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Galilean anti-de-Sitter spacetime in Romans theory

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ABSTRACT

The Romans type IIA theory is the only known example of 10-dimensional maximal supergravity where (tensor) fields are explicitly massive. We provide an example of a non-relativistic anti-de-Sitter $NRadS_4 \times S^6$ background as a solution in massive type IIA. A compactification of which on S^6 gives immediately the prototype NRadS background in $D = 4$ which is proposed to be dual to 'cold atoms' or unitary fermions on a wire.

1. Introduction

Recently, the applications of AdS/CFT holography [1–3] to strongly coupled condensed matter systems, which show scaling behaviour near quantum critical points, have taken a big leap [4–16]. In studying the scaling behaviour near critical points one considers the non-relativistic limit of AdS/CFT which exhibits a reduced conformal symmetry or 'Schrödinger group' [17]. Its main applications to study strongly coupled fermionic systems at finite density has been called as 'AdS/Atoms' cold and hot [4,5].[1] For the study of finite temperature properties like phase transitions, transport and viscosity, etc., one, however, needs to include black holes in AdS backgrounds [6,7].[2] For superconductivity the dual non-relativistic AdS geometry generally involves spontaneously broken Higgs phases where the Abelian gauge field becomes massive.

Recently, there have been many examples where non-relativistic anti-de-Sitter (NRadS) geometries can be embedded in type IIB and M-theory, see for example [8–11,15,16]. While there have been no attempts to our knowledge where the same has been worked out for massive type IIA supergravity. The Romans' massive IIA theory is the only known example of a 10-dimensional maximal supergravity where (tensor) fields are explicitly massive to begin with. Thus massive type IIA provides a unique case to look for a NRadS solutions and study dual Galilean field theory.

In this short note we provide an example of $NRadS_4 \times S^6$ background as a solution in massive type IIA supergravity. A compactification of which on S^6 gives immediately the prototype NRadS background in $D = 4$ which is proposed to be dual to non-relativistic field theory of cold atoms. The Letter is organised as follows. In Section 2 first we review the relevant aspects of massive type IIA sugra action and then obtain the non-relativistic AdS solution. We also discuss its reduction to four dimensions in Section 3. The brief discussion is given in Section 4.

2. Galilean solution of Romans theory

The massive type IIA sugra action [19] is given by

$$S = \frac{1}{2k^2}\int d^{10}x\sqrt{-g}\Big[R - \frac{1}{2}(\partial_\mu\phi)^2 - \frac{e^{-\phi}}{2}(H_3)^2 - \frac{e^{3\phi/2}}{2}(G_2)^2 - \frac{e^{\phi/2}}{2}(G_4)^2 - \frac{e^{5\phi/2}}{2}m^2\Big] \quad (1)$$

where we have left out the topological terms as those are vanishing for the type of backgrounds we are going to study, for details see [21,23]. Various field strengths are

$$H_3 = dB_2, \qquad G_2 = dC_1 + mB_2,$$
$$G_4 = dC_3 + B_2 \wedge dC_1 + \frac{m}{2}B_2 \wedge B_2 \quad (2)$$

where m is the mass parameter in the theory. The 2-rank tensor field is explicitly massive with mass square m^2, and there is a potential term $\propto m^2$ for the dilaton field. The potential is due to the requirement of maximal supersymmetry in the 10-dimensional theory [19]. However, as soon as the mass vanishes the potential altogether vanishes and the theory reduces to ordinary type IIA supergravity in ten dimensions.

The massive type IIA theory does not admit any Minkowskian vacuum solution [19]. Instead it is known that theory admits 1/2-supersymmetric domain-wall solutions, also called $D8$-branes [20,21], and other supersymmetric flux vacua like ($D6$, $D8$) and ($D4$, $D6$, $D8$) bound states [23,24]. Particularly interesting solution for this Letter are the Freund–Rubin solutions $AdS_4 \times S^6$ which are supported by a constant 4-form flux [19]

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[1] A closely related framework on Galilean symmetry was studied much earlier by [18].

