

EUROSENSORS 2014, the XXVIII edition of the conference series

Modeling and Experimental Investigation of Resonant Viscosity and Mass Density Sensors Considering their Cross-Sensitivity to Temperature

Martin Heinisch^{a,*}, Erwin K. Reichel^a, Isabelle Dufour^b, Bernhard Jakoby^a

^a*Institute for Microelectronics and Microsensors, Johannes Kepler University, Linz, Austria*

^b*Laboratoire IMS, Université de Bordeaux, Talence, France*

Abstract

In this contribution we discuss a generalized, reduced order model for resonant viscosity and mass density sensors which considers also the devices' cross sensitivities to temperature. The applicability of the model is substantiated by experimental results from measurements obtained with a circular steel tuning fork in various liquids and temperatures. Advantages of this model are its simplicity, its general applicability for resonant mass density and viscosity sensors which furthermore facilitates the comparison of different sensors.

© 2014 The Authors. Published by Elsevier Ltd. This is an open access article under the CC BY-NC-ND license

(<http://creativecommons.org/licenses/by-nc-nd/3.0/>).

Peer-review under responsibility of the scientific committee of Eurosensors 2014

Keywords: General model, temperature, viscosity, mass density, viscosity sensor, resonator

1. Introduction

Miniaturized viscosity sensors are attractive devices for condition monitoring applications involving fluid media. Recently introduced devices utilize vibrating resonant mechanical structures interacting with the fluid where the resonance frequency f_r and the quality factor Q are affected by the fluid's viscosity [1]. These devices have been of great interest during the past two decades and the amount of reported principles seems to be still increasing. This interest results not only from the obvious advantages such as the devices' applicability for miniaturized (handheld) devices which furthermore allows low-cost production rather than the possibility for measurements which cannot be performed with conventional laboratory experiments, e.g. the rheometric characterization at frequencies higher than 100 Hz. Many principles are aiming to achieve similar or even better accuracies than provided by high precision laboratory instruments.

An extensive literature review of resonant devices for viscosity and mass density showed, that in most cases, the devices' cross-sensitivities of f_r and Q to temperature are not investigated. However, such an investigation and a precise modeling of this temperature dependence is extremely important, particularly when aiming at high accurate measurement results.

*Corresponding author. Tel.: +43-732-2468-6266 ; fax: +43-732-2468-6250

E-mail address: martin.heinisch@jku.at

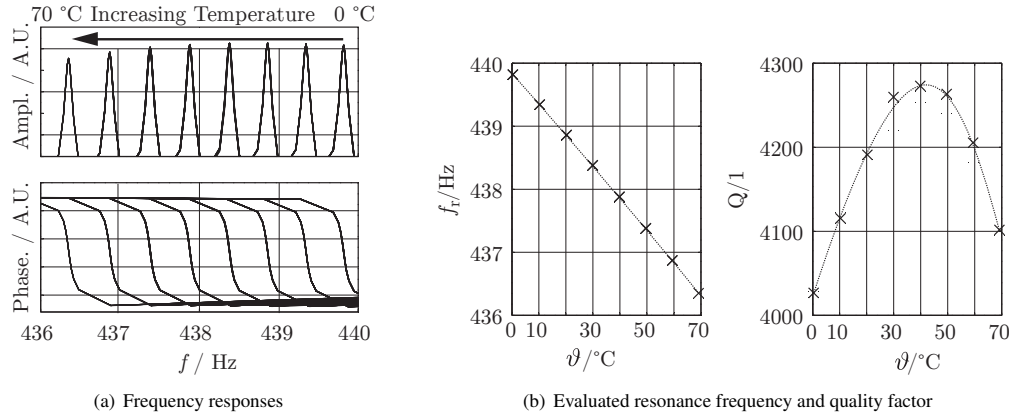


Figure 1: a) Cross-sensitivity of the frequency response to temperature (in air): The frequency responses of a steel tuning fork have been recorded in a temperature range from 0 °C to 70 °C in 10 °C steps. After the temperature has been controlled to the desired value, the resonator was kept at this temperature for an additional hour to ensure stable measuring conditions. The measurements were performed back and forth, i.e. going from 0 °C to higher temperatures, and once 70 °C were reached the temperatures were gradually degraded (in 10 °C steps) until 0 °C were reached again. b) Evaluated resonance frequencies and quality factors from the frequency responses

