Computers and Mathematics with Applications 58 (2009) 2060–2068

Contents lists available at ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

Soft p-ideals of soft BCI-algebras

Young Bae Jun [a], Kyoung Ja Lee [b,*], Jianming Zhan [c]

[a] Department of Mathematics Education (and RINS), Gyeongsang National University, Chinju 660-701, Republic of Korea

[b] Department of Mathematics Education, Hannam University, Daejeon 306-791, Republic of Korea

[c] Department of Mathematics, Hubei Institute for Nationalities, Enshi, Hubei Province 445000, PR China

ARTICLE INFO

Article history:
Received 28 August 2008
Received in revised form 30 July 2009
Accepted 30 July 2009

Keywords:
Soft set
(p-idealistic) soft BCI-algebra
Soft ideal
Soft p-ideal

ABSTRACT

Molodtsov [D. Molodtsov, Soft set theory – First results, Comput. Math. Appl. 37 (1999) 19–31] introduced the concept of soft set as a new mathematical tool for dealing with uncertainties that is free from the difficulties that have troubled the usual theoretical approaches. Jun [Y. B. Jun, Soft BCK/BCI-algebras, Comput. Math. Appl. 56 (2008) 1408–1413] applied first the notion of soft sets by Molodtsov to the theory of BCK/BCI-algebras. In this paper we introduce the notion of soft p-ideals and p-idealistic soft BCI-algebras, and then investigate their basic properties. Using soft sets, we give characterizations of (fuzzy) p-ideals in BCI-algebras. We provide relations between fuzzy p-ideals and p-idealistic soft BCI-algebras.

1. Introduction

To solve complicated problems in economics, engineering, and environment, we cannot successfully use classical methods because of various uncertainties typical for those problems. There are three theories: theory of probability, theory of fuzzy sets, and the interval mathematics which we can consider as mathematical tools for dealing with uncertainties. But all these theories have their own difficulties. Uncertainties cannot be handled using traditional mathematical tools but may be dealt with using a wide range of existing theories such as the probability theory, the theory of (intuitionistic) fuzzy sets, the theory of vague sets, the theory of interval mathematics, and the theory of rough sets. However, all of these theories have their own difficulties which are pointed out in [1]. Molodtsov [1] and Maji et al. [2] suggested that one reason for these difficulties may be due to the inadequacy of the parametrization tool of the theory. To overcome these difficulties, Molodtsov [1] introduced the concept of soft set as a new mathematical tool for dealing with uncertainties that is free from the difficulties that have troubled the usual theoretical approaches. Molodtsov pointed out several directions for the applications of soft sets. At present, works on the soft set theory are progressing rapidly. Maji et al. [2] described the application of soft set theory to a decision making problem. Maji et al. [3] also studied several operations on the theory of soft sets. Chen et al. [4] presented a new definition of soft set parametrization reduction, and compared this definition to the related concept of attributes reduction in rough set theory. The algebraic structure of set theories dealing with uncertainties has been studied by some authors. The most appropriate theory for dealing with uncertainties is the theory of fuzzy sets developed by Zadeh [5]. The author (together with colleagues) applied the fuzzy set theory to BCK-algebras [6,7], BCC-algebras [8], B-algebras [9], hyper BCK-algebras [10], MTL-algebras [11], hemirings [12], implicative algebras [13], lattice implication algebras [14], and incline algebras [15]. In [16], Jun applied the notion of soft sets by Molodtsov to the theory of BCK/BCI-algebras. He introduced the notion of soft BCK/BCI-algebras and soft subalgebras, and then derived their basic

* Corresponding author.
E-mail addresses: skywine@gmail.com (Y.B. Jun), kjlee@hnu.kr, lsj1109@hotmail.com (K.J. Lee), zhanjianming@hotmail.com (J. Zhan).

0898-1221/$ – see front matter © 2009 Elsevier Ltd. All rights reserved.
doi:10.1016/j.camwa.2009.07.072

properties. In [17], Jun and Park dealt with the algebraic structure of BCK/BCI-algebras by applying soft set theory. They discussed the algebraic properties of soft sets in BCK/BCI-algebras. They introduced the notion of soft ideals and idealistic soft BCK/BCI-algebras, and gave several examples. They investigated relations between soft BCK/BCI-algebras and idealistic soft BCK/BCI-algebras. In this paper we apply the notion of soft sets by Molodtsov to p-ideals in BCI-algebras. We introduce the notion of soft p-ideals and p-idealistic soft BCI-algebras, and then derive their basic properties. Using soft sets, we give characterizations of (fuzzy) p-ideals in BCI-algebras. We provide relations between fuzzy p-ideals and p-idealistic soft BCI-algebras.

2. Basic results on BCI-algebras

A BCK-algebra is an important class of logical algebras introduced by K. Iséki and was extensively investigated by several researchers.

An algebra $(X; *, 0)$ of type $(2, 0)$ is called a *BCI-algebra* if it satisfies the following conditions:

(I) $(\forall x, y, z \in X)(((x * y) * (x * z)) * (z * y) = 0)$,
(II) $(\forall x, y \in X)((x * (x * y)) * y = 0)$,
(III) $(\forall x \in X)(x * x = 0)$,
(IV) $(\forall x, y \in X)(x * y = 0, y * x = 0 \Rightarrow x = y)$.

If a BCI-algebra X satisfies the following identity:

(V) $(\forall x \in X)(0 * x = 0)$,

then X is called a *BCK-algebra*. In any BCK/BCI-algebra X one can define a partial order "$\leq$" by putting $x \leq y$ if and only if $x * y = 0$. Every BCK/BCI-algebra X satisfies:

$$(\forall x, y, z \in X)((x * y) * z = (x * z) * y). \tag{2.1}$$

A nonempty subset S of a BCI-algebra X is called a *subalgebra* of X if $x * y \in S$ for all $x, y \in S$. A subset H of a BCI-algebra X is called an *ideal* of X if it satisfies the following axioms:

(I1) $0 \in H$,
(I2) $(\forall x \in X)(\forall y \in H)(x * y \in H \Rightarrow x \in H)$.

Any ideal H of a BCI-algebra X satisfies the following implication:

$$(\forall x \in X)\ (\forall y \in H) \quad (x \leq y \Rightarrow x \in H). \tag{2.2}$$

A subset H of a BCI-algebra X is called a *p-ideal* (see [18]) of X if it satisfies (I1) and

(I3) $(\forall x, z \in X)(\forall y \in H)((x * z) * (y * z) \in H \Rightarrow x \in H)$.