[2] See for example [6,10] for a list of many previous works.

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$$ds^2 = L^2\left(\frac{-2\,dx^+\,dx^- + dy^2 + dz^2}{z^2} + \frac{5}{2}\,d\Omega_6^2\right),$$
$$\phi = \phi_0, \qquad G_{+-yz} = cL^4 z^{-4} \tag{3}$$

with $2L^{-2} = m^2 g_s^{5/2}$, $c^2 = 5m^2 g_s^2$, and $g_s = e^{\phi_0}$ being the string coupling constant. The AdS_4 metric is given with light-cone coordinates while $d\Omega_6^2$ is the metric of unit six-sphere. This is an example of a non-supersymmetric solution.

2.1. *Massive string (dust)*

Recently, there have been several examples of non-relativistic geometries [4,5] which are proposed to be dual to condensed matter phenomenon like unitary fermion systems, superfluidity and superconductivity, etc. An observed common feature has been that in order to obtain non-relativistic solutions one needs to introduce some massive (vector) fields propagating in the geometry. Hence in massive type IIA theory we must consider a 2-rank tensor, the only massive field in the theory, to be nontrivial. The above action in the tensorial notation is given by

$$S = \frac{1}{2k^2}\int d^{10}x\sqrt{-g}\left[R - \frac{1}{2}(\partial_\mu\phi)^2 - \frac{e^{-\phi}}{2.3!}(H_{\mu\nu\lambda})^2 - \frac{e^{3\phi/2}}{2.2!}(G_{\mu\nu})^2 - \frac{e^{\phi/2}}{2.4!}(G_{\mu\nu\lambda\sigma})^2 - \frac{e^{5\phi/2}}{2}m^2\right], \tag{4}$$

with

$$H_{\mu\nu\lambda} = 3\partial_{[\mu}B_{\nu\lambda]},\ G_{\mu\nu} = 2\partial_{[\mu}C_{\nu]} + mB_{\mu\nu},$$
$$G_{\mu\nu\lambda\sigma} = 4\partial_{[\mu}C_{\nu\lambda\sigma]} + 6B_{[\mu\nu}G_{\nu\sigma]} - 3mB_{[\mu\nu}B_{\nu\sigma]}. \tag{5}$$

To solve the equations of motion obtained from action (4) we make the following ansatz for $NRadS_4 \times S^6$ geometry and fields

$$ds^2 = L^2\left(-\frac{2}{z^{2a}}(dx^+)^2 + \frac{-2\,dx^+\,dx^- + dy^2 + dz^2}{z^2} + \frac{5}{2}\,d\Omega_6^2\right),$$
$$\phi = \phi_0, \qquad G_{+-yz} = cL^4/z^4,$$
$$B_{+y} = f(z), \qquad C_+ = g(z), \tag{6}$$

where $f(z)$, $g(z)$ are functions which are to be determined next. Note that with this choice of the tensor fields $SO(1,3)$ Lorentz invariance is explicitly broken. The background will involve $D0$-, $D8$-branes with 4-form flux alongwith fundamental strings stretched along the y direction. (One may, however, consider B_{+z} to be nonzero and set $C_+ = 0$ by using the Stueckelberg gauge invariance.) Given the choice of $B_{\mu\nu}$ and C_μ as in (6), one finds that the invariants $(G_{\mu\nu})^2$ and $(H_{\mu\nu\lambda})^2$ vanish identically. Due to this the dilaton equation of motion remains unchanged and is simply written as

$$\nabla\phi - V'(\phi) = 0 \tag{7}$$

with the dilaton potential as

$$V(\phi) = -\frac{c^2}{2}e^{\phi/2} + \frac{m^2}{2}e^{5\phi/2}. \tag{8}$$

Hence a constant dilaton solution $\phi = \phi_0$ will still be fixed by the 4-form flux and the mass parameter as in the relativistic case (3), that is $c^2 = 5m^2 g_s^2$. It means that by making above non-relativistic deformations, the vital parameter like string coupling constant g_s is not affected. Notice that these parameters determine the overall curvature of the spacetime which we have to keep small irrespective of the non-relativistic limit (deformation). Next the Einstein equations for the metric (6) along the six-sphere also remain unchanged and those are solved if we set $2L^{-2} = m^2 g_s^{5/2}$ as earlier. The equation involving R_{++} component however gets modified now because $T_{++} \propto g_{++}$ receives contributions from the gauge potentials B_2 and C_1.