This statement is explained by the following example: In [2] a device featuring a mass density resolution of 0.01 kg/cm³ with a sensitivity of 16 Hz/(kg/m³) at $f_r \approx 130$ kHz is reported. Considering the relative cross-sensitivity of similar devices which typically is $\Delta f_r / (f_r \Delta T) \approx (30 \dots 100) \cdot 10^6 / \text{K}$ (see [3], [4]), implies a cross-sensitivity to temperature of $\Delta f_r / \Delta T = (3.9 \dots 13) \text{ Hz/K}$ for this particular device. This reveals that for facilitating the aforementioned sensitivity, an accuracy of (0.01 ... 0.04) K of the temperature measurement would be necessary, considering the modeling of the temperature dependence to be exact at 100 %. Furthermore, a thorough investigation of the temperature characteristics of resonant viscosity and mass density sensors is indispensable, as all liquids' viscosities (and mass densities) are significantly temperature dependent [5]. Thus measuring these quantities without precisely measuring the ambient temperature and without knowing the sensors temperature characteristics yield non-reliable results.

For a first attempt to evaluate the resonator's temperature dependence, frequency responses in air have been recorded at various temperatures. Figure 1 shows frequency responses as well as evaluated resonance frequencies and quality factors in a temperature range from 0 °C to 70 °C in air which have been recorded using a steel tuning fork with circular cross-section. The almost linear dependence of f_r on temperature ϑ can be explained considering thermal expansion as well as the change of the Young's modulus. However, the supposedly parabolic dependence of Q is difficult to explain. For this reason further investigations in air were not refined, but frequency responses in four different liquids (ethanol, isopropanol, DI-water and a mixture of 61 % glycerol in DI-water) at five temperatures (15, 20, 25, 30 and 35 °C) have been performed and analyzed. The results obtained from these experiments in liquids allowed to formulate an empiric temperature related extension of a previously introduced model.

2. Model

2.1. Temperature-independent model

Recently, we devised a reduced order model applicable for immersed resonant viscosity and mass density sensors relating a liquid's viscosity η and mass density ρ to the resonator's resonance frequency $f_r = \omega_0 / (2\pi)$ and quality factor Q which are determined from its frequency response, see [6]. This model has been successfully applied for several types of resonant viscosity and mass density sensors found in literature and sensors of our own work, see [7] and reads

$$\frac{1}{\omega_0^2} = \frac{m}{k_0} \approx \frac{1}{\omega_{0m}} = m_{0k} + m_{\rho k} \rho + m_{\eta \rho k}^* \sqrt{\eta \rho}, \quad \text{and} \quad (1)$$

$$\frac{1}{Q} = \frac{c}{\sqrt{k_0 m}} \approx \frac{1}{Q_m} = \frac{c_{0k} + c_{\eta k} \eta + c_{\eta \rho k}^* \sqrt{\eta \rho}}{\sqrt{m_{0k} + m_{\rho k} \rho + m_{\eta \rho k}^* \sqrt{\eta \rho}}},$$