We know that every p-ideal of a BCI-algebra X is also an ideal of X.

We refer the reader to the books [19,20] for further information regarding BCK/BCI-algebras.

3. Basic results on soft sets

Molodtsov [1] defined the soft set in the following way: Let U be an initial universe set and E be a set of parameters. Let $\mathscr{P}(U)$ denotes the power set of U and $A \subset E$.

Definition 3.1 ([1]). A pair $(\mathscr{F}, A)$ is called a *soft set* over U, where $\mathscr{F}$ is a mapping given by

$$\mathscr{F} : A \to \mathscr{P}(U).$$

In other words, a soft set over U is a parameterized family of subsets of the universe U. For $\varepsilon \in A$, $\mathscr{F}(\varepsilon)$ may be considered as the set of ε-approximate elements of the soft set $(\mathscr{F}, A)$. Clearly, a soft set is not a set. For illustration, Molodtsov considered several examples in [1].

Definition 3.2 ([3]). Let $(\mathscr{F}, A)$ and $(\mathscr{G}, B)$ be two soft sets over a common universe U. The *intersection* of $(\mathscr{F}, A)$ and $(\mathscr{G}, B)$ is defined to be the soft set $(\mathscr{H}, C)$ satisfying the following conditions:

(i) $C = A \cap B$,
(ii) $(\forall e \in C)\ (\mathscr{H}(e) = \mathscr{F}(e)$ or $\mathscr{G}(e)$, (as both are same sets)).

In this case, we write $(\mathscr{F}, A)\widetilde{\cap}(\mathscr{G}, B) = (\mathscr{H}, C)$.

Definition 3.3 ([3]). Let $(\mathscr{F}, A)$ and $(\mathscr{G}, B)$ be two soft sets over a common universe U. The *union* of $(\mathscr{F}, A)$ and $(\mathscr{G}, B)$ is defined to be the soft set $(\mathscr{H}, C)$ satisfying the following conditions:

(i) $C = A \cup B$,
(ii) for all $e \in C$,

$$\mathscr{H}(e) = \begin{cases} \mathscr{F}(e) & \text{if } e \in A \setminus B, \\ \mathscr{G}(e) & \text{if } e \in B \setminus A, \\ \mathscr{F}(e) \cup \mathscr{G}(e) & \text{if } e \in A \cap B. \end{cases}$$

In this case, we write $(\mathscr{F}, A)\widetilde{\cup}(\mathscr{G}, B) = (\mathscr{H}, C)$.

Definition 3.4 ([3]). If $(\mathscr{F}, A)$ and $(\mathscr{G}, B)$ are two soft sets over a common universe U, then "$(\mathscr{F}, A)$ AND $(\mathscr{G}, B)$" denoted by $(\mathscr{F}, A)\widetilde{\wedge}(\mathscr{G}, B)$ is defined by $(\mathscr{F}, A)\widetilde{\wedge}(\mathscr{G}, B) = (\mathscr{H}, A \times B)$, where $\mathscr{H}(\alpha, \beta) = \mathscr{F}(\alpha) \cap \mathscr{G}(\beta)$ for all $(\alpha, \beta) \in A \times B$.

Definition 3.5 ([3]). If $(\mathscr{F}, A)$ and $(\mathscr{G}, B)$ are two soft sets over a common universe U, then "$(\mathscr{F}, A)$ OR $(\mathscr{G}, B)$" denoted by $(\mathscr{F}, A)\widetilde{\vee}(\mathscr{G}, B)$ is defined by $(\mathscr{F}, A)\widetilde{\vee}(\mathscr{G}, B) = (\mathscr{H}, A \times B)$, where $\mathscr{H}(\alpha, \beta) = \mathscr{F}(\alpha) \cup \mathscr{G}(\beta)$ for all $(\alpha, \beta) \in A \times B$.

Definition 3.6 ([3]). For two soft sets $(\mathscr{F}, A)$ and $(\mathscr{G}, B)$ over a common universe U, we say that $(\mathscr{F}, A)$ is a *soft subset* of $(\mathscr{G}, B)$, denoted by $(\mathscr{F}, A)\widetilde{\subset}(\mathscr{G}, B)$, if it satisfies:

(i) $A \subset B$,
(ii) For every $\varepsilon \in A$, $\mathscr{F}(\varepsilon)$ and $\mathscr{G}(\varepsilon)$ are identical approximations.

The most appropriate theory for dealing with uncertainties is the theory of fuzzy sets developed by Zadeh [5].

4. Soft *p*-ideals

In what follows let X and A be a BCI-algebra and a nonempty set, respectively, and R will refer to an arbitrary binary relation between an element of A and an element of X, that is, R is a subset of $A \times X$ without otherwise specified. A set-valued function $\mathscr{F} : A \to \mathscr{P}(X)$ can be defined as $\mathscr{F}(x) = \{y \in X \mid (x, y) \in R\}$ for all $x \in A$. The pair $(\mathscr{F}, A)$ is then a soft set over X.

Definition 4.1 ([17]). Let S be a subalgebra of X. A subset I of X is called an *ideal* of X related to S (briefly, S-ideal of X), denoted by $I \lhd S$, if it satisfies:

(i) $0 \in I$,
(ii) $(\forall x \in S)(\forall y \in I)(x * y \in I \Rightarrow x \in I)$.

Definition 4.2. Let S be a subalgebra of X. A subset I of X is called a *p-ideal* of X related to S (briefly, S-*p-ideal* of X), denoted by $I \lhd_p S$, if it satisfies:

(i) $0 \in I$,
(ii) $(\forall x, z \in S)(\forall y \in I)((x * z) * (y * z) \in I \Rightarrow x \in I)$.

Example 4.3. Let $X = \{0, 1, 2, a, b\}$ be a BCI-algebra with the following Cayley table:

$*$	0	1	2	a	b
0	0	0	0	a	a
1	1	0	1	b	a
2	2	2	0	a	a
a	a	a	a	0	0
b	b	a	b	1	0

Then $S = \{0, 1, a, b\}$ is a subalgebra of X, and $I = \{0, 1, 2\}$ is an S-p-ideal of X.

Note that every S-p-ideal of X is an S-ideal of X.

Definition 4.4 ([16]). Let $(\mathscr{F}, A)$ be a soft set over X. Then $(\mathscr{F}, A)$ is called a *soft BCI-algebra* over X if $\mathscr{F}(x)$ is a subalgebra of X for all $x \in A$.