The B-field equations are solved if we take

$$g(z) = \frac{\sqrt{5}m}{a}\frac{q}{z^a}, \qquad f(z) = \frac{q}{z^{a+1}}, \tag{9}$$

for the exponents being $a = 3$ and $a = -4$ only. The last remaining constant q is determined by the equation involving R_{++}. We do find that the non-relativistic solutions (6) exist provided

$$q = \pm\sqrt{2g_s}L^2, \quad \text{for } a = 3,$$
$$q = \pm 2\sqrt{\frac{7g_s}{3}}L^2, \quad \text{for } a = -4. \tag{10}$$

From this we clearly see that the non-relativistic matter (dust) responsible for the AdS deformation is nothing but made up of the massive *strings* stretched along y direction. As the boundary of $NRadS_4$ space is located at $z = 0$, near the boundary the metric and fields are divergent for $a = 3$ case but everything is fine in the interior of the spacetime. While for $a = -4$ it is other way round. Nevertheless the invariants appearing in the action (4) stay finite every where in both the cases.

Although our solutions (6) have a scaling (dilatation) symmetry

$$x^+ \to \lambda^a x^+, \qquad x^- \to \lambda^{2-a}x^-, \qquad y \to \lambda y, \qquad z \to \lambda z \tag{11}$$

with exponents $a = 3, -4$. But these solutions have no special conformal symmetry as that arises only when the exponent is $a = 2$ [4,5].

3. Compactification

The four-dimensional AdS supergravity can be obtained by compactification of the action (4) on a six-sphere. On consistent truncation and keeping only the relevant spacetime components, it will give us following $(3+1)$-dimensional effective action

$$S_4 \sim \int d^4x\sqrt{-g}\left[R - \frac{1}{2.3!}\frac{1}{g_s}(H_{\mu\nu\lambda})^2 - \frac{1}{2.2!}g_s^{\frac{3}{2}}(G_{\mu\nu})^2 + \Lambda\right] - \frac{\sqrt{5}m}{2!2!}g_s^{\frac{3}{2}}\int d^4x\,\epsilon^{\mu\nu\lambda\rho}\left(B_{\mu\nu}G_{\lambda\rho} - \frac{m}{2}B_{\mu\nu}B_{\lambda\rho}\right), \tag{12}$$

where we have kept only the tensor field and the 1-form and have integrated out the 4-form field strength which is a volume form over spacetime. We have normalised $\epsilon^{0123} = 1$. The four-dimensional cosmological constant expressed in 10-dimensional variables is given by $\Lambda = 3m^2 g_s^{5/2} = \frac{6}{L^2}$. It is important to note that from 10-dimensional point of view, m is fixed by the 4-form flux as $c^2 = 5m^2 g_s^2$.[3] But the curvature of spacetime will remain small so long as string coupling remains weak ($g_s < 1$).

As it is obvious that $G_{\mu\nu}$ is an auxiliary field strength. We first integrate it out using its field equation

$$\star G_2 + \sqrt{5}mB_2 = 0 \tag{13}$$

where $\star$ is a Hodge-dual in four dimensions. Upon integration the action (12) reduces to

$$S_4 \sim \int d^4x\sqrt{-g}\left[R - \frac{1}{2.3!}\frac{1}{g_s}(H_{\mu\nu\lambda})^2 - \frac{5m^2}{2.2!}g_s^{\frac{3}{2}}(B_{\mu\nu})^2 + \Lambda\right] + \frac{\sqrt{5}m^2}{2(2!)^2}g_s^{\frac{3}{2}}\int d^4x\,\epsilon^{\mu\nu\lambda\rho}B_{\mu\nu}B_{\lambda\rho}. \tag{14}$$

[3] In other words, by the number of N_c $D2$- and N_f $D8$-branes (usually in type I′ picture ($N_f \leqslant 16$) [22]).