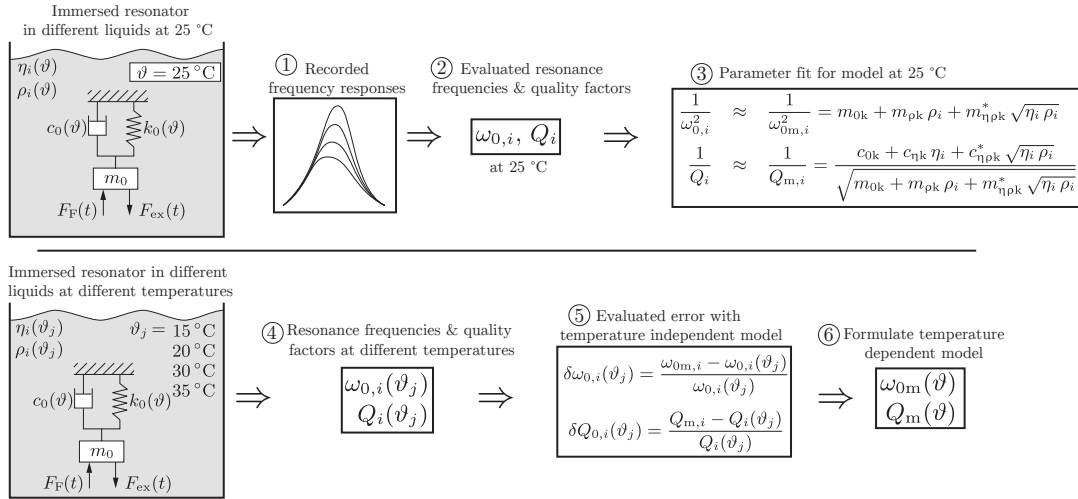


Figure 2: Modeling approach: The frequency responses of a mechanical resonator are recorded upon immersion in liquids (denoted with the index i) with viscosity η_i and mass density ρ_i at 25 °C. The angular resonance frequencies $\omega_{0,i}$ and quality factors Q_i are determined fitting the frequency response of a second order resonator into the recorded frequency response. $\omega_{0,i}$ and Q_i from at least three liquids are necessary to identify the parameters of a temperature-independent model relating ω_0 and Q to η and ρ . The evaluation of relative deviations $\delta\omega_{0m,i}(\theta_j)$ and $\delta Q_{m,i}(\theta_j)$ of the modeled temperature-independent quantities $\omega_{0m,i}$ and $Q_{m,i}$ from measured quantities $\omega_{0,i}(\theta_j)$ and $Q_i(\theta_j)$ at temperatures θ_j reveals the failing of the temperature-independent model. With the knowledge of this error, a temperature dependence can be introduced in the existing model.

where m_{0k} , $m_{\rho k}$, $m_{\eta\rho k}^*$, c_{0k} , $c_{\eta k}$ and $c_{\eta\rho k}^*$ are parameters, which can be determined through a fit with at least three different resonance frequencies and quality factors which have been evaluated from recorded frequency responses in different liquids. Measurements in ethanol, isopropanol, DI-water and a mixture of 61 % glycerol in DI-water at 25 °C were performed to determine these parameters with a linear least square fit, see also Fig. 2. The viscosities $\eta_i(\theta_j)$ and mass densities $\rho_i(\theta_j)$ of the aforementioned liquids were evaluated with an Anton Paar SVM 3000 viscometer at temperatures θ_j of 15, 20, 25, 30 and 35 °C. With the knowledge of the temperature dependent viscosities $\eta_i(\theta_j)$ and mass densities $\rho_i(\theta_j)$, resonance frequencies $\omega_{0m,i}$ and quality factors $Q_{m,i}$ were calculated using the identified model, see Eq. 1. With these parameters (resulting from the temperature-independent model) the relative deviations $\delta\omega_{0,i}(\theta_j)$ and $\delta Q_i(\theta_j)$ from measured resonance frequencies $\omega_{0,i}(\theta_j)$ and quality factors $Q_i(\theta_j)$ were evaluated for every liquid at each temperature and are depicted in Fig. 3(a).

2.2. Temperature-dependent model

The evaluation of δf_r depicted in Fig. 3(a) suggests that the influence of θ on f_r is (almost) linear and independent of the liquid's viscosity or mass density. This behavior is considered by an additive term $m_{0k\theta} \cdot (\theta - 25^\circ\text{C})$, where $m_{0k\theta}$ is a parameter which is fitted such that the relative deviation δf_r reaches a minimum. (Note: the notation of the factor $m_{0k\theta}$ might be misleading as it suggests an increased mass instead of an decreased spring constant. In fact, the result is the same although not intuitive.)

The interpretation of δQ shows that for low viscous liquids (ethanol) the temperature gradient on δQ is negative, whereas it is positive in case of higher viscous liquids (61 % Glycerol in DI-water). The value for δQ for water at 35 °C is considered as an outlier and thus, is not respected for interpretation. To model this behavior, two terms $c_{0k\theta} \cdot (\theta - 25^\circ\text{C})$ and $c_{\eta k\theta} \cdot (\theta - 25^\circ\text{C}) \cdot \eta$ are added in the the expression for Q in Eq. 1. Thus the equations for the temperature dependent model read:

$$\begin{aligned} \frac{1}{\omega_0^2(\theta)} &= \frac{m}{k_0(\theta)} \approx m_{0k} + m_{0k\theta}(\theta - 25^\circ\text{C}) + m_{\rho k} \rho + m_{\eta\rho k}^* \sqrt{\eta\rho}, \quad \text{and} \\ \frac{1}{Q(\theta)} &= \frac{c(\theta)}{\sqrt{k_0(\theta)m}} \approx \frac{c_{0k} + c_{0k\theta}(\theta - 25^\circ\text{C}) + (c_{\eta k} + c_{\eta k\theta}(\theta - 25^\circ\text{C}))\eta + c_{\eta\rho k}^* \sqrt{\eta\rho}}{\sqrt{m_{0k} + m_{0k\theta}(\theta - 25^\circ\text{C}) + m_{\rho k} \rho + m_{\eta\rho k}^* \sqrt{\eta\rho}}}. \end{aligned} \quad (2)$$

The relative deviations of temperature dependent, modeled results from the measured values are depicted in Fig. 3(b).