Definition 4.5 ([17]). Let $(\mathscr{F}, A)$ be a soft BCI-algebra over X. A soft set $(\mathscr{G}, I)$ over X is called a *soft ideal* of $(\mathscr{F}, A)$, denoted by $(\mathscr{G}, I)\widetilde{\lhd}(\mathscr{F}, A)$, if it satisfies:

(i) $I \subset A$,
(ii) $(\forall x \in I)(\mathscr{G}(x) \lhd \mathscr{F}(x))$.

Definition 4.6. Let $(\mathscr{F}, A)$ be a soft BCI-algebra over X. A soft set $(\mathscr{G}, I)$ over X is called a *soft p-ideal* of $(\mathscr{F}, A)$, denoted by $(\mathscr{G}, I) \widetilde{\lhd}_p (\mathscr{F}, A)$, if it satisfies:

(i) $I \subset A$,
(ii) $(\forall x \in I)(\mathscr{G}(x) \lhd_p \mathscr{F}(x))$.

Let us illustrate this definition using the following example.

Example 4.7. Consider a BCI-algebra $X = \{0, 1, 2, a, b\}$ which is given in Example 4.3. Let $(\mathscr{F}, A)$ be a soft set over X, where $A = \{0, 1, 2, a\} \subset X$ and $\mathscr{F} : A \to \mathscr{P}(X)$ is a set-valued function defined by

$$\mathscr{F}(x) = \{0\} \cup \{y \in X \mid y * (y * x) \in \{0, 1\}\}$$

for all $x \in A$. Then $\mathscr{F}(0) = \mathscr{F}(1) = X$, $\mathscr{F}(2) = \{0, 1, a, b\}$ and $\mathscr{F}(a) = \{0\}$, which are subalgebras of X. Hence $(\mathscr{F}, A)$ is a soft BCI-algebra over X. Let $I = \{0, 1, 2\} \subset A$ and $\mathscr{G} : I \to \mathscr{P}(X)$ be a set-valued function defined by

$$\mathscr{G}(x) = \begin{cases} Z(\{0, 1\}) & \text{if } x = 2, \\ \{0\} & \text{if } x \in \{0, 1\}, \end{cases}$$

where $Z(\{0, 1\}) := \{x \in X \mid 0 * (0 * x) \in \{0, 1\}\}$. Then $\mathscr{G}(0) \lhd_p \mathscr{F}(0)$, $\mathscr{G}(1) \lhd_p \mathscr{F}(1)$ and $\mathscr{G}(2) \lhd_p \mathscr{F}(2)$. Hence $(\mathscr{G}, I)$ is a soft p-ideal of $(\mathscr{F}, A)$.

Note that every soft p-ideal is a soft ideal. But the converse is not true as seen in the following example.

Example 4.8. Let $X = \{0, a, b, c, d\}$ be a BCK-algebra, and hence a BCI-algebra, with the following Cayley table:

$*$	0	a	b	c	d
0	0	0	0	0	0
a	a	0	a	a	a
b	b	b	0	b	b
c	c	c	c	0	c
d	d	d	d	d	0

For $A = X$, define a set-valued function $\mathscr{F} : A \to \mathscr{P}(X)$ by

$$\mathscr{F}(x) = \{y \in X \mid y * (y * x) \in \{0, a\}\}$$

for all $x \in A$. Then $(\mathscr{F}, A)$ is a soft BCI-algebra over X (see [16]).

(1) Let $(\mathscr{G}, I)$ be a soft set over X, where $I = \{a, b, c\}$ and $\mathscr{G} : I \to \mathscr{P}(X)$ is a set-valued function defined by

$$\mathscr{G}(x) = \{y \in X \mid y * (y * x) \in \{0, d\}\}$$

for all $x \in I$. Then $\mathscr{G}(a) = \{0, b, c, d\} \lhd X = \mathscr{F}(a)$, $\mathscr{G}(b) = \{0, a, c, d\} \lhd \{0, a, c, d\} = \mathscr{F}(b)$ and $\mathscr{G}(c) = \{0, a, b, d\} \lhd \{0, a, b, d\} = \mathscr{F}(c)$. Hence $(\mathscr{G}, I)$ is a soft ideal of $(\mathscr{F}, A)$ (see [17]). But $(\mathscr{G}, I)$ is not a soft p-ideal of $(\mathscr{F}, A)$ since $(a * a) * (0 * a) = 0 \in \mathscr{G}(a)$ and $a \notin \mathscr{G}(a)$.

(2) For $I = \{a, b, c\}$, let $\mathscr{H} : I \to \mathscr{P}(X)$ be a set-valued function defined by

$$\mathscr{H}(x) = \{0\} \cup \{y \in X \mid x \leq y\}$$

for all $x \in I$. Then $\mathscr{H}(a) = \{0, a\} \lhd X = \mathscr{F}(a)$, $\mathscr{H}(b) = \{0, b\} \lhd \{0, a, c, d\} = \mathscr{F}(b)$ and $\mathscr{H}(c) = \{0, c\} \lhd \{0, a, b, d\} = \mathscr{F}(c)$. Therefore $(\mathscr{H}, I)$ is a soft ideal of $(\mathscr{F}, A)$ (see [17]). But $(\mathscr{H}, I)$ is not a soft p-ideal of $(\mathscr{F}, A)$ since $(b * b) * (0 * b) = 0 \in \mathscr{H}(a)$ and $b \notin \mathscr{H}(a)$.

Theorem 4.9. *Let $(\mathscr{F}, A)$ be a soft BCI-algebra over X. For any soft sets $(\mathscr{G}_1, I_1)$ and $(\mathscr{G}_2, I_2)$ over X where $I_1 \cap I_2 \neq \emptyset$, we have*

$$(\mathscr{G}_1, I_1)\widetilde{\lhd}_p(\mathscr{F}, A),\ (\mathscr{G}_2, I_2)\widetilde{\lhd}_p(\mathscr{F}, A) \Rightarrow (\mathscr{G}_1, I_1)\widetilde{\cap}(\mathscr{G}_2, I_2)\widetilde{\lhd}_p(\mathscr{F}, A).$$

