Renormalisation group improved leptogenesis in family symmetry models

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Abstract

We study renormalisation group (RG) corrections relevant for leptogenesis in the case of family symmetry models such as the Altarelli–Feruglio A_4 model of tri-bimaximal lepton mixing or its extension to tri-maximal mixing. Such corrections are particularly relevant since in large classes of family symmetry models, to leading order, the CP violating parameters of leptogenesis would be identically zero at the family symmetry breaking scale, due to the form dominance property. We find that RG corrections violate form dominance and enable such models to yield viable leptogenesis at the scale of right-handed neutrino masses. More generally, the results of this paper show that RG corrections to leptogenesis cannot be ignored for any family symmetry model involving sizeable neutrino and τ Yukawa couplings.
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1. Introduction

One of the most important and well-studied questions in particle physics is why the observable Universe has a tiny but non-zero ratio of baryons to photons without which there would be no stars, planets or life. The measurement of cosmic microwave background (CMB) anisotropies and the successful prediction of light element abundances from big bang nucleosynthesis (BBN), both lead to a consistent value of this ratio at the recombination time when atoms are formed [1],

$$\eta = \frac{n_B}{n_\gamma} \approx 6.2 \times 10^{-10}, \tag{1}$$

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where n_B and n_γ are baryon and photon number densities respectively.[1] Any theory which successfully produces such a baryon asymmetry must fulfil the famous Sakharov conditions [2] of C and CP violation, B violation and departure from thermal equilibrium. One of the most popular of these is known as leptogenesis [3], which takes advantage of the fact that non-perturbative, $B - L$ conserving, $B + L$ violating sphaleron processes can convert a lepton number asymmetry into a B asymmetry. The lepton number asymmetry is obtained from the decays of heavy Majorana neutrinos and so leptogenesis is intimately linked to neutrino mass, mixing and CP violation.

The discovery of neutrino mass and mixing is arguably one of the most influential observations in particle physics in the last 15 years. It has inspired a large number of works aimed at explaining both the extremely small mass and, in particular, the striking mixing pattern (which is very different from the close to diagonal quark sector) [4–21]. Seesaw mechanisms provide the most common explanation for small neutrino masses; a heavy particle is introduced which has a Yukawa coupling with the lepton doublet. When this particle is integrated out, the effective theory has a Majorana mass term for the left-handed neutrinos which is suppressed by the large mass of the new particle. This new particle must be a colour singlet but can be a weak fermionic singlet with zero hypercharge (type I) [22], a weak scalar triplet with two units of hypercharge (type II) [23] or a weak fermionic triplet with zero hypercharge (type III) [24]. The issue of neutrino mixing is also very well studied, and a popular technique is to postulate the existence of an extra family symmetry at high energies. This symmetry is then broken in a specific way, by the vacuum expectation value (VEV) of heavy scalars called flavons. The remnants of the breaking pattern show up in the observed mixing of neutrinos at low energies.

Many of these models of neutrino mixing (predominantly employing the type I seesaw) exhibit a property known as form dominance (FD) [25], defined by the condition that the columns of the neutrino Yukawa matrix are proportional to columns of the mixing matrix in a particular basis corresponding to diagonal charged lepton and right-handed neutrino mass matrices. As discussed in several papers [26–32], models with family symmetry typically predict vanishing CP violating lepton asymmetry parameters ϵ and hence zero leptogenesis.[2] As pointed out in [26], this can be understood very simply from the FD property that the columns of the neutrino Yukawa matrix are mutually orthogonal since they are proportional to the columns of the mixing matrix which is unitary.[3] However in family symmetry models the Yukawa matrices are predicted at the scale of family symmetry breaking, which may be close to the grand unified theory (GUT) scale, and above the mass scale of right-handed neutrinos. Therefore in such models the Yukawa matrix will be subject to renormalisation group (RG) running from the family symmetry breaking scale down to the scale of right-handed neutrino masses relevant for leptogenesis. To illustrate the effects of RG corrections, we analyse two specific models involving sizeable neutrino and τ Yukawa couplings and satisfying FD at leading order (LO): the first model [7] reproduces the well-studied tri-bimaximal (TB) mixing pattern [34]; and the second model [35] reproduces the tri-maximal (TM) mixing pattern [36] consistent with the results from T2K [37]. Although in both models RG running occurs over only one or two orders of magnitude in the energy scale, we shall show that this leads to sufficient violation of FD to allow successful leptogenesis in each case.

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[1] Corresponding to a portion of comoving volume containing 1 photon at temperatures where the right-handed neutrinos are relativistic.

[2] For a discussion of how to achieve leptogenesis in the flavour symmetric phase, see e.g. [33].

[3] The vanishing of leptogenesis due to the orthogonality of the columns of the neutrino Yukawa matrix was first observed in the case of hierarchical neutrinos and constrained sequential dominance with tri-bimaximal mixing in [27] and was subsequently generalised to the case of FD with any neutrino mass pattern and any mixing pattern in [26].

One could ask why RG effects should be considered when higher order (HO) operators in the A_4 model have been shown to produce a realistic value of η [29]. The answer is that RG effects turn out to be of equal importance to HO operators in determining leptogenesis and so in general both effects should be considered together. Here we choose to drop the effect of HO operators for clarity: we want to study the effects of RG corrections to leptogenesis in isolation in order to illustrate the magnitude of the effect. Moreover, there are ultraviolet completions of the A_4 model of both TB [38] and TM mixing [35] in which HO operators play a negligible role, and the viability of leptogenesis in such cases then relies exclusively on the effects of RG corrections considered here.

We emphasise that this paper represents the first study which takes into account RG corrections to leptogenesis in non-Abelian discrete family symmetry models with FD present. The results in this paper show that RG corrections have a large impact on leptogenesis in any family symmetry models involving neutrino Yukawa couplings of order unity. Therefore, when considering leptogenesis in such models, RG corrections should not be ignored even when corrections arising from HO operators are also present. It should be pointed out that the phrases "RG effect" and "RG corrections" are taken to mean those between the family symmetry scale and the leptogenesis scale, and to those which help to generate a non-zero η. RG effects in evolving parameters from the leptogenesis scale to the electroweak (EW) scale are well studied (e.g. in [39] or [40]) and are a generic consideration for all models which explain neutrino mixings using a family symmetry broken at high energies. Furthermore, in the A_4 models considered here, such effects are expected to be small.

The rest of the paper is organised as follows. Section 2 briefly outlines the process of calculating the baryon asymmetry of the universe η arising from leptogenesis. Then in Section 3, the idea of FD is introduced and it is shown that the CP violating parameter in leptogenesis is indeed zero under the condition of FD. Section 4 introduces the Altarelli–Feruglio A_4 model of TB neutrino mixing, while Section 5 introduces the parameters of the A_4 model of TM mixing. In Section 6 we analytically estimate the RG running of the neutrino Yukawa matrices in leading log approximation. Numerical results for the baryon asymmetry of the universe arising from leptogenesis in both TB and TM models are presented in Section 7 including contour plots of input parameters reproducing the physical value of η. Section 8 concludes the paper.

