In this contribution I have briefly summarized some of the results obtained for perfect and random ferromagnets quenched below their critical point. For the autoresponse function a perfect agreement is found between the numerical data and the theory of local scale-invariance, and this even for very large values of the dynamical exponent \(z\). This greatly extends the range of applicability of the theory and shows that the space-time symmetries underlying the theory are indeed realized in systems undergoing phase ordering. I have also discussed for the disordered model the autocorrelation that displays strong finite-time corrections. Simple scaling is recovered after subtracting off this correction term. It is therefore not needed to invoke an unusual superaging scenario, as done recently for the random-site Ising model, in order to describe the autocorrelation in disordered ferromagnets undergoing coarsening.

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Figure 2: (a) Autocorrelation $C(t, s)$ vs $t/s$ for various waiting times $s$ in the random-bond Ising model with $\varepsilon = 2$ and $T = 1$. (b) Plot of $C(y_s, s)$ for the same parameters and $y = 5, 10, 15, 20$ (from top to bottom). The data can be perfectly fitted by the scaling form (10) with $b' = 0.075$, which illustrates the existence of important finite-time corrections. (c) The scaling function $f_C(t/s)$ obtained after subtracting off the finite-time corrections from the autocorrelation, again for $\varepsilon = 2$ and $T = 1$. The resulting scaling function is in agreement with simple aging of the autocorrelation.