Proof. Using Definition 3.2, we can write

$$(\mathscr{G}_1, I_1)\widetilde{\cap}(\mathscr{G}_2, I_2) = (\mathscr{G}, I),$$

where $I = I_1 \cap I_2$ and $\mathscr{G}(x) = \mathscr{G}_1(x)$ or $\mathscr{G}_2(x)$ for all $x \in I$. Obviously, $I \subset A$ and $\mathscr{G} : I \to \mathscr{P}(X)$ is a mapping. Hence $(\mathscr{G}, I)$ is a soft set over X. Since $(\mathscr{G}_1, I_1)\widetilde{\lhd}_p(\mathscr{F}, A)$ and $(\mathscr{G}_2, I_2)\widetilde{\lhd}_p(\mathscr{F}, A)$, we know that $\mathscr{G}(x) = \mathscr{G}_1(x) \lhd_p \mathscr{F}(x)$ or $\mathscr{G}(x) = \mathscr{G}_2(x) \lhd_p \mathscr{F}(x)$ for all $x \in I$. Hence

$$(\mathscr{G}_1, I_1)\widetilde{\cap}(\mathscr{G}_2, I_2) = (\mathscr{G}, I)\widetilde{\lhd}_p(\mathscr{F}, A).$$

This completes the proof. □

Corollary 4.10. *Let $(\mathscr{F}, A)$ be a soft BCI-algebra over X. For any soft sets $(\mathscr{G}, I)$ and $(\mathscr{H}, I)$ over X, we have*

$$(\mathscr{G}, I)\widetilde{\lhd}_p(\mathscr{F}, A),\ (\mathscr{H}, I)\widetilde{\lhd}_p(\mathscr{F}, A) \Rightarrow (\mathscr{G}, I)\widetilde{\cap}(\mathscr{H}, I)\widetilde{\lhd}_p(\mathscr{F}, A).$$

Proof. Straightforward. □

Theorem 4.11. *Let $(\mathscr{F}, A)$ be a soft BCI-algebra over X. For any soft sets $(\mathscr{G}, I)$ and $(\mathscr{H}, J)$ over X in which I and J are disjoint, we have*

$$(\mathscr{G}, I)\tilde{\lhd}_p(\mathscr{F}, A),\ (\mathscr{H}, J)\tilde{\lhd}_p(\mathscr{F}, A) \Rightarrow (\mathscr{G}, I)\tilde{\cup}(\mathscr{H}, J)\tilde{\lhd}_p(\mathscr{F}, A).$$

Proof. Assume that $(\mathscr{G}, I)\tilde{\lhd}_p(\mathscr{F}, A)$ and $(\mathscr{H}, J)\tilde{\lhd}_p(\mathscr{F}, A)$. By means of Definition 3.3, we can write $(\mathscr{G}, I)\tilde{\cup}(\mathscr{H}, J) = (\mathscr{K}, U)$ where $U = I \cup J$ and for every $x \in U$,

$$\mathscr{K}(x) = \begin{cases} \mathscr{G}(x) & \text{if } x \in I \setminus J, \\ \mathscr{H}(x) & \text{if } x \in J \setminus I, \\ \mathscr{G}(x) \cup \mathscr{H}(x) & \text{if } x \in I \cap J. \end{cases}$$

Since $I \cap J = \emptyset$, either $x \in I \setminus J$ or $x \in J \setminus I$ for all $x \in U$. If $x \in I \setminus J$, then $\mathscr{K}(x) = \mathscr{G}(x) \lhd_p \mathscr{F}(x)$ since $(\mathscr{G}, I)\tilde{\lhd}_p(\mathscr{F}, A)$. If $x \in J \setminus I$, then $\mathscr{K}(x) = \mathscr{H}(x) \lhd_p \mathscr{F}(x)$ since $(\mathscr{H}, J)\tilde{\lhd}_p(\mathscr{F}, A)$. Thus $\mathscr{K}(x) \lhd_p \mathscr{F}(x)$ for all $x \in U$, and so $(\mathscr{G}, I)\tilde{\cup}(\mathscr{H}, J) = (\mathscr{K}, U)\tilde{\lhd}_p(\mathscr{F}, A)$. $\square$

If I and J are not disjoint in Theorem 4.11, then Theorem 4.11 is not true in general as seen in the following example.

Example 4.12. Let $X = \{0, 1, a, b, c\}$ be a BCI-algebra with the following Cayley table:

$*$	0	1	a	b	c
0	0	0	a	b	c
1	1	0	a	b	c
a	a	a	0	c	b
b	b	b	c	0	a
c	c	c	b	a	0

For $A = \{0, 1\} \subset X$, let $\mathscr{F} : A \to \mathscr{P}(X)$ be a set-valued function defined by

$$\mathscr{F}(x) = \{y \in X \mid y * x = y\}$$

for all $x \in A$. Then $\mathscr{F}(0) = X$ and $\mathscr{F}(1) = \{0, a, b, c\}$, which are subalgebras of X, and hence $(\mathscr{F}, A)$ is a soft BCI-algebra over X. If we take $I = A$ and define a set-valued function $\mathscr{G} : I \to \mathscr{P}(X)$ by

$$\mathscr{G}(x) = \{y \in X \mid x * (x * y) \in \{0, b\}\}$$

for all $x \in I$, then we obtain that

$$\mathscr{G}(0) = \{0, 1, b\} \lhd_p \mathscr{F}(0) \text{ and } \mathscr{G}(1) = \{0, 1, b\} \lhd_p \mathscr{F}(1).$$

This means that $(\mathscr{G}, I)\tilde{\lhd}_p(\mathscr{F}, A)$. Now, consider $J = \{0\}$ which is not disjoint with I, and let $\mathscr{H} : J \to \mathscr{P}(X)$ be a set-valued function defined by

$$\mathscr{H}(x) = \{y \in X \mid x * (x * y) \in \{0, c\}\}$$

for all $x \in J$. Then $\mathscr{H}(0) = \{0, 1, c\} \lhd_p \mathscr{F}(0)$, and so $(\mathscr{H}, J)\tilde{\lhd}_p(\mathscr{F}, A)$. But if $(\mathscr{K}, U) := (\mathscr{G}, I)\tilde{\cup}(\mathscr{H}, J)$, then $\mathscr{K}(0) = \mathscr{G}(0) \cup \mathscr{H}(0) = \{0, 1, b, c\}$, which is not a p-ideal of X related to $\mathscr{F}(0)$ since $(a * 0) * (b * 0) = c \in \mathscr{K}(0)$ and $a \notin \mathscr{K}(0)$. Hence $(\mathscr{K}, U) = (\mathscr{G}, I)\tilde{\cup}(\mathscr{H}, J)$ is not a soft p-ideal of $(\mathscr{F}, A)$.

5. *p*-idealistic soft BCI-algebras

Definition 5.1 (*[17]*). Let $(\mathscr{F}, A)$ be a soft set over X. Then $(\mathscr{F}, A)$ is called an *idealistic soft BCI-algebra* over X if $\mathscr{F}(x)$ is an ideal of X for all $x \in A$.