2. Leptogenesis

Leptogenesis takes advantage of the heavy right-handed neutrinos introduced in many models to account for the smallness of the left-handed neutrino mass. The addition of these right-handed neutrino fields N_i introduces two new terms into the superpotential

$$W_\nu = (Y_\nu)_{\alpha i}(l_\alpha \cdot H_u)N_i + \frac{1}{2}N_i(M_{RR})_{ij}N_j, \tag{2}$$

which then lead to an effective light neutrino mass once the heavy degrees of freedom are integrated out. In (2), N_i are the heavy right-handed neutrinos with Majorana mass matrix M_{RR}; l_α are the lepton doublets and H_u the (hypercharge $+1/2$) Higgs doublet, which interact with N_i through the Yukawa couplings $(Y_\nu)_{\alpha i}$. These interactions also fulfil the well-known Sakharov conditions [2] required to generate a baryon asymmetry: (1) C and CP violation (coming from the complex Yukawa coupling); (2) B violation (the Majorana mass of Ns violates L; sphalerons convert $\sim \frac{1}{3}$ of this into B violation); (3) Departure from thermal equilibrium (due to out-of-equilibrium decays of the right-handed neutrinos). The procedure for calculation of
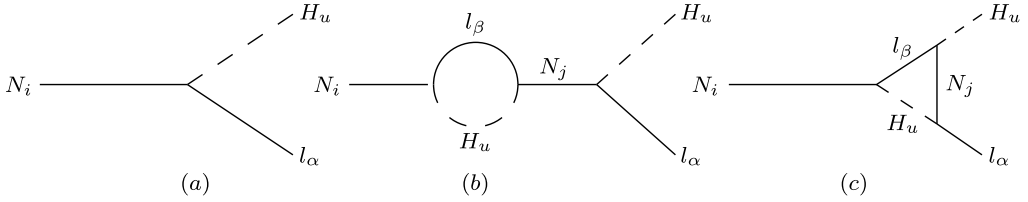
Fig. 1. Diagrams contributing to the CP violating parameter $\epsilon_{i,\alpha i}$; it is the interference of (a) with (b) and (c) which gives rise to non-zero $\epsilon_{i,\alpha i}$.

this asymmetry is first to calculate the amount of CP violation in the decays of the right-handed neutrinos. This is then used as an input parameter to find the $B - L$ asymmetry through integration of the Boltzmann equations [41]. These equations take into account the evolution of a $B - L$ asymmetry generated by N decays against the background of N inverse decays partially washing it out. Finally, this $B - L$ asymmetry is converted into a B asymmetry using previously calculated results for sphaleron processes [42,43].

2.1. Unflavoured asymmetry

To one-loop order, the CP asymmetry arises from the interference of the diagrams in Fig. 1. Using the standard supersymmetric Feynman rules, one can calculate the decay widths for the decay $N_i \to l_\alpha + H_u$, $\Gamma_i = \sum_\alpha \Gamma_{\alpha i}$; these are then used to find the CP asymmetry for N_i by summing over all lepton flavours α [44],

$$\epsilon_i = \frac{\Gamma_i - \overline{\Gamma}_i}{\Gamma_i + \overline{\Gamma}_i} = \frac{1}{8\pi (Y_\nu^\dagger Y_\nu)_{ii}} \sum_{j \neq i} \mathrm{Im}\left((Y_\nu^\dagger Y_\nu)_{ij}^2\right) f\left(\frac{M_j^2}{M_i^2}\right). \tag{3}$$

Here, M_i are the real mass eigenvalues of M_{RR}, and [26,27,45]

$$f(x_{ij}) = f_{ij} = \sqrt{x_{ij}}\left(\frac{2}{1 - x_{ij}} - \ln\left(\frac{1 + x_{ij}}{x_{ij}}\right)\right), \tag{4}$$

with $x_{ij} = \frac{M_j^2}{M_i^2}$, is the loop factor. Note that ϵ_i is summed over all flavours of the outgoing lepton and is called the *unflavoured* asymmetry. This formula implicitly assumes that the N_i are not degenerate (since this would lead to an infinite self-energy contribution unless one considers resonance effects); for studies of leptogenesis with nearly degenerate neutrinos, see e.g. [46] or, in the context of Abelian family symmetries, [47].

2.2. Flavoured asymmetry

The above discussion and formula for ϵ_i is relevant when the lepton doublets produced are a coherent superposition of the three flavours. This is only the case above a certain energy when the expansion rate of the universe is greater than all charged lepton interaction rates. However, as the universe cools, the τ lepton Yukawa coupling will start to come in to equilibrium at an energy of around [27] $(1 + \tan^2 \beta) \times 10^{12}$ GeV,[4] breaking the coherence of the single state superposition $e + \mu + \tau$ down into two states: the τ and the remaining coherent combination $e + \mu$. Thus, if

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[4] $\tan \beta$ here is the ratio of MSSM Higgs VEVs, and will be defined algebraically in Section 4.

the dynamics of leptogenesis occur below this temperature,[5] one should take such differences into account in the calculations. The CP parameter taking into account such flavour effects is [26,27,45]

$$\epsilon_{\alpha i} = \frac{1}{8\pi (Y_\nu^\dagger Y_\nu)_{ii}} \sum_{j \neq 1} \left(\text{Im}\left(Y_{\alpha i}^* Y_{\alpha j} (Y_\nu^\dagger Y_\nu)_{ij}\right) f(x_{ij}) + \text{Im}\left(Y_{\alpha i}^* Y_{\alpha j} (Y_\nu^\dagger Y_\nu)_{ji}\right) g(x_{ij}) \right), \qquad (5)$$

with $g(x_{ij}) = g_{ij} = \frac{1}{(1-x_{ij})}$ and f_{ij} as above.

2.3. Final asymmetry

Ultimately we want to estimate a value for the baryon to photon ratio at recombination; this is related to the $B - L$ asymmetry N_{B-L} at the leptogenesis scale by [49]

$$\eta = 0.89 \times 10^{-2} N_{B-L}. \qquad (6)$$

The numerical coefficient above has two contributions: (1) from the $B - L$ conserving sphaleron processes (which are only $\sim 33\%$ efficient at converting $B - L$ into B); (2) from scaling by photon number density in the relevant comoving volume (recall that we are calculating the baryon to photon ratio at recombination). The sphalerons convert part of the L asymmetry into a B asymmetry via a suppressed dimension 18 operator active at the energies we consider, $\gg M_{EW}$. The CP asymmetries calculated in the previous section are then related to N_{B-L} via

$$N_{B-L} = \sum_{\alpha, i} \epsilon_{\alpha i} \kappa_{\alpha i}, \qquad (7)$$

which defines the efficiency parameters $\kappa_{\alpha i}$; these encode how efficiently the decays of N produce a $B - L$ asymmetry at the leptogenesis scale. In the strong washout regime, the $\kappa_{\alpha i}$ are approximated analytically by (up to superpartner effects which increase N_{B-L} by a factor of $\sqrt{2}$; see, for example, [49]):

$$\kappa_{\alpha i} \approx \frac{2}{K_{\alpha i} z_B(K_{\alpha i})} \left(1 - \exp\left(-\frac{1}{2} K_{\alpha i} z_B(K_{\alpha i}) \right) \right), \qquad (8)$$

with

$$z_B(K_{\alpha i}) \approx 2 + 4(K_{\alpha i})^{0.13} \exp\left(-\frac{2.5}{K_{\alpha i}} \right), \qquad (9)$$

the decay parameter

$$K_{\alpha i} = \frac{\widetilde{m}_{\alpha i}}{m_{MSSM}^*}, \qquad (10)$$

and effective neutrino mass

$$\widetilde{m}_{\alpha i} = (Y_\nu^\dagger)_{i\alpha} (Y_\nu)_{\alpha i} \frac{v_u^2}{M_i}. \qquad (11)$$

The $\widetilde{m}_{\alpha i}$ are model specific and are presented below for the models in question (see Section 7), while $m_{MSSM}^* = 1.58 \times 10^{-3} \sin^2 \beta$ eV [27] is the equilibrium neutrino mass. The up-type Higgs

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[5] Strictly speaking the τ interaction rate must be faster than the N inverse decay rate to overcome the Quantum Zeno effect [48], but this is a small effect and beyond the scope of this paper.

