3rd World Conference on Learning, Teaching and Educational Leadership – WCLTA 2012

Enhancement of 3D imagination in the 1st and 2nd grade

Jaroslava Kloboučková*, Darina Jirotková, Jana Slezáková

Faculty of Education, Charles University in Prague, M.D.Rettigové 4, 116 39 Praha 1, Czech Republic

Abstract

International comparison studies (TIMSS and PISA) and teachers’ expressed views indicate a decreasing trend in Czech pupils’ competence in geometry. The present study describes an innovative educational strategy for teaching 3D geometry - the didactical environment Cube Buildings and how it fosters the development of spatial sense in the earliest school age group. The results of one experiment in this environment are included as a partial result of a longitudinal action research project of one of the authors. The findings illustrate how a pupil can build problem-solving strategies and how knowledge is spread among pupils.

Keywords: Schema-oriented education; didactical environment; cube building; cube solid; geometrical language.

1. Introduction

Both international comparison studies (TIMSS and PISA) and teachers’ expressed views indicate a decreasing trend in Czech pupils’ competence in geometry. Secondary school pupils’ conceptions of geometrical objects and relationships are not well-established. Naturally, this situation is rooted already in their primary school experience. Although curricular documents such as the national curricular guidelines talk about spatial imagination development and understanding both 2D and 3D geometrical structures in primary school, the majority of Czech textbook series tasks and problems to practice application of these.

Hejný at al. (2006-2010) have elaborated a series of textbooks for primary school pupils, based on the building of mental schemata through work in a variety of didactical environments (Wittmann, 2001, Hejný, 2011a) and on the application of the theory of generic models (Hejný, 2012). Such conception allows pupils to construct mathematical knowledge while they are engaged in suitable activities. The geometrical content builds on pupils’ experience and is tightly linked to other areas of mathematics.

It is obvious that neither curricular documents nor textbooks can ensure effective teaching practice. The educational process comprises of many factors and different aspects of the notorious didactical triangle have been given different importance throughout history. The triangle expresses relationships between the pupil and his learning and his development and the teacher, his professional tools, and the content. Janík (2009) says:

There are numerous didactic transformations, transpositions or reconstructions taking place between the vertices of the didactic triangle. These are the subject of specific subject didactics.... It has become apparent that teacher’s didactic content knowledge is a prerequisite for a well mastered didactical transformation. (p. 8)

* Corresponding Author: Jaroslava Kloboučková. Tel.: ++420-221-900-226
E-mail address: jaroslava.klobouckova@pedf.cuni.cz
Lately, the notion of a teacher’s pedagogical beliefs and their role in the individual teaching style of a teacher has become a frequent subject in mathematics education research. A teacher’s belief develops and grows more mature the longer he/she teaches. It is formed by his/her education, lifestyle, demands of the society; it reflects the opportunities of life-long education and many other circumstances (Hejný, Kuřina, 2009).

Most teachers don’t doubt the importance of teaching arithmetic or algebra. At the same time, they seem to view geometry in a less straightforward way. Jirotková, (2010) describes the results of a survey among teachers: Geometry as a school subject is significantly less favored than arithmetic. Most teachers perceives geometry as work on construction tasks on one hand, and a pupil’s precision in such work is the main indicator of his or her geometrical competence. On the other hand, geometry is perceived as a set of formulas to calculate volumes and areas of geometric shapes. (Jirotková, 2010, p. 28).

We understand geometry as an area of mathematics that provides unique opportunities to cultivate thinking and creativity. While in 2D this may mean especially the forming of understanding and perceptions of shapes, pattern discovery, forming of hypotheses, and argumentation, 3D geometry cultivates spatial sense and the ability to use several languages in different situations, i.e. representations of 3D objects.

In the following section we will describe the Cube building didactical environment as didactically developed in the above mentioned textbooks. This environment formed the backbone of a few selected experiments. These experiments were conducted within a longitudinal action research project of the first author. Her teaching style is conducted in the framework of schema-oriented education, which places strong emphasis on the premise that the teacher facilitates the pupils’ construction of knowledge rather than explaining things to them. The teacher is also genuinely partaking in the positive emotions originating in discovery and views a pupil’s error as a gateway to further learning.