Definition 5.2. Let $(\mathscr{F}, A)$ be a soft set over X. Then $(\mathscr{F}, A)$ is called a *p-idealistic soft BCI-algebra* over X if $\mathscr{F}(x)$ is a p-ideal of X for all $x \in A$.

Example 5.3. Consider a BCI-algebra $X = \{0, 1, 2, a, b\}$ which is given in Example 4.3. Let $(\mathscr{F}, A)$ be a soft set over X, where $A = X$ and $\mathscr{F} : A \to \mathscr{P}(X)$ is a set-valued function defined by

$$\mathscr{F}(x) = \begin{cases} Z(\{0, 1\}) & \text{if } x \in \{2, a, b\}, \\ X & \text{if } x \in \{0, 1\}, \end{cases}$$

where $Z(\{0, 1\}) := \{x \in X \mid 0 * (0 * x) \in \{0, 1\}\}$. Then $(\mathscr{F}, A)$ is a p-idealistic soft BCI-algebra over X.

For any element x of a BCI-algebra X, we define the order of x, denoted by $o(x)$, as

$$o(x) = \min\{n \in \mathbb{N} \mid 0 * x^n = 0\},$$

where $0 * x^n = (\cdots((0 * x) * x) \cdots) * x$ in which x appears n-times.

Example 5.4. Let $X = \{0, a, b, c, d, e, f, g\}$ and consider the following Cayley table:

$*$	0	a	b	c	d	e	f	g
0	0	0	0	0	d	d	d	d
a	a	0	0	0	e	d	d	d
b	b	b	0	0	f	f	d	d
c	c	b	a	0	g	f	e	d
d	d	d	d	d	0	0	0	0
e	e	d	d	d	a	0	0	0
f	f	f	d	d	b	b	0	0
g	g	f	e	d	c	b	a	0

Then $(X; *, 0)$ is a BCI-algebra (see [21]). Let $(\mathscr{F}, A)$ be a soft set over X, where $A = \{a, b, c\} \subset X$ and $\mathscr{F} : A \to \mathscr{P}(X)$ is a set-valued function defined as follows:

$$\mathscr{F}(x) = \{y \in X \mid o(x) = o(y)\}$$

for all $x \in A$. Then $\mathscr{F}(a) = \mathscr{F}(b) = \mathscr{F}(c) = \{0, a, b, c\}$ is a p-ideal of X. Hence $(\mathscr{F}, A)$ is a p-idealistic soft BCI-algebra over X. But, if we take $B = \{a, b, d, f\} \subset X$ and define a set-valued function $\mathscr{G} : B \to \mathscr{P}(X)$ by

$$\mathscr{G}(x) = \{0\} \cup \{y \in X \mid o(x) = o(y)\}$$

for all $x \in B$, then $(\mathscr{G}, B)$ is not a p-idealistic soft BCI-algebra over X since $\mathscr{G}(d) = \{0, d, e, f, g\}$ is not a p-ideal of X.

Example 5.5. Consider a BCI-algebra $X = \{0, a, b, c\}$ with the following Cayley table:

$*$	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

Let $A = X$ and $\mathscr{F} : A \to \mathscr{P}(X)$ be a set-valued function defined by

$$\mathscr{F}(x) = \{0, x\}$$

for all $x \in A$. Then $\mathscr{F}(0) = \{0\}$, $\mathscr{F}(a) = \{0, a\}$, $\mathscr{F}(b) = \{0, b\}$ and $\mathscr{F}(c) = \{0, c\}$ which are ideals of X. Hence $(\mathscr{F}, A)$ is an idealistic soft BCI-algebra over X (see [17]). Note that $\mathscr{F}(x)$ is a p-ideal of X for all $x \in A$. Hence $(\mathscr{F}, A)$ is a p-idealistic soft BCI-algebra over X.

Obviously, every p-idealistic soft BCI-algebra over X is an idealistic soft BCI-algebra over X, but the converse is not true in general as seen in the following example.

Example 5.6. Consider a BCI-algebra $X := Y \times \mathbb{Z}$, where $(Y, *, 0)$ is a BCI-algebra and $(\mathbb{Z}, -, 0)$ is the adjoint BCI-algebra of the additive group $(\mathbb{Z}, +, 0)$ of integers. Let $\mathscr{F} : X \to \mathscr{P}(X)$ be a set-valued function defined as follows:

$$\mathscr{F}(y, n) := \begin{cases} Y \times \mathbb{N}_0 & \text{if } n \in \mathbb{N}_0, \\ \{(0, 0)\} & \text{otherwise,} \end{cases} \tag{5.1}$$

for all $(y, n) \in X$, where $\mathbb{N}_0$ is the set of all nonnegative integers. Then $(\mathscr{F}, X)$ is an idealistic soft BCI-algebra over X (see [17]). But it is not a p-idealistic soft BCI-algebra over X since $\{(0, 0)\}$ may not be a p-ideal of X.

Proposition 5.7. *Let $(\mathscr{F}, A)$ and $(\mathscr{F}, B)$ be soft sets over X where $B \subseteq A \subseteq X$. If $(\mathscr{F}, A)$ is a p-idealistic soft BCI-algebra over X, then so is $(\mathscr{F}, B)$.*

Proof. Straightforward. □

The converse of Proposition 5.7 is not true in general as seen in the following example.

Example 5.8. Consider a p-idealistic soft BCI-algebra $(\mathscr{F}, A)$ over X which is described in Example 5.4. If we take $B = \{a, b, c, d\} \supseteq A$, then $(\mathscr{F}, B)$ is not a p-idealistic soft BCI-algebra over X since $\mathscr{F}(d) = \{d, e, f, g\}$ is not a p-ideal of X.