VEV is denoted by v_u. The main point to address is then the form that the Yukawa matrices take. This is discussed in the context of family symmetries [4–21] which we turn to now.

3. Form dominance

In order to explain the observed pattern of neutrino mixing, many models invoke the idea that a high energy family symmetry unifying the three flavours is spontaneously broken in a specific way that leaves some imprint in the neutrino sector at low energies. This method introduces relationships between the parameters of Y_ν leading to predictions for $\epsilon_{\alpha i}$ and ϵ_i. As discussed in the introduction, it is a striking fact that many of these family symmetry models exhibit a property known as FD [25], which constrains the CP violating parameter of leptogenesis to be identically zero [26], as we now discuss.

The FD [25] condition is that the columns of Y_ν in Eq. (2) are proportional to the columns of U,

$$A_i = \alpha U_{i1}, \qquad B_i = \beta U_{i2}, \qquad C_i = \gamma U_{i3}, \tag{12}$$

where U is the unitary Pontecorvo–Maki–Nakagawa–Sakata (PMNS) matrix which is parameterised by three mixing angles and three complex phases (one Dirac and two Majorana). The consequences of such FD on leptogenesis is then very simple to understand: since U is unitary, the columns of Y_ν must be mutually orthogonal. This means that the contraction $(Y_\nu^\dagger Y_\nu)_{ij}$, with $i \neq j$, appearing in Eqs. (3) and (5) are identically zero and so leptogenesis gives $\eta = 0$.

The FD condition also greatly simplifies the form of the effective neutrino mass matrix arising from the type I seesaw formula. In terms of parameters in Eq. (2), the effective neutrino mass matrix can be written,

$$m_\nu = -v_u^2 Y_\nu M_{RR}^{-1} Y_\nu^T. \tag{13}$$

In the basis where the right-handed neutrinos are diagonal, i.e. that in which $M_{RR} = \text{diag}(M_A, M_B, M_C)$, and writing $Y_\nu = (A, B, C)$, Eq. (13) gives

$$m_\nu = -v_u^2 \left(\frac{AA^T}{M_A} + \frac{BB^T}{M_B} + \frac{CC^T}{M_C} \right). \tag{14}$$

In the charged lepton diagonal basis, m_ν is diagonalised by U. Assuming FD, m_ν is diagonalisable independently of the parameters α, β, γ, and, from (14) and (12), one finds

$$m_\nu^{diag} = v_u^2 \, \text{diag} \left(\frac{\alpha^2}{M_A}, \frac{\beta^2}{M_B}, \frac{\gamma^2}{M_C} \right). \tag{15}$$

A particularly well-studied case is that of TB mixing [34]. However, as emphasised in [32], TB mixing is not linked to FD. Indeed we shall study two A_4 family symmetry models, one with TB mixing and one with TM mixing, where FD is present in both cases, leading to zero leptogenesis at LO, before RG corrections are included.

4. Parameters of the A_4 model of TB mixing

We first consider the parameters of the Altarelli–Feruglio A_4 model of TB mixing with renormalisable neutrino Dirac coupling [7],

$$W_\nu = y(lN)H_u + (x_A \xi + \widetilde{x}_A \widetilde{\xi})(NN) + x_B(\varphi_S NN), \tag{16}$$

where, under A_4, the conventionally defined fields transform as $N \sim \mathbf{3}$, $l \sim \mathbf{3}$, $\varphi_S \sim \mathbf{3}$, while $H_u \sim \mathbf{1}$ and $\xi, \widetilde{\xi} \sim \mathbf{1}$; the x_i are constant complex parameters. Since this model is well known, we refer the reader to [7] for more details. The charged lepton mass matrix in the basis used in [7] is diagonal so the mixing structure in the neutrino sector will not receive corrections from charged lepton rotations. The TB structure in the neutrino sector arises from the flavon fields obtaining vacuum expectation values (VEVs) in particular directions,

$$\langle \varphi_S \rangle = v_s \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \qquad \langle \xi \rangle = u \quad \text{and} \quad \langle \widetilde{\xi} \rangle = 0, \tag{17}$$

where the dynamics responsible for vacuum alignment has been extensively studied (for instance, in [7] for F-term alignment or in [12] for D-term alignment).

The TB structure arises in the Majorana sector of Eq. (16), explicitly

$$M_{RR} = \begin{pmatrix} a + \frac{2b}{3} & -\frac{b}{3} & -\frac{b}{3} \\ -\frac{b}{3} & \frac{2b}{3} & a - \frac{b}{3} \\ -\frac{b}{3} & a - \frac{b}{3} & \frac{2b}{3} \end{pmatrix}, \tag{18}$$

where we define $a = 2x_A u$, $b = 2x_B v_s$ as complex parameters with phase $\phi_{a,b}$. For the purposes of leptogenesis it is convenient to rotate the N such that their mass matrix is diagonal. The resulting neutrino Yukawa matrix in the diagonal N basis is then,

$$Y_{TB} = y \begin{pmatrix} \frac{-2}{\sqrt{6}} e^{i\phi_A} & \frac{1}{\sqrt{3}} e^{i\phi_B} & 0 \\ \frac{1}{\sqrt{6}} e^{i\phi_A} & \frac{1}{\sqrt{3}} e^{i\phi_B} & \frac{-1}{\sqrt{2}} e^{i\phi_C} \\ \frac{1}{\sqrt{6}} e^{i\phi_A} & \frac{1}{\sqrt{3}} e^{i\phi_B} & \frac{1}{\sqrt{2}} e^{i\phi_C} \end{pmatrix}. \tag{19}$$

One can see explicitly that FD is present in this model, since the columns of Y_{TB} are manifestly proportional to the columns of the TB mixing matrix, and thus it immediately follows that $\epsilon_i = \epsilon_{\alpha i} = 0$ at the scale of A_4 breaking. The phases defined in (19) are given as,

$$\phi_A = -\frac{1}{2}\left(\phi_b + \tan^{-1}\left(\frac{-|a|\sin(\phi_b - \phi_a)}{|b| + |a|\cos(\phi_b - \phi_a)}\right)\right), \tag{20}$$

$$\phi_B = -\frac{1}{2}\phi_a, \tag{21}$$

$$\phi_C = -\frac{1}{2}\left(\phi_b + \tan^{-1}\left(\frac{|a|\sin(\phi_b - \phi_a)}{|b| - |a|\cos(\phi_b - \phi_a)}\right)\right). \tag{22}$$

Therefore, there are actually only two phases (ϕ_a and ϕ_b) and two magnitudes ($|a|$ and $|b|$) in the model, although only phase differences appear when considering physical quantities. This means that we may set one phase to zero without loss of generality; for our calculations we choose $\phi_a = 0$.