2. Learning environment Cube Buildings

Most pupils as young as preschool age have some experience with cube buildings. This intuitive knowledge and real-life experience need to be gradually turned into knowledge in school. We can attain this goal by linking three activities: solving problems involving manipulation, verbal description of the activity, and the introduction of symbolic language that will simply and clearly describe more complex buildings. The introduction of various symbolic languages enables children to create a large amount of problems graded by their level of difficulty. This environment is didactically effective also in the second grade.

2.1. Description of the geometric environment Cube Building

By a cube building we understand a 3D object built, following a certain set of rules, with a final number of identical cubes. The rules are simple: we start placing a cube on the “floor”, then we add a second cube so that the cubes now share one face, and we continue adding more cubes to one or more of the cubes that are already part of the building, until we use up all the provided cubes.

2.2. The languages of the Cube Building environment.

We will elaborate here on five ways we can represent a cube building. These five languages are incorporated in the above introduced textbooks and we are working with them at the primary school level. We wish to stress that these are introduced separately and at a pace that allows pupils to feel the need and usefulness of using a new language in describing a solution or creating new problems.

1) Physical model. A real building constructed from blocks – cube models.

2) Portrait. Either hand-drawn, computer-drawn, or a photo of the physical model. Figure 1. shows two of such buildings - A and B. Each consists of 4 cubes.
3) Dotted plan. We inscribe dots into the squares of the buildings’ floor projection: the number of dots is equal to the height (in cubes) of the respective part of the building (Fig. 1). The dots will be later replaced by numbers.

![Figure 1. Cube buildings and their dotted plans](image)

4) Triple projection. A cube building B is represented in three orthogonal projections (Fig. 2). Looking at the building from above, we see a triomino $B_p$ – the floor plan of building B; looking from the front, we can see triomino $B_n$ – its frontal projection; looking from a side, we can see triomino $B_b$ – the lateral projection of building.

![Figure 2. Cube building B and three orthogonal projections](image)

These four languages capture the finished building, i.e. the concept. Each of them has a certain advantages and disadvantages. For example, a portrait is visually string but technically difficult. A blueprint is technically easy but does not give the same visual detail. A triple projection depicts the building best but is the most difficult one to work with. If we want to work on the process of building, these languages will become cumbersome. Therefore, a new language was added to capture the processual part of the building. It helps pupils to understand processually the concept of a cube building. According to Gray and Tall (1994) the pupil reaches a more quality understanding of the concept of a cube building, i.e. a procept is created in his or her mind.

5) Construction account. We give a detailed step-by-step account of the building’s construction. Construction account uses a total of six iconic symbols: □ = put down a cube, ← = go West, → = go East, ↓ = go South, ↑ = go North, ≡ = go up one floor.

Building A (Fig. 1) is then represented in the following way: □ → □ ≡ □ ≡ □. Building B (Fig. 1) is then represented in the following way: □ ↑ □ ← □ ≡ □.

3. Action research

The partial experiments are part of a longitudinal action research project that started in September 2010 when one of the authors began teaching a first grade class in a primary school in Prague. The entire project is planned for another three years.

3.1. Research methodology

The teaching content is given by the School Educational Program (SEP). A rough draft of a lesson plan is usually prepared by the teacher-researcher for the whole week and a detailed lesson protocol is done for each upcoming
lesson. Each lesson is videotaped and the elaboration of a detailed protocol for the following lesson is based on reflection upon viewing the video recording. In the protocols much attention is paid to pupil-differentiated instruction. The participants in this research are all pupils attending the class. There was no initial selection. The following research documents are being collected and compiled: framework weekly program, detailed updated protocol for every lesson, video recording of each lesson, transcripts of selected video recordings, pupils’ written production, including individual work, pair work and whole class work. A teaching journal is kept to record the first reflection based purely on the teacher-researcher own daily observations. Once a week a second reflection is done based on the week’s video recordings. In this reflection stage, some interesting phenomena are identified, and relevant samples from the video recordings are transcribed, a protocol is drawn up and archived. The theoretical framework for this analysis is specifically Hejný’s (2012) generic model theory. The teaching is guided by the principles of constructivist approach to teaching and focuses on the building of schemata as understood in the didactic framework of schema-oriented education (Hejný, 2007). The content and teaching approach are guided by the textbooks and teacher manual as developed and authored by Hejný et al. (2007-2011).