Theorem 5.9. *Let $(\mathscr{F}, A)$ and $(\mathscr{G}, B)$ be two p-idealistic soft BCI-algebras over X. If $A \cap B \neq \emptyset$, then the intersection $(\mathscr{F}, A)\widetilde{\cap}(\mathscr{G}, B)$ is a p-idealistic soft BCI-algebra over X.*

Proof. Using Definition 3.2, we can write $(\mathscr{F}, A)\widetilde{\cap}(\mathscr{G}, B) = (\mathscr{H}, C)$, where $C = A \cap B$ and $\mathscr{H}(x) = \mathscr{F}(x)$ or $\mathscr{G}(x)$ for all $x \in C$. Note that $\mathscr{H} : C \to \mathscr{P}(X)$ is a mapping, and therefore $(\mathscr{H}, C)$ is a soft set over X. Since $(\mathscr{F}, A)$ and $(\mathscr{G}, B)$ are p-idealistic soft BCI-algebras over X, it follows that $\mathscr{H}(x) = \mathscr{F}(x)$ is a p-ideal of X, or $\mathscr{H}(x) = \mathscr{G}(x)$ is a p-ideal of X for all $x \in C$. Hence $(\mathscr{H}, C) = (\mathscr{F}, A)\widetilde{\cap}(\mathscr{G}, B)$ is a p-idealistic soft BCI-algebra over X. □

Corollary 5.10. *Let $(\mathscr{F}, A)$ and $(\mathscr{G}, A)$ be two p-idealistic soft BCI-algebras over X. Then their intersection $(\mathscr{F}, A)\widetilde{\cap}(\mathscr{G}, A)$ is a p-idealistic soft BCI-algebra over X.*

Proof. Straightforward. □

Theorem 5.11. *Let $(\mathscr{F}, A)$ and $(\mathscr{G}, B)$ be two p-idealistic soft BCI-algebras over X. If A and B are disjoint, then the union $(\mathscr{F}, A)\widetilde{\cup}(\mathscr{G}, B)$ is a p-idealistic soft BCI-algebra over X.*

Proof. Using Definition 3.3, we can write $(\mathscr{F}, A)\widetilde{\cup}(\mathscr{G}, B) = (\mathscr{H}, C)$, where $C = A \cup B$ and for every $x \in C$,

$$\mathscr{H}(x) = \begin{cases} \mathscr{F}(x) & \text{if } x \in A \setminus B, \\ \mathscr{G}(x) & \text{if } x \in B \setminus A, \\ \mathscr{F}(x) \cup \mathscr{G}(x) & \text{if } x \in A \cap B. \end{cases}$$

Since $A \cap B = \emptyset$, either $x \in A \setminus B$ or $x \in B \setminus A$ for all $x \in C$. If $x \in A \setminus B$, then $\mathscr{H}(x) = \mathscr{F}(x)$ is a p-ideal of X since $(\mathscr{F}, A)$ is a p-idealistic soft BCI-algebra over X. If $x \in B \setminus A$, then $\mathscr{H}(x) = \mathscr{G}(x)$ is a p-ideal of X since $(\mathscr{G}, B)$ is a p-idealistic soft BCI-algebra over X. Hence $(\mathscr{H}, C) = (\mathscr{F}, A)\widetilde{\cup}(\mathscr{G}, B)$ is a p-idealistic soft BCI-algebra over X. □

Theorem 5.12. *If $(\mathscr{F}, A)$ and $(\mathscr{G}, B)$ are p-idealistic soft BCI-algebras over X, then $(\mathscr{F}, A)\widetilde{\wedge}(\mathscr{G}, B)$ is a p-idealistic soft BCI-algebra over X.*

Proof. By means of Definition 3.4, we know that

$$(\mathscr{F}, A)\widetilde{\wedge}(\mathscr{G}, B) = (\mathscr{H}, A \times B),$$

where $\mathscr{H}(x, y) = \mathscr{F}(x) \cap \mathscr{G}(y)$ for all $(x, y) \in A \times B$. Since $\mathscr{F}(x)$ and $\mathscr{G}(y)$ are p-ideals of X, the intersection $\mathscr{F}(x) \cap \mathscr{G}(y)$ is also a p-ideal of X. Hence $\mathscr{H}(x, y)$ is a p-ideal of X for all $(x, y) \in A \times B$, and therefore $(\mathscr{F}, A)\widetilde{\wedge}(\mathscr{G}, B) = (\mathscr{H}, A \times B)$ is a p-idealistic soft BCI-algebra over X. □

Definition 5.13. A p-idealistic soft BCI-algebra $(\mathscr{F}, A)$ over X is said to be *trivial* (resp., *whole*) if $\mathscr{F}(x) = \{0\}$ (resp., $\mathscr{F}(x) = X$) for all $x \in A$.

Example 5.14. Let X be a BCI-algebra which is given in Example 5.5, and let $\mathscr{F} : X \to \mathscr{P}(X)$ be a set-valued function defined by

$$\mathscr{F}(x) = \{0\} \cup \{y \in X \mid o(x) = o(y)\}$$

for all $x \in X$. Then $\mathscr{F}(0) = \{0\}$ and $\mathscr{F}(a) = \mathscr{F}(b) = \mathscr{F}(c) = X$. We can check that $\{0\} \lhd_p X$ and $X \lhd_p X$. Hence $(\mathscr{F}, \{0\})$ is a trivial p-idealistic soft BCI-algebra over X and $(\mathscr{F}, X \setminus \{0\})$ is a whole p-idealistic soft BCI-algebra over X.

The proofs of the following three lemmas are straightforward, so they are omitted.

Lemma 5.15. *Let $f : X \to Y$ be an onto homomorphism of BCI-algebras. If I is an ideal of X, then $f(I)$ is an ideal of Y.*

Lemma 5.16. *Let $f : X \to Y$ be an isomorphism of BCI-algebras. If I is a p-ideal of X, then $f(I)$ is a p-ideal of Y.*

Let $f : X \to Y$ be a mapping of BCI-algebras. For a soft set $(\mathscr{F}, A)$ over X, $(f(\mathscr{F}), A)$ is a soft set over Y where $f(\mathscr{F}) : A \to \mathscr{P}(Y)$ is defined by $f(\mathscr{F})(x) = f(\mathscr{F}(x))$ for all $x \in A$.

Lemma 5.17. *Let $f : X \to Y$ be an isomorphism of BCI-algebras. If $(\mathscr{F}, A)$ is a p-idealistic soft BCI-algebra over X, then $(f(\mathscr{F}), A)$ is a p-idealistic soft BCI-algebra over Y.*

Theorem 5.18. *Let $f : X \to Y$ be an isomorphism of BCI-algebras and let $(\mathscr{F}, A)$ be a p-idealistic soft BCI-algebra over X.*

(1) *If $\mathscr{F}(x) \subseteq \ker(f)$ for all $x \in A$, then $(f(\mathscr{F}), A)$ is a trivial p-idealistic soft BCI-algebra over Y.*
(2) *If $(\mathscr{F}, A)$ is whole, then $(f(\mathscr{F}), A)$ is a whole p-idealistic soft BCI-algebra over Y.*

Proof. (1) Assume that $\mathscr{F}(x) \subseteq \ker(f)$ for all $x \in A$. Then $f(\mathscr{F})(x) = f(\mathscr{F}(x)) = \{0_Y\}$ for all $x \in A$. Hence $(f(\mathscr{F}), A)$ is a trivial p-idealistic soft BCI-algebra over Y by Lemma 5.17 and Definition 5.13.