In this basis, the Majorana neutrino mass matrix is real and diagonal and is given by

$$M_{RR}^{diag} = \text{diag}(M_1, M_2, M_3) = \begin{pmatrix} |a + b| & 0 & 0 \\ 0 & |a| & 0 \\ 0 & 0 & |-a + b| \end{pmatrix}. \tag{23}$$

The effective left-handed neutrino masses are then given by[6]

$$m_i = \frac{y_\beta^2 v^2}{M_i},$$ (24)

which incorporates the SUSY parameter $\tan\beta$

$$y_\beta = y\sin\beta, \qquad \tan\beta = \frac{v_u}{v_d}, \qquad v = \sqrt{v_u^2 + v_d^2} = \sqrt{\langle H_u\rangle^2 + \langle H_d\rangle^2} \approx 174\,\text{GeV}.$$ (25)

5. Parameters of the A_4 model of TM mixing

In light of results from T2K [37] indicating a sizeable reactor angle, models predicting TB mixing can potentially be ruled out. Instead, schemes such as TM mixing remain viable [36]:

$$U_{TM} = \begin{pmatrix} \frac{2}{\sqrt{6}}\cos\theta & \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{6}}\sin\theta e^{i\rho} \\ -\frac{1}{\sqrt{6}}\cos\theta - \frac{1}{\sqrt{2}}\sin\theta e^{-i\rho} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}}\cos\theta - \frac{1}{\sqrt{6}}\sin\theta e^{i\rho} \\ -\frac{1}{\sqrt{6}}\cos\theta + \frac{1}{\sqrt{2}}\sin\theta e^{-i\rho} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}}\cos\theta - \frac{1}{\sqrt{6}}\sin\theta e^{i\rho} \end{pmatrix}.$$ (26)

Here $\frac{2}{\sqrt{6}}\sin\theta = \sin\theta_{13}$ and ρ is related to the Dirac phase. It is possible to minimally extend the Altarelli–Feruglio model above by adding a flavon in the $\mathbf{1'}$ representation of A_4 which reproduces this pattern [35]:

$$W_{\mathbf{1'}} = x_C\xi'NN,$$ (27)

where we define the complex parameter $c = x_C\langle\xi'\rangle$, with phase ϕ_c. It has been shown in [35] that the addition of this flavon doesn't affect the right-handed neutrino masses to first order, and so the parameters in common with the previous section will be unaffected. Analogously to Eq. (19), in the basis where charged leptons are diagonal and right-handed neutrinos are real and diagonal, the Yukawa matrix for TM mixing is,

$$Y_{TM} = y\begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{6}}\alpha_{13}^* \\ -\frac{1}{\sqrt{6}} - \frac{1}{\sqrt{2}}\alpha_{13} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{6}}\alpha_{13}^* \\ -\frac{1}{\sqrt{6}} + \frac{1}{\sqrt{2}}\alpha_{13} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{6}}\alpha_{13}^* \end{pmatrix}\begin{pmatrix} \exp(i\phi_A) & 0 & 0 \\ 0 & \exp(i\phi_B) & 0 \\ 0 & 0 & \exp(i\phi_C) \end{pmatrix},$$ (28)

where the $\phi_{A,B,C}$ are as in Eq. (20). We can see that the columns of this matrix are proportional to columns of U_{TM} and therefore the model respects FD. Therefore, as for the previous model of TB mixing, this model of TM mixing also gives zero leptogenesis and $\eta = 0$, to leading order. The parameter α_{13} measures the deviation from TB mixing and is given by [35]

$$\alpha_{13} = \frac{\sqrt{3}}{2}\left(\text{Re}\,\frac{c}{2(a - \frac{c}{2})} + \text{Im}\,\frac{c}{2(a - \frac{c}{2})}\frac{\text{Im}\,\frac{b}{a - \frac{c}{2}}}{\text{Re}\,\frac{b}{a - \frac{c}{2}}} - i\,\frac{\text{Im}\,\frac{c}{2(a - \frac{c}{2})}}{\text{Re}\,\frac{b}{a - \frac{c}{2}}}\right).$$ (29)

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[6] Note that in this paper we consider a normal ordering of light neutrino masses, therefore M_1 is the heaviest right-handed neutrino mass. This means that ϵ_3 and $\epsilon_{\alpha3}$ will be dominant contributions to leptogenesis, coming from the lightest right-handed neutrino. This is simply a notational consideration, and does not affect the physics.

6. Renormalisation group evolution of the Yukawa couplings

In order to generate a non-zero $\epsilon_{\alpha i}$ and ϵ_i, we now consider the effects of running the neutrino Yukawa couplings from the scale at which A_4 is broken down to the scale at which leptogenesis takes place. At one-loop, the RG equation for the neutrino Yukawa couplings in the MSSM above the scale of right-handed neutrino masses is given by [39,50],[7]

$$\frac{dY_\nu}{dt} = \frac{1}{16\pi^2}\big[N_l \cdot Y_\nu + Y_\nu \cdot N_\nu + (N_{H_u})Y_\nu\big], \qquad (30)$$

where

$$N_l = Y_e Y_e^\dagger + Y_\nu Y_\nu^\dagger - \left(\frac{3}{2}g_2^2 + \frac{3}{10}g_1^2\right)\cdot I_3, \qquad (31)$$

$$N_\nu = 2Y_\nu^\dagger Y_\nu, \qquad (32)$$

$$N_{H_u} = 3\,\mathrm{Tr}\big(Y_u^\dagger Y_u\big) + \mathrm{Tr}\big(Y_\nu^\dagger Y_\nu\big) - \left(\frac{3}{2}g_2^2 + \frac{3}{10}g_1^2\right). \qquad (33)$$

In these equations, $t = \log(\frac{Q_1}{Q_0})$ with Q_1 being the renormalisation scale and Q_0 the family symmetry breaking scale; $Y_{e,u}$ are the charged lepton and up-type quark Yukawa couplings respectively; $g_{1,2}$ are the[8] $U(1)_Y$ and $SU(2)_L$ gauge couplings respectively; and I_3 is the 3×3 identity matrix. Each N_X arises from all one-loop insertions allowed by gauge symmetry on the X-leg of the vertex.

In leading log approximation, taking the continuous derivatives to be approximately equal to a single discrete step, Eq. (30) may be approximated as:

$$\frac{dY_\nu}{dt} \approx \frac{\Delta Y_\nu}{\Delta t} = \frac{Y_\nu(Q_0) - Y_\nu(Q_1)}{t(Q_0) - t(Q_1)} \equiv Z, \qquad (34)$$

yielding the solution,

$$Y_\nu(Q_1) \approx Y_\nu(Q_0) - Z\Delta t. \qquad (35)$$

As an example, we demonstrate the RG evolution of the TB Yukawa matrix in (19) $Y_\nu = Y_{TB}$ (the case of Y_{TM} is completely analogous). Inserting (19) into (30) and using the third family approximation then gives

$$\frac{dY_{TB}}{dt} \approx \frac{y}{16\pi^2}\left(\left(J + 3|y|^2\right)\begin{pmatrix} \frac{-2}{\sqrt{6}}e^{i\phi_A} & \frac{1}{\sqrt{3}}e^{i\phi_B} & 0 \\ \frac{1}{\sqrt{6}}e^{i\phi_A} & \frac{1}{\sqrt{3}}e^{i\phi_B} & \frac{-1}{\sqrt{2}}e^{i\phi_C} \\ \frac{1}{\sqrt{6}}e^{i\phi_A} & \frac{1}{\sqrt{3}}e^{i\phi_B} & \frac{1}{\sqrt{2}}e^{i\phi_C} \end{pmatrix}\right.$$
$$\left. + y_\tau^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{1}{\sqrt{6}}e^{i\phi_A} & \frac{1}{\sqrt{3}}e^{i\phi_B} & \frac{1}{\sqrt{2}}e^{i\phi_C} \end{pmatrix}\right) \equiv Z_{TB} \qquad (36)$$

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[7] Note that, as has been pointed out before, we do not consider running from the leptogenesis scale down to M_{EW} which would be necessary if one wanted to consider leptogenesis effects on neutrino mass bounds as studied in [40,51]. Also in [40] the importance of RG corrections in calculating leptogenesis predictions in the framework of a generic GUT scale theory was emphasised, although specific models were not considered.