3.2. Partial experiment

In March 2011, an interesting phenomenon was observed in the pupil Vena. (All pupils’ names have been changed).

In the introductory stage of the first lesson the pupils were working in groups of five and six. Each group had at their disposal an unlimited number of cubes and a square grid. The size of the squares in the grid corresponded to the measurements of the cube. The pupils were given these oral instructions: “Make a cube building using exactly four cubes and draw its plan into the square grid. Carry on with this activity and try to build as many buildings as possible.” The groups could work at their own pace.

Vena was working in a group with three girls. He took up the role of a coordinator. The girls were engaged in the building process and Vena was deciding which building should be recorded. After a moment, he reported that they were finished with the task. Responding to the teacher’s question: “How many buildings do you have?”, Vena said: “Well, we got twelve. And that’s all.” The teacher-researcher tried to dissuade Vena from his conclusive statement but he was quite sure about his answer; “But it’s really impossible to have more”. And he went on justifying his conviction: “Well, if I take from the tall one (pointing at the tallest building) the top cube, I must place it here next to it. And we already have that here (pointing at the corresponding building). And if I take the top one here, then we can place it (still pointing at the original tall building) here, here, here, or here (he demonstrates all possibilities of moving the cube around). And we already have all these here (pointing at the rest of the buildings). And it’s the same in case of the lower ones.” However, it was apparent that the rest of the pupils do not completely comprehend his explanation and wanted to find out for themselves that another building cannot be built.

In the final stages of the lesson, each group presented their findings and the rest of the class was checking them. They found five, eight and twelve (ten, after the others corrected the result) buildings. Vena presented his group’s findings. As he started speaking, the bell rang and it was impossible to hear his words. However, from his gestures we could infer that he was showing a way to exhaust all possibilities by sorting out the buildings by their height and constructing all buildings of the same height. We believe that Vena had created his generic model (Hejný, 2012) of strategy for constructing all cube buildings out of four cubes. This fact inspired further experimenting. The idea was to investigate the persistence of the generic model in Vena’s mind, his ability to modify it for another context and its transfer to other pupils; we call this latter process cognitive osmosis (Hejný, 2011b).

In the next class, then, all children, with the exception of Vena, were given this task: “Take out exactly four cubes, not more than that. Construct a building from all of these cubes and draw its plan into a square grid. Using the same cubes, construct another building and also draw it. Try to find as many buildings as possible.”

Some pupils from Vena’s group (Fig. 3) were influenced by the results of their team’s work and remembered how many buildings they could make. They did not adopt Vena’s strategy. Vena probably did not have time to communicate the “discovery” to his classmates and they did not have time to discover it for themselves (or with Vena’s advice).
Vena worked on his own now, on a slightly modified assignment: “Take all your cubes. Construct as many buildings as possible but only those that are made of exactly five cubes and which have not more than 3 cubes on the first floor. Draw their blueprints into the square grid.”

It was expected that Vena would use “his” strategy. This expectation was met only partly. Clearly, Vena approached this new situation as a completely new problem and repeated his process of discovery through creating isolated models. This time, however, was the process of discovery much faster.

Figure 3. Vena’s group

4. Conclusion

In conclusion we would like to emphasize the fact that each experiment represents a valuable enrichment in terms of not only our experience with teaching children but also working with teacher-students at the primary school level. We find such experiments consistently to be an effective tool in an individual’s professional development.

One of the aims of the action research is to describe, using video analysis, a teaching style of the teacher (the first author), reveal any shifts or changes and find their causes. Meticulous record-keeping of class activities will enable us to observe any developments in the teacher-researcher’s strategies. In spite of the fact that the study is in its early stages, we have been able to detect shifts in at least the following four directions:

- The voice dominance of the teacher is receding; pupils are given more and more space in discussing their solutions, ideas and opinions.
- The teacher does not interfere even when there is an incorrect construct.
- The work environment in class has been improving, pupils often work in groups and their work has been progressing from working together to collaboration. Mistakes and misconceptions are dealt with by pupils.
- The teacher is more aware of differentiating between her pupils. She selects tasks so that the pupil is both capable of solving the problem and appropriately challenged by it at the same time.

Acknowledgements

The paper was supported by the research projects No. P407/11/1740 Critical areas of primary school mathematics – analysis of teachers’ didactic practices and No. MSM 0021620862, Teaching profession in the environment of changing education requirements.

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