(2) Suppose that $(\mathscr{F}, A)$ is whole. Then $\mathscr{F}(x) = X$ for all $x \in A$, and so $f(\mathscr{F})(x) = f(\mathscr{F}(x)) = f(X) = Y$ for all $x \in A$. It follows from Lemma 5.17 and Definition 5.13 that $(f(\mathscr{F}), A)$ is a whole p-idealistic soft BCI-algebra over Y. □

Definition 5.19 ([22]). A fuzzy μ in X is a *fuzzy p-ideal* of X if it satisfies the following assertions:

(i) $(\forall x \in X)(\mu(0) \geq \mu(x))$,
(ii) $(\forall x, y, z \in X)(\mu(x) \geq \min\{\mu((x * z) * (y * z)), \mu(y)\})$.

Lemma 5.20 ([22]). *A fuzzy set μ in X is a fuzzy p-ideal of X if and only if it satisfies:*

$$(\forall t \in [0,1])(U(\mu;t) \neq \emptyset \Rightarrow U(\mu;t) \text{ is a p-ideal of } X).$$

Theorem 5.21. *For every fuzzy p-ideal μ of X, there exists a p-idealistic soft BCI-algebra $(\mathscr{F}, A)$ over X.*

Proof. Let μ be a fuzzy p-ideal of X. Then $U(\mu;t) := \{x \in X \mid \mu(x) \geq t\}$ is a p-ideal of X for all $t \in \operatorname{Im}(\mu)$. If we take $A = \operatorname{Im}(\mu)$ and consider a set-valued function $\mathscr{F} : A \to \mathscr{P}(X)$ given by $\mathscr{F}(t) = U(\mu;t)$ for all $t \in A$, then $(\mathscr{F}, A)$ is a p-idealistic soft BCI-algebra over X. □

Conversely, the following theorem is straightforward.

Theorem 5.22. *For any fuzzy set μ in X, if a p-idealistic soft BCI-algebra $(\mathscr{F}, A)$ over X is given by $A = \operatorname{Im}(\mu)$ and $\mathscr{F}(t) = U(\mu;t)$ for all $t \in A$, then μ is a fuzzy p-ideal of X.*

Let μ be a fuzzy set in X and let $(\mathscr{F}, A)$ be a soft set over X in which $A = \operatorname{Im}(\mu)$ and $\mathscr{F} : A \to \mathscr{P}(X)$ is a set-valued function defined by

$$(\forall t \in A) \quad (\mathscr{F}(t) = \{x \in X \mid \mu(x) + t > 1\}). \tag{5.2}$$

Then there exists $t \in A$ such that $\mathscr{F}(t)$ is not a p-ideal of X as seen in the following example.

Example 5.23. For any BCI-algebra X, define a fuzzy set μ in X by $\mu(0) = t_0 < 0.5$ and $\mu(x) = 1 - t_0$ for all $x \neq 0$. Let $A = \operatorname{Im}(\mu)$ and $\mathscr{F} : A \to \mathscr{P}(X)$ be a set-valued function given by (5.2). Then $\mathscr{F}(1 - t_0) = X \setminus \{0\}$, which is not a p-ideal of X.

Theorem 5.24. *Let μ be a fuzzy set in X and let $(\mathscr{F}, A)$ be a soft set over X in which $A = [0, 1]$ and $\mathscr{F} : A \to \mathscr{P}(X)$ is given by (5.2). Then the following assertions are equivalent:*

(1) *μ is a fuzzy p-ideal of X,*
(2) *for every $t \in A$ with $\mathscr{F}(t) \neq \emptyset$, $\mathscr{F}(t)$ is a p-ideal of X.*

Proof. Assume that μ is a fuzzy p-ideal of X. Let $t \in A$ be such that $\mathscr{F}(t) \neq \emptyset$. If we select $x \in \mathscr{F}(t)$, then $\mu(0) + t \geq \mu(x) + t > 1$, and so $0 \in \mathscr{F}(t)$. Let $t \in A$ and $x, y, z \in X$ be such that $y \in \mathscr{F}(t)$ and $(x*z)*(y*z) \in \mathscr{F}(t)$. Then $\mu(y) + t > 1$ and $\mu((x*z)*(y*z)) + t > 1$. Since μ is a fuzzy p-ideal of X, it follows that

$$\begin{aligned} \mu(x) + t &\geq \min\{\mu((x*z)*(y*z)), \mu(y)\} + t \\ &= \min\{\mu((x*z)*(y*z)) + t, \mu(y) + t\} \\ &> 1, \end{aligned}$$

so that $x \in \mathscr{F}(t)$. Hence $\mathscr{F}(t)$ is a p-ideal of X for all $t \in A$ with $\mathscr{F}(t) \neq \emptyset$. Conversely, suppose that (2) is valid. If there exists $a \in X$ such that $\mu(0) < \mu(a)$, then we can select $t_a \in A$ such that $\mu(0) + t_a \leq 1 < \mu(a) + t_a$. It follows that $a \in \mathscr{F}(t_a)$ and $0 \notin \mathscr{F}(t_a)$, which is a contradiction. Hence $\mu(0) \geq \mu(x)$ for all $x \in X$. Now, assume that

$$\mu(a) < \min\{\mu((a*c)*(b*c)), \mu(b)\}$$

for some $a, b, c \in X$. Then

$$\mu(a) + s_0 \leq 1 < \min\{\mu((a*c)*(b*c)), \mu(b)\} + s_0,$$

for some $s_0 \in A$, which implies that $(a*c)*(b*c) \in \mathscr{F}(s_0)$ and $b \in \mathscr{F}(s_0)$, but $a \notin \mathscr{F}(s_0)$. This is a contradiction. Therefore

$$\mu(x) \geq \min\{\mu((x*z)*(y*z)), \mu(y)\}$$

for all $x, y, z \in X$, and thus μ is a fuzzy p-ideal of X. □

Corollary 5.25. *Let μ be a fuzzy set in X such that $\mu(x) > 0.5$ for some $x \in X$, and let $(\mathscr{F}, A)$ be a soft set over X in which*

$$A := \{t \in \operatorname{Im}(\mu) \mid t > 0.5\}$$

and $\mathscr{F} : A \to \mathscr{P}(X)$ is given by (5.2). If μ is a fuzzy p-ideal of X, then $(\mathscr{F}, A)$ is a p-idealistic soft BCI-algebra over X.