[8] Note that g_1 is the GUT normalised hypercharge coupling, related to the standard hypercharge coupling g' by $g_1 = \sqrt{\frac{5}{3}}g'$.

Table 1
Flavoured asymmetries and washout parameters.

	Asymmetry	$\widetilde{m}_{\alpha i}$
$\epsilon_{\alpha 1}$	$\frac{1}{8\pi A^\dagger A}[\text{Im}(A_\alpha^* B_\alpha (A^\dagger B))f_{12} + \text{Im}(A_\alpha^* B_\alpha (B^\dagger A))g_{12} + \text{Im}(A_\alpha^* C_\alpha (A^\dagger C))f_{13} + \text{Im}(A_\alpha^* C_\alpha (C^\dagger A))g_{13}]$	$\frac{|A_\alpha|^2}{M_1}v_u^2$
$\epsilon_{\alpha 2}$	$\frac{1}{8\pi B^\dagger B}[\text{Im}(B_\alpha^* A_\alpha (B^\dagger A))f_{21} + \text{Im}(B_\alpha^* A_\alpha (A^\dagger B))g_{21} + \text{Im}(B_\alpha^* C_\alpha (B^\dagger C))f_{23} + \text{Im}(B_\alpha^* C_\alpha (C^\dagger B))g_{23}]$	$\frac{|B_\alpha|^2}{M_2}v_u^2$
$\epsilon_{\alpha 3}$	$\frac{1}{8\pi C^\dagger C}[\text{Im}(C_\alpha^* A_\alpha (C^\dagger A))f_{31} + \text{Im}(C_\alpha^* A_\alpha (A^\dagger C))g_{31} + \text{Im}(C_\alpha^* B_\alpha (C^\dagger B))f_{32} + \text{Im}(C_\alpha^* B_\alpha (B^\dagger C))g_{32}]$	$\frac{|C_\alpha|^2}{M_3}v_u^2$

where $J = N_{H_u} - (\frac{3}{2}g_2^2 + \frac{3}{10}g_1^2)$ and y_τ is the Yukawa coupling of the τ lepton. This shows that the contributions from the charged lepton Yukawa couplings breaks the orthogonality of the columns, appearing as they do in only the third component of each column. This is the effect which gives rise to a non-zero CP violating parameter. The leading log solution for the TB case is then given by

$$Y_{TB}(Q_1) \approx Y_{TB}(Q_0) - Z_{TB}\Delta t. \tag{37}$$

One must also consider how the charged lepton Yukawa coupling runs; the relevant RGE is [39,50]

$$\frac{\mathrm{d}Y_e}{\mathrm{d}t} = \frac{1}{16\pi^2}\big[N_l \cdot Y_e + Y_e \cdot N_e + (N_{H_d})Y_e\big], \tag{38}$$

with N_l as before and

$$N_e = 2Y_e^\dagger Y_e - \frac{6}{5}g_1^2 \cdot I_3, \tag{39}$$

$$N_{H_d} = 3\,\text{Tr}\big(Y_d^\dagger Y_d\big) + \text{Tr}\big(Y_e^\dagger Y_e\big) - \left(\frac{3}{2}g_2^2 + \frac{3}{10}g_1^2\right). \tag{40}$$

Here Y_d is the down quark Yukawa coupling matrix. Since Y_ν is unitary for both models, specifically see Eqs. (19) and (28), there will be no off-diagonal entries in Eq. (38). Using again the third family and leading log approximations gives small corrections to y_τ dependent upon y, y_b the bottom quark Yukawa and y_τ at the GUT scale; taking values of $y = 2\sqrt{\pi}$, $y_b = 1$ and $y_\tau = 0.5$ gives a correction of $\sim 10\%$ to the value of y_τ and therefore we neglect this effect (notice that the chosen values of y and y_τ are at the extreme end of the ranges we scan over and so this correction is the largest we expect).

7. Results

In this section we present the results of our analysis for both the TB and TM models in leading log approximation. The use of leading log approximation is justified by the small interval of energies over which the running takes place. Since we are using this approximation and since the neutrino hierarchy is not very strong (using work in [29] to fix neutrino parameters), we do not consider threshold effects from successive decoupling of the right-handed neutrinos. For a detailed analysis of such effects one can consult [39] or [40]. As before, one can represent the Yukawa matrix derived in (35) as $Y_\nu(Q_1) = (A(Q_1), B(Q_1), C(Q_1))$ where $A(Q_1)$, $B(Q_1)$ and $C(Q_1)$ are the RG evolved versions of the column vectors in Section 3, which, as clearly seen in (36), (37) are no longer orthogonal after RG corrections are included. This allows us to write the flavoured asymmetries as in Table 1. Using Eq. (5) one notices immediately that $\epsilon_{13} = 0$ since $C_1(Q_1) = 0$. We can see that the $\epsilon_{\alpha i}$ receive a correction from RG running since, e.g.
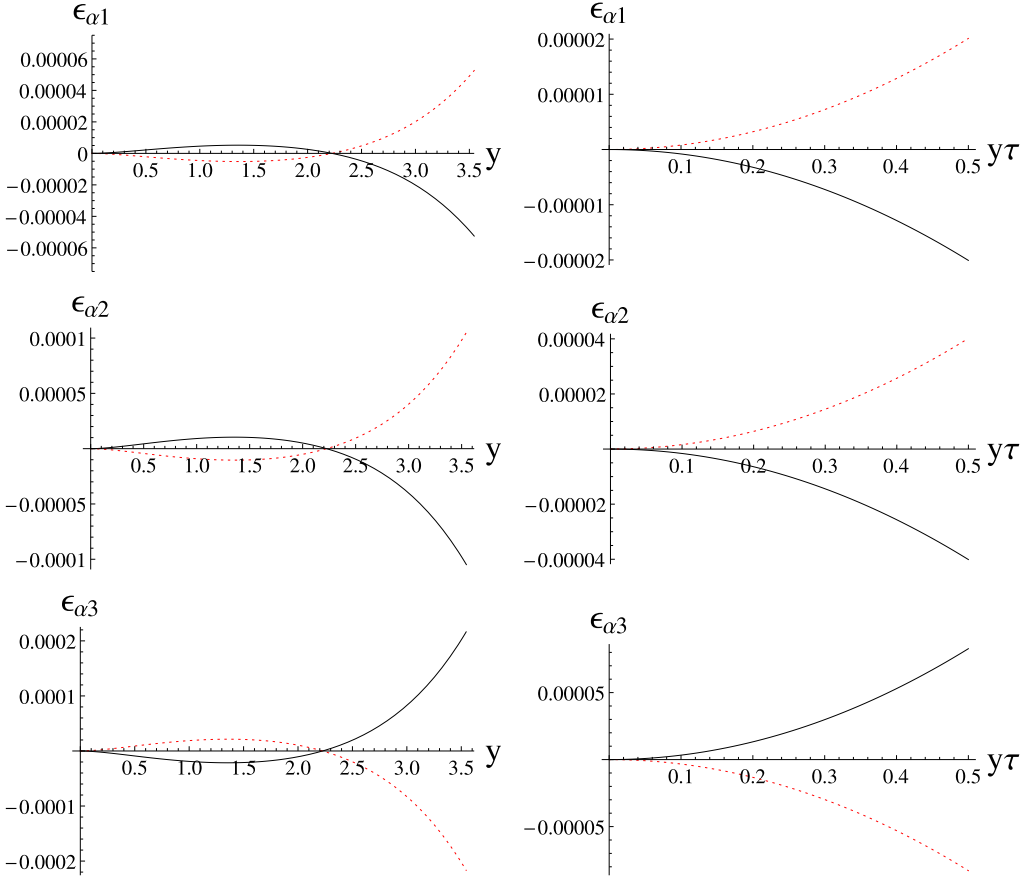
Fig. 2. Flavoured asymmetries plotted against neutrino Yukawa y and tau lepton Yukawa y_τ in the two-flavour regime (i.e. $e - \mu$ and τ) for the TB model. In the y_τ graphs, y is fixed to be 3, while in the y graphs, y_τ is fixed to be 0.5. $\epsilon_{e\mu,i}$ are black solid lines while $\epsilon_{\tau,i}$ are red dashed lines. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