Let us write down the equation of motion of B_2 obtained from varying the action (14)

$$\frac{1}{g_s} d\star H_3 - m^2 g_s^{\frac{3}{2}}(5\star B_2 - \sqrt{5}B_2) = 0. \tag{15}$$

In order to see that it gives the same effective action as studied by [4,5] we will define a massive gauge (Proca) field through a duality relation in four-dimensions as

$$\frac{1}{g_s}\star H_3 = dX + m_0 A_1, \tag{16}$$

where X is the axion field and parameter m_0 will get specified next. It is the Goldstone mode introduced so that there is a shift (Stueckelberg) invariance

$$\delta X = -m_0\lambda, \qquad \delta A_1 = d\lambda. \tag{17}$$

With this gauge symmetry the axion can always be eaten up by the gauge field. Eqs. (16) and (15) imply

$$F_2 \equiv dA_1 = \frac{m^2 g_s^{\frac{3}{2}}}{m_0}(5\star B - \sqrt{5}B). \tag{18}$$

It leads to a Proca equation

$$d\star F_2 + 6m^2 g_s^{\frac{5}{2}}\star A = 0. \tag{19}$$

Thus the gauge field has the mass square $6m^2 g_s^{\frac{5}{2}}$. We can also get this equation from a simple action

$$S_4 \sim \int d^4x\sqrt{-g}\left[R - \frac{1}{2.2!}(F_{\mu\nu})^2 - \frac{g_s}{2}(\partial_\mu X + m_0 A_\mu)^2 + \frac{6}{L^2}\right] \tag{20}$$

provided we identify $m_0^2 = 6m^2 g_s^{\frac{3}{2}} = \frac{12}{g_s L^2}$. This kind of action has been the starting point of many non-relativistic AdS geometries holographically dual to the strongly coupled condensed matter phenomena [4,5]. The crucial difference however is that the mass of the Proca field is fine tuned to the negative cosmological constant in the action. This comes from the consistent embedding of the theory in the Romans supergravity in ten dimensions.

The 4-dimensional non-relativistic vacua of (20) can be written as

$$ds^2 = L^2\left(-\frac{2}{z^{2a}}(dx^+)^2 + \frac{-2\,dx^+\,dx^- + dy^2 + dz^2}{z^2}\right),$$
$$A_+ = \frac{q(a+1)}{\sqrt{12g_s}L}\frac{1}{z^a}, \qquad X = 0 \tag{21}$$

where q is as in Eq. (10).

3.1. Hierarchy of scales

While truncating to the action (12) we kept dilaton fixed to its vacuum value $\phi = \phi_0$. Now if we allow infinitesimal perturbations in the dilaton field around its anti-de-Sitter minima, say $\phi = \phi_0 + \rho$, keeping everything else fixed, we find from (7) that the fluctuation ρ has got positive mass square given by $5/L^2$. Including the fluctuating mode of dilaton we may also write to the leading order an action

$$S_4 \sim \int d^4x\sqrt{-g}\left[R + \frac{6}{L^2} - \frac{1}{2.2!}(F_{\mu\nu})^2 - \frac{1}{2}(\partial_\mu\chi + m_A A_\mu)^2 - \frac{1}{2}(\partial_\mu\rho)^2 - \frac{5}{L^2}\frac{\rho^2}{2}\right] \tag{22}$$

where by simple scaling $\chi = \sqrt{g_s}X$, $m_A^2 = g_s m_0^2$. The above action is nothing but represents an $U(1)$ field coupled to a complex scalar field with real components (ρ, χ) in a spontaneously broken Higgs vacua. It is interesting in the sense that broken phases like this represent dual superconductors [7]. It is remarkable that it could also be realised in Romans' massive theory. We note down the characteristic ratio

$$\frac{m_A^2}{m_\rho^2} = \frac{12}{5}. \tag{23}$$

The hierarchy of mass scales goes as

$$m_\rho^2 > m_{BF}^2. \tag{24}$$

The Breitenlohner–Freedman (BF) bound emphasizes that the mass square of a scalar in AdS_{d+1} should be larger than $-\frac{d^2}{4L^2}$. Thus the mass hierarchy is satisfactory from the AdS stability point of view.