Proof. Straightforward. □

Theorem 5.26. *Let μ be a fuzzy set in X and let $(\mathscr{F}, A)$ be a soft set over X in which $A = (0.5, 1]$ and $\mathscr{F} : A \to \mathscr{P}(X)$ is defined by*

$$(\forall t \in A) \quad (\mathscr{F}(t) = U(\mu;t)).$$

Then $\mathscr{F}(t)$ is a p-ideal of X for all $t \in A$ with $\mathscr{F}(t) \neq \emptyset$ if and only if the following assertions are valid:

(1) $(\forall x \in X)(\max\{\mu(0), 0.5\} \geq \mu(x))$,
(2) $(\forall x, y, z \in X)(\max\{\mu(x), 0.5\} \geq \min\{\mu((x*z)*(y*z)), \mu(y)\})$.

Proof. Assume that $\mathscr{F}(t)$ is a p-ideal of X for all $t \in A$ with $\mathscr{F}(t) \neq \emptyset$. If there exists $x_0 \in X$ such that $\max\{\mu(0), 0.5\} < \mu(x_0)$, then we can select $t_0 \in A$ such that $\max\{\mu(0), 0.5\} < t_0 \leq \mu(x_0)$. It follows that $\mu(0) < t_0$, so that $x_0 \in \mathscr{F}(t_0)$ and $0 \notin \mathscr{F}(t_0)$. This is a contradiction, and so (1) is valid. Suppose that there exist $a, b, c \in X$ such that

$$\max\{\mu(a), 0.5\} < \min\{\mu((a * c) * (b * c)), \mu(b)\}.$$

Then

$$\max\{\mu(a), 0.5\} < u_0 \leq \min\{\mu((a * c) * (b * c)), \mu(b)\}$$

for some $u_0 \in A$. Thus $(a * c) * (b * c) \in \mathscr{F}(u_0)$ and $b \in \mathscr{F}(u_0)$, but $a \notin \mathscr{F}(u_0)$. This is a contradiction, and so (2) is valid.

Conversely, suppose that (1) and (2) are valid. Let $t \in A$ with $\mathscr{F}(t) \neq \emptyset$ For any $x \in \mathscr{F}(t)$, we have

$$\max\{\mu(0), 0.5\} \geq \mu(x) \geq t > 0.5$$

and so $\mu(0) \geq t$, i.e., $0 \in \mathscr{F}(t)$. Let $x, y, z \in X$ be such that $y \in \mathscr{F}(t)$ and $(x * z) * (y * z) \in \mathscr{F}(t)$. Then $\mu(y) \geq t$ and $\mu((x * z) * (y * z)) \geq t$. It follows from the second condition that

$$\max\{\mu(x), 0.5\} \geq \min\{\mu((x * z) * (y * z)), \mu(y)\} \geq t > 0.5,$$

so that $\mu(x) \geq t$, i.e., $x \in \mathscr{F}(t)$. Therefore $\mathscr{F}(t)$ is a p-ideal of X for all $t \in A$ with $\mathscr{F}(t) \neq \emptyset$. $\square$

Acknowledgements

The authors are very grateful to referees for their valuable comments and suggestions for improving this paper.

References

[1] D. Molodtsov, Soft set theory — First results, Comput. Math. Appl. 37 (1999) 19–31.
[2] P.K. Maji, A.R. Roy, R. Biswas, An application of soft sets in a decision making problem, Comput. Math. Appl. 44 (2002) 1077–1083.
[3] P.K. Maji, R. Biswas, A.R. Roy, Soft set theory, Comput. Math. Appl. 45 (2003) 555–562.
[4] D. Chen, E.C.C. Tsang, D.S. Yeung, X. Wang, The parametrization reduction of soft sets and its applications, Comput. Math. Appl. 49 (2005) 757–763.
[5] L.A. Zadeh, Fuzzy sets, Inform. Control 8 (1965) 338–353.
[6] Y.B. Jun, X.L. Xin, Involutory and invertible fuzzy BCK-algebras, Fuzzy Sets and Systems 117 (2001) 463–469.
[7] Y.B. Jun, X.L. Xin, Fuzzy prime ideals and invertible fuzzy ideals in BCK-algebras, Fuzzy Sets and Systems 117 (2001) 471–476.
[8] W.A. Dudek, Y.B. Jun, Z. Stojakovic, On fuzzy ideals in BCC-algebras, Fuzzy Sets and Systems 123 (2001) 251–258.
[9] Y.B. Jun, E.H. Roh, H.S. Kim, On fuzzy B-algebras, Czechoslovak Math. J. 52 (127) (2002) 375–384.
[10] Y.B. Jun, W.H. Shim, Fuzzy strong implicative hyper BCK-ideals of hyper BCK-algebras, Inform. Sci. 170 (2005) 351–361.
[11] Y.B. Jun, Y. Xu, X.H. Zhang, Fuzzy filters of MTL-algebras, Inform. Sci. 175 (2005) 120–138.
[12] Y.B. Jun, M.A. Ozturk, S.Z. Song, On fuzzy *h*-ideals in hemirings, Inform. Sci. 162 (2004) 211–226.
[13] Y.B. Jun, S.S. Ahn, H.S. Kim, Quotient structures of some implicative algebras via fuzzy implicative filters, Fuzzy Sets and Systems 121 (2001) 325–332.
[14] Y.B. Jun, Fuzzy positive implicative and fuzzy associative filters of lattice implication algerbas, Fuzzy Sets and Systems 121 (2001) 353–357.
[15] Y.B. Jun, S.S. Ahn, H.S. Kim, Fuzzy subincline (ideals) of incline algebras, Fuzzy Sets and Systems 123 (2001) 217–225.
[16] Y.B. Jun, Soft BCK/BCI-algebras, Comput. Math. Appl. 56 (2008) 1408–1413.
[17] Y.B. Jun, C.H. Park, Applications of soft sets in ideal theory of BCK/BCI-algebras, Inform. Sci. 178 (2008) 2466–2475.
[18] X.H. Zhang, H. Jiang, S.A. Bhatti, On *p*-ideals of a BCI-algebra, Punjab Univ. J. Math. 27 (1994) 121–128.
[19] Y.S. Huang, BCI-Algebras, Science Press, Beijing, 2006.
[20] J. Meng, Y.B. Jun, BCK-Algebras, Kyungmoon Sa Co, Seoul, 1994.
[21] M.A. Chaudhry, Weakly positive implicative and weakly implicative BCI-algebras, Math. Jpn. 35 (1990) 141–151.
[22] Y.B. Jun, J. Meng, Fuzzy *p*-ideals in BCI-algebras, Math. Japn. 40 (1994) 271–282.