$$A(Q_1)^\dagger B(Q_1) = \left(A(Q_0) - (Z\Delta t)_{\alpha 1}\right)^\dagger \left(B(Q_0) - (Z\Delta t)_{\alpha 2}\right), \tag{41}$$

where the leading term on the right-hand side vanishes since FD implies that $A(Q_0)$ and $B(Q_0)$ (and $C(Q_0)$) are orthogonal.

In order to progress further, one will need to insert specific values for the parameters in the matrix, which are model dependent. Here, we make use of work presented in [29] to fix the parameters consistently with experimental data. We take the leptogenesis scale Q_1 to be approximately the seesaw scale, $Q_1 \sim (1.74^2 y^2) \times 10^{14}$ GeV (using the basic seesaw formula).[9] This indicates that for small y we are in the two-flavour regime for $\tan\beta > 10$; for larger values of y,

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[9] This mass scale may look quite large especially when compared to the upper bound on the reheating temperature due to the over-production of late-decaying gravitinos [52]. However, heavy gravitinos with masses $m_{3/2} > 40$ TeV, will decay before nucleosynthesis. Assuming dark matter to have a significant axion/axino component, then allows reheat temperatures to be sufficiently high to produce right-handed neutrinos of mass $\sim 10^{14}$ GeV, as recently discussed in e.g. [53,54] (and references therein).
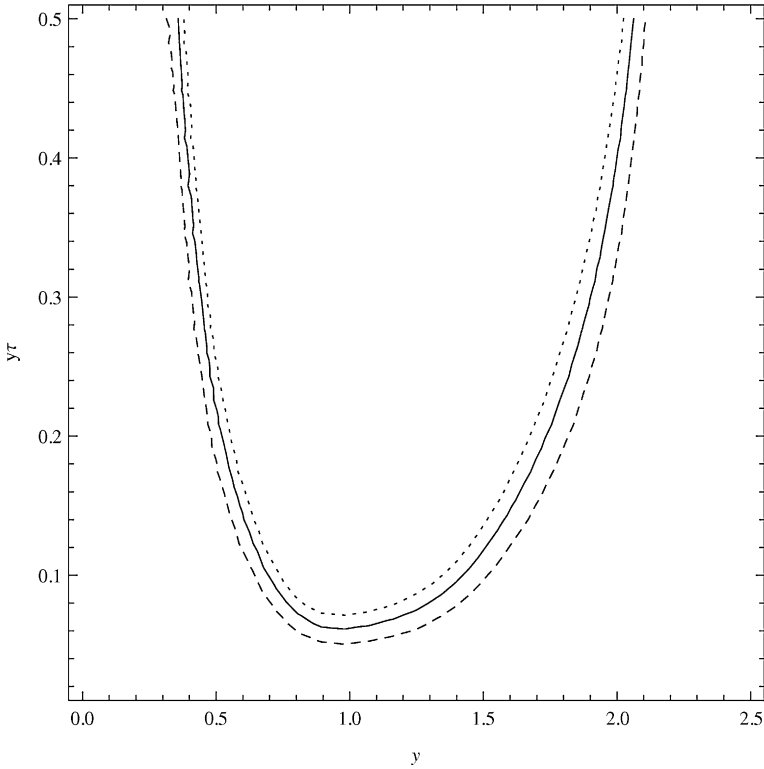
Fig. 3. A plot showing the contours of the baryon to photon ratio η in the tau Yukawa, y_τ, versus neutrino Yukawa, y, plane. The dotted and dashed lines are $\eta = 8.2 \times 10^{-10}$ and $\eta = 4.2 \times 10^{-10}$ while the solid line is the measured value of $\eta = 6.2 \times 10^{-10}$.

$\tan\beta$ needs to be larger for us to be in the two-flavour regime. However in the forthcoming plots, parts of the contour existing at large y correspond also to larger y_τ and so sufficiently large $\tan\beta$. The family symmetry scale is around an order of magnitude below the GUT scale, roughly $Q_0 \sim 1.5 \times 10^{15}$ GeV; and $y_t \sim 1$. We then calculate the asymmetry for $0 < y < 2\sqrt{\pi}$ (to keep the coupling perturbative) and $0 < y_\tau < 0.5$ (to remain within bounds for $\tan\beta$ [55]).

7.1. TB mixing

We now specialise to the case of RG improved leptogenesis in the TB model, where the TB Yukawa couplings are given in (37), repeated below,

$$Y_{TB}(Q_1) \approx Y_{TB}(Q_0) - Z_{TB}\Delta t. \tag{42}$$

The results for the flavoured asymmetries versus y and y_τ are presented in Fig. 2, in the two-flavour regime. It can be seen that the contributions from $\epsilon_{\alpha 3}$ are the dominant ones, as expected.