3.2. Scaling solution

We now look for the scaling solutions near the boundary. The isometry direction X^- can be taken periodic and the corresponding Kaluza–Klein momentum, $P^+ = M$, modes will generate a spectrum. In a given momentum sector M, there are no scaling solutions near the boundary for NRadS geometries if the exponents $a > 2$ [5]. However in zero light-cone momentum sector we will still have a spectrum of non-normalisable operators. We find these operators with non-relativistic geometry exponent $a = 3$. If the bulk gauge field $A_\mu(x;z)$ behaves as $\sim z^{-\lambda_+}$ near the boundary, the corresponding conformal operator is the conserved current $J^\mu(x)$ in the boundary theory having conformal dimension $\Delta_+ = d - 1 + \lambda_+$ [3]. We can easily read from our $NRadS_4$ solution $\lambda_+ = a = 3$. The operator dimension for a spin-1 field can also be obtained as

$$\Delta_\pm = \frac{d}{2} \pm \sqrt{\left(\frac{d-2}{2}\right)^2 + m^2L^2}. \tag{25}$$

With mass as $m^2 = 12L^{-2}$ and $d = 3$, we get the conformal dimension of the current operator to be $\Delta_+ = 5$. Thus the inclusion of massive gauge fields in Romans theory gives rise to an irrelevant non-relativistic deformation of the original $(2+1)$-dimensional (relativistic) CFT. With NR deformation the theory however looses special conformal symmetry while scale invariance (dilatation) still exists.

4. Discussion: CFT of cold atoms

In this note we provided an example of a non-relativistic anti-de-Sitter background $NRadS_4 \times S^6$ as the solution of massive type IIA supergravity. A compactification on S^6 provides immediately the prototype NRadS background in four dimensions, which are generally proposed to be dual to a critical phenomenon involving strongly coupled matter (fermions) in non-relativistic boundary theory [4,5].

According to the proposal [4,5] the dual field theory will be a $(2+1)$-dimensional quantum theory describing the non-relativistic dynamics of strongly coupled fermions along a *wire*. The AdS–CFT holography works well when the spacetime curvature and the coupling constant are kept small in string theory, i.e. Λ has to be small. As we saw for the $NRadS_4$ solution, Λ is tightly related to the mass m_A of the $U(1)$ field and the string coupling constant g_s. We are allowed to have $m \to \infty$ only if $g_s \to 0$ keeping $m^2 g_s^{5/2}$ or m_A^2 fixed and small. In general, m is related to the number of $D2$-branes (via 4-form flux) and/or $D8$-branes present in the background. The number of $D2$- and $D8$-branes is pretty much interlinked in massive type IIA as is evident from the solution (6) ($c^2 = 5m^2 g_s^2$). In any situation large m would also mean large N_c. To recall we know that usual $(2+1)$-dimensional (relativistic) super-Yang–Mills theory is UV complete but the theory flows to a strongly coupled fixed point in the IR. A non-relativistic deformation of this theory as in this work would then describe non-relativistic phenomena on a wire. However there remains only a Galilean symmetry in IR and not the full Schrödinger group. We expect the theory will be having strongly coupled phases like unitary fermions on wire. It would be worthwhile to explore such a non-relativistic field theory as a dual of $NRadS_4 \times S^6$. Incidentally the theory will be possessing global $SO(7)$ symmetry. This global symmetry can reduce if, instead of S^6, we select other compact (Einstein) spaces such as CP^3, $S^2 \times S^2 \times S^2$, $S^2 \times S^4$, or $S^3 \times S^3$ [19].

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