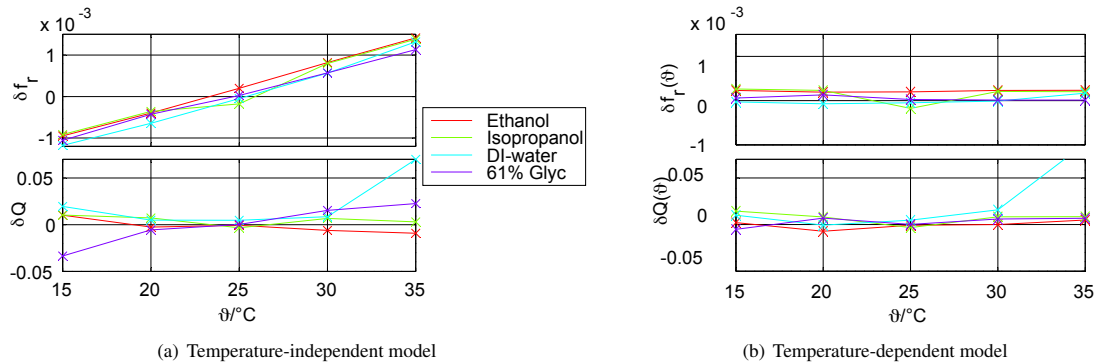


Figure 3: Relative deviation δQ and δf_r between measured and modeled results. The value for water at 35°C is considered to be an outlier. a) deviation between measured values and results calculated with the temperature-independent model. b) deviation between measured values and results calculated with the temperature-dependent model.

In both cases (i.e. for δf_r and δQ) the extension of the equations by the aforementioned temperature terms shows a clear improvement of the model (except for the outlier). The main advantage of such a model which can be used for many resonant mass density and viscosity sensors is that it allows the estimation of achievable sensitivities, and comparability of different sensor designs but also maximum achievable accuracies. The inaccuracy in the temperature measurement (which is present in any case) directly limits the sensor's accuracy in viscosity and mass density. With this model this accuracy limit can be estimated.

3. Conclusion and Outlook

The parameters of a recently developed, reduced order model relating an immersed mechanical resonator's resonance frequency and quality factor to the liquid's mass density and viscosity were determined through measurements obtained in four different liquids at 25°C . With this identified model, quality factors and resonance frequencies were calculated for the liquids' viscosities and mass densities at 15, 20, 30 and 35°C . By evaluating the relative deviations of these calculated results from the measured values, a temperature dependence was empirically introduced in the model.

Regarding future work, further measurements have to be performed (in a larger temperature and viscosity range) to experimentally analyze these findings. Furthermore, a physically motivated model accounting for the cross-sensitivity to temperature has to be developed.

Acknowledgment

We are indebted to the Austrian COMET program (Austrian Centre of Competence in Mechatronics, ACCM) for the financial support.

References

- [1] B. Jakoby, R. Beigelbeck, F. Keplinger, F. Lucklum, A. Niedermayer, E. K. Reichel, C. Riesch, T. Voglhuber-Brunnmaier, B. Weiss, Miniaturized sensors for the viscosity and density of liquids – performance and issues, *IEEE transactions on ultrasonics, ferroelectrics, and frequency control* 57 (1) (2010) 111–120. doi:10.1109/TUFFC.2010.1386.
- [2] M. F. Khan, S. Schmid, P. E. Larsen, Z. J. Davis, W. Yan, E. H. Stenby, A. Boisen, Online measurement of mass density and viscosity of pl fluid samples with suspended microchannel resonator, *Sensors and Actuators B: Chemical* 185 (2013) 456–461.
- [3] H. S. Wasisto, S. Merzsch, A. Waag, E. Uhde, T. Salthammer, E. Peiner, Airborne engineered nanoparticle mass sensor based on a silicon resonant cantilever, *Sensors and Actuators B: Chemical* 180 (2013) 77–89.
- [4] R. Sandberg, W. Svendsen, K. Mølhave, A. Boisen, Temperature and pressure dependence of resonance in multi-layer microcantilevers, *Journal of Micromechanics and Microengineering* 15 (8) (2005) 1454.
- [5] C. J. Seaton, Viscosity-temperature correlation for liquids, in: *STLE/ASME 2006 International Joint Tribology Conference*, American Society of Mechanical Engineers, 2006, pp. 131–142.
- [6] M. Heinisch, T. Voglhuber-Brunnmaier, E. K. Reichel, I. Dufour, B. Jakoby, Reduced order models for resonant viscosity and mass density sensors, Submitted at *Sens. Actuators A: Phys.*
- [7] M. Heinisch, T. Voglhuber-Brunnmaier, I. Dufour, B. Jakoby, Validity of describing resonant viscosity and mass density sensors by linear 2nd order resonators, *These Proceedings*.