Following the procedure set out in Section 2.3, we then calculate the baryon to photon ratio η. Fig. 3 displays the contour matching the experimentally measured value of 6.2×10^{-10}, along with two others, demonstrating the sensitivity of the required Yukawa couplings to the value of η. This shows that there is a definite range of Yukawa couplings for which a realistic
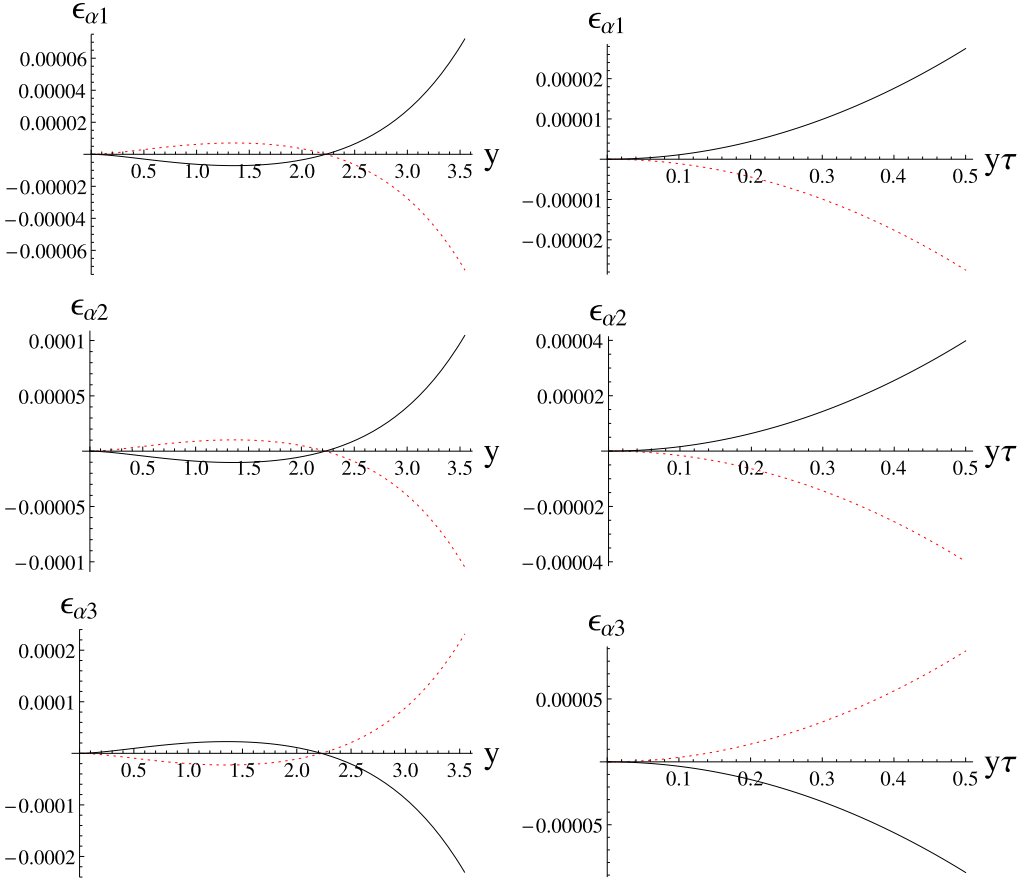
Fig. 4. Flavoured asymmetries plotted against neutrino Yukawa y and tau lepton Yukawa y_τ in the two-flavour regime (i.e. $e - \mu$ and τ) for the TM model with $\theta_{13} = 8°$ and a real parameter $c = x_C \langle \xi' \rangle$. In the y_τ graphs, y is fixed to be 3, while in the y graphs, y_τ is fixed to be 0.5. $\epsilon_{e\mu,i}$ are black solid lines while $\epsilon_{\tau,i}$ are red dashed lines. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

matter–antimatter asymmetry can be obtained purely by considering RG evolution of the neutrino Yukawa matrix, without the need for any extra particles or HO operators to be considered.

7.2. TM mixing

We now perform a similar analysis in the TM model, using the RG improved Yukawa matrix analogous to (37), namely,

$$Y_{TM}(Q_1) \approx Y_{TM}(Q_0) - Z_{TM}\Delta t, \tag{43}$$

where the high energy Yukawa matrix $Y_{TM}(Q_0)$ is given in (28), with Z_{TM} analogous to (36) and otherwise assuming similar parameters to the case of TB mixing. However one must choose the new complex parameter c carefully in order to satisfy the relation [35]

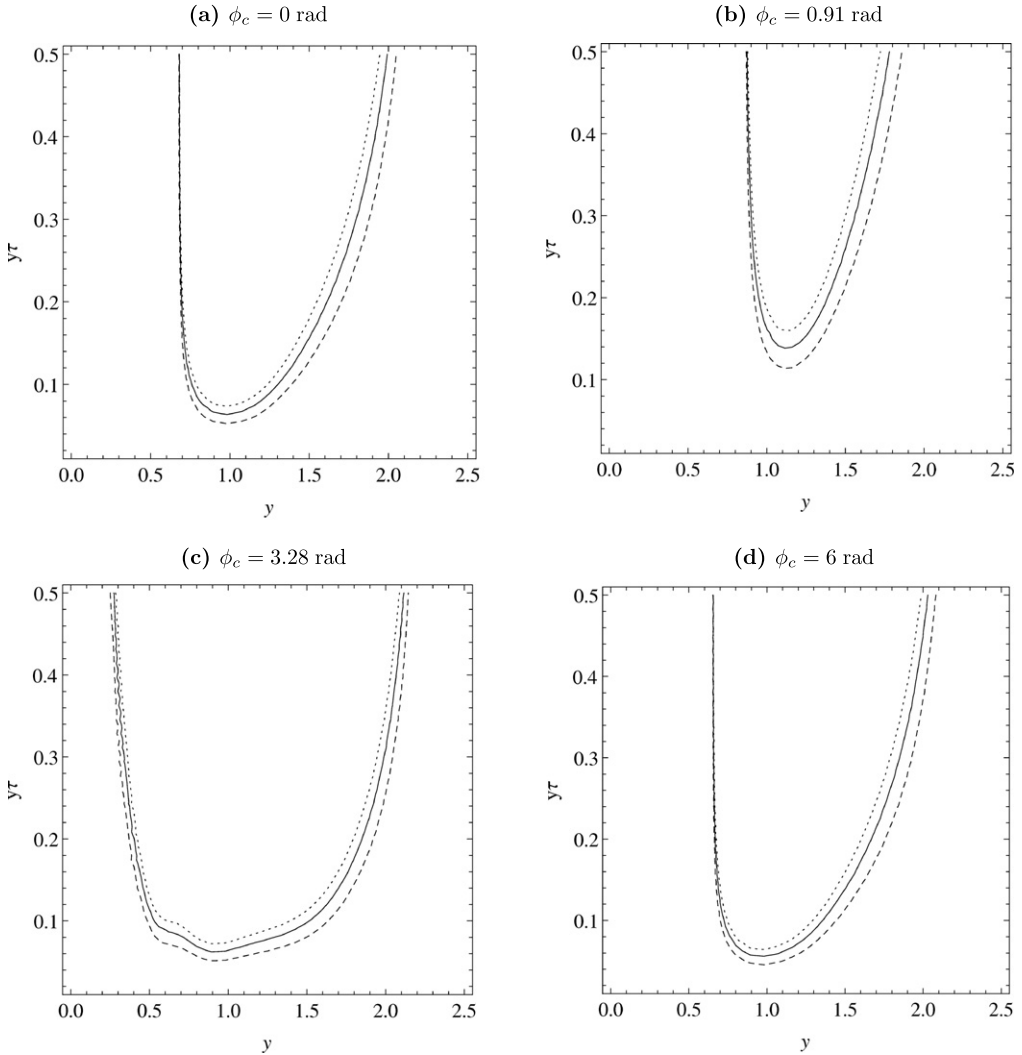$$\frac{\sqrt{6}}{2} \sin \theta_{13} = |\alpha_{13}|. \tag{44}$$

(a) $\phi_c = 0$ rad

(b) $\phi_c = 0.91$ rad

(c) $\phi_c = 3.28$ rad

(d) $\phi_c = 6$ rad

Fig. 5. These plots show contours of the baryon to photon ratio η from the TM model with $\theta_{13} = 8°$ in the tau Yukawa, y_τ, versus neutrino Yukawa, y, plane. The dotted and dashed lines are $\eta = 8.2 \times 10^{-10}$ and $\eta = 4.2 \times 10^{-10}$ while the solid line is the measured value of $\eta = 6.2 \times 10^{-10}$. Each plot is for a different value of the phase of $c = x_C \langle \xi' \rangle$, given above the relevant panel.

We present flavoured asymmetries for $\theta_{13} = 8°$ (the current T2K central value [37]) and $\phi_c = 0$ in Fig. 4. We also plot contours of $\eta = 4.2 \times 10^{-10}$, 6.2×10^{-10}, 8.2×10^{-10} for $\theta_{13} = 8°$ and $\eta = 6.2 \times 10^{-10}$ for $\theta_{13} = 0.1°$, $3°$, $6°$, $9°$, $12°$; and for each value of θ_{13}, we present four different choices of phase and modulus of c which satisfy (44). These can be seen in Figs. 5 and 6. For small θ_{13} and therefore small c, the results are very similar to those for TB mixing (cf. Figs. 3 and 6 purple line), which is expected since the only difference between the two models is the presence of the ξ' flavon. For the larger values of θ_{13}, it is clear that changing c has a significant effect as one can see from the variation of contours in Fig. 6; for instance
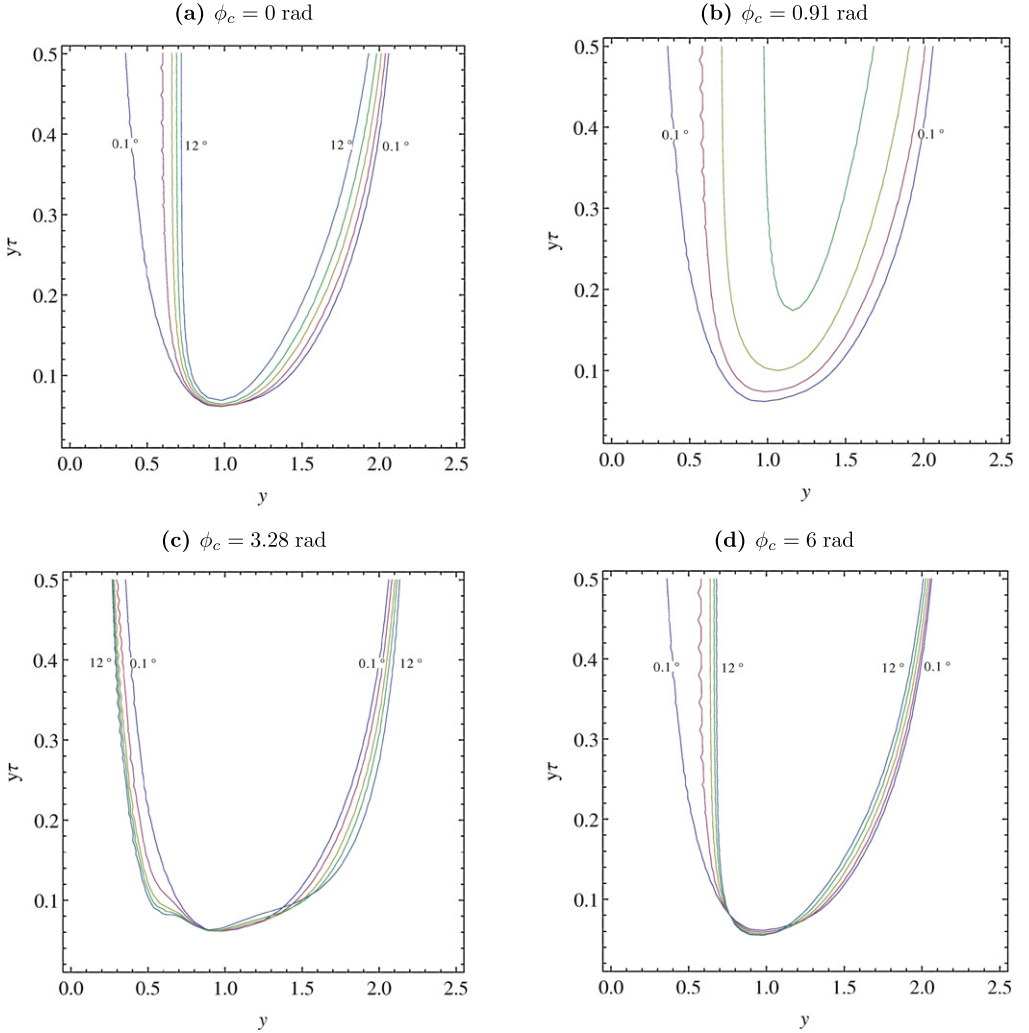
(a) $\phi_c = 0$ rad

(b) $\phi_c = 0.91$ rad

(c) $\phi_c = 3.28$ rad

(d) $\phi_c = 6$ rad

Fig. 6. These plots show contours of the baryon to photon ratio $\eta = 6.2 \times 10^{-10}$ from the TM model with $\theta_{13} = 0.1°$, $3°$, $6°$, $9°$, $12°$ (purple, red, yellow, green, blue respectively) in the tau Yukawa, y_τ, versus neutrino Yukawa, y, plane. Each plot is for a different value of the phase of $c = x_C \langle \xi' \rangle$, given above the relevant panel. Note that the $\theta_{13} = 12°$ contour is not possible for $\phi_c = 0.91$ radians. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

the $12°$ contour for a phase of $\phi_c = 0.91$ rad doesn't show up across the whole displayed plane.

Because we are using experimental (i.e. low energy) input here, one should briefly mention again running between the leptogenesis scale and the electroweak scale. As with all models of neutrino mixing the obtained high-energy (in our case leptogenesis scale) parameters have to be RG evolved down to the EW scale before one compares them with data. However these effects have been well studied, [39,40] and shown to be possible to control with respect to fitting the data. This discussion also applies for the TB case presented in the previous subsection. Finally,

one should not let these considerations detract from the main goal of this paper which is to obtain a non-zero value for ϵ in the presence of FD.

8. Conclusion

In this paper we have studied RG corrections relevant for leptogenesis in the case of family symmetry models such as the Altarelli–Feruglio A_4 model of tri-bimaximal lepton mixing or its extension to tri-maximal mixing. Such corrections are particularly relevant since in large classes of family symmetry models, to leading order, the CP violating parameters of leptogenesis would be identically zero at the family symmetry breaking scale, due to the form dominance property. We have used the third family approximation, keeping only the largest Yukawa couplings, subject to the constraint of perturbativity. In addition, the τ Yukawa coupling is related to the SUSY parameter $\tan\beta$, which has had experimental bounds placed upon it.

Our results demonstrate that it is possible to obtain the observed value for the baryon asymmetry of the Universe in models with FD by exploiting RG running of the neutrino Yukawa matrix over the small energy interval between the family symmetry breaking scale and the right-handed neutrino mass scale $\sim 10^{14}$ GeV. Of course, the importance of RG corrections applies more generally than to the particular models we have considered here for illustrative purposes, and the right-handed neutrino masses may be lower in some models.

In conclusion, the results in this paper show that RG corrections have a large impact on leptogenesis in any family symmetry models involving neutrino and charged lepton Yukawa couplings of order unity, even though the range of RG running between the flavour scale and the leptogenesis scale may be only one or two orders of magnitude in energy. Therefore, when considering leptogenesis in such models, RG corrections should not be ignored, even when corrections arising from HO operators are also present.

Acknowledgements

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