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The Doppler effect and the three most famous experiments for special relativity

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ABSTRACT

Using the general formula for the Doppler effect at any arbitrary angle, the three famous experiments for special theory of relativity will be examined. Explanation of the experiments of Michelson, Kennedy–Thorndike and Ives–Stilwell will be given in a precise and elegant way without postulates, arbitrary assumptions or approximations.

Introduction

Let's assume an observer is at rest somewhere in a frame of reference K (Fig. 1). On the x axis we assume a light source with a velocity v moving away from the observer. Another frame of reference, K', is connected to this light source so that the axes x and x' are aligned. If the observer is located at point A of frame K, then the angle between the direction in which the observer receives the signal and the direction of source motion is ϑ. For the triangle ACO′ we obtain by the law of cosines:

$$r_s^2 = r_o^2 + v^2t^2 - 2r_o vt\cos\vartheta \tag{1}$$

The distances r_s and r_o represent the n -fold of the wavelength in the corresponding reference frame: $r_s = n\lambda_s$ and $r_o = n\lambda_o$; i.e. the source within the time t emits n wavefronts, which in the source's frame of reference have the wavelength λ_s and in the observer's frame of reference λ_o. The distance vt likewise represents the n-fold of the product of velocity v and the period T_s with which the source emits: $vt = nvT_s$; and the period T_s can be written as: $T_s = \lambda_s/c$. If we insert these into Eq. (1) and if we solve it by wavelength λ_o, we obtain:

$$\lambda_o^2 - 2\lambda_o\lambda_s\frac{v}{c}\cos\vartheta + \frac{v^2}{c^2}\lambda_s^2 - \lambda_s^2 = 0 \tag{2}$$

Eq. (2) is a quadratic equation and if we solve it by λ_o we obtain:

$$\lambda_o = \lambda_s\left(\frac{v}{c}\cos\vartheta + \sqrt{1 - \frac{v^2}{c^2}\sin^2\vartheta}\right) \tag{3}$$

Eq. (3) should be called the general Doppler effect at any arbitrary angle. From this equation we obtain the values of observed wavelengths for three specified positions of an observer relative to the velocity of the source ($\vartheta = 0^\circ$, $\vartheta = 90^\circ$, $\vartheta = 180^\circ$), which we have set out in Fig. 2.

Michelson experiment

If we solve Eq. (3) according to the emitted wavelength we obtain:

$$\lambda_s = \frac{\lambda_o}{\frac{v}{c}\cos\vartheta + \sqrt{1 - \frac{v^2}{c^2}\sin^2\vartheta}} \tag{4}$$

or, if we solve Eq. (4) according to the emitting period of source ($T_s = \frac{\lambda_s}{c}$) we obtain:

$$T_s = \frac{\lambda_o}{v\cos\vartheta + \sqrt{c^2 - v^2\sin^2\vartheta}} \tag{5}$$

Let us now consider the Michelson experiment (Fig. 3a). The length of the both arms of interferometer is L. On the transverse arm of the Michelson interferometer the light propagates upwards and downwards and conforms to Eq. (5) for the angle $\vartheta = 90^\circ$. On the other arm the light propagates backwards and forwards and conforms to Eq. (5) for angles $\vartheta = 0^\circ$ and $\vartheta = 180^\circ$.

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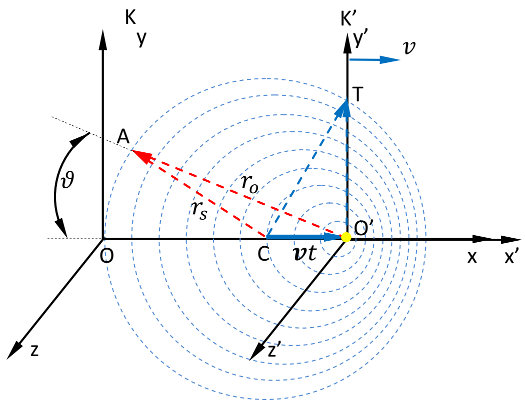

Fig. 1. Doppler effect.

The general solution of the Michelson experiment with Eq. (5) is:

$$T_s = \frac{\lambda_o(0^\circ)}{c+v} = \frac{\lambda_o(90^\circ)}{\sqrt{c^2-v^2}} = \frac{\lambda_o(180^\circ)}{c-v} \tag{6}$$

More concretely, let us call the time within which the light propagates back and forth along the horizontal arm of the interferometer $t_\parallel$, and let us call the time of the propagation up and down on the interferometer's transverse arm $t_\perp$. For these times the following equation system is derived from Eq. (6):

$$\left.\begin{array}{l} t_\parallel = n\frac{\lambda_o(0^\circ)}{c+v} + n\frac{\lambda_o(180^\circ)}{c-v} \\ t_\perp = 2n\frac{\lambda_o(90^\circ)}{\sqrt{c^2-v^2}} \end{array}\right\} \tag{7}$$

It can clearly be seen from Eq. (6) that the subtraction of these two times is in accordance with the result of the Michelson experiment [1]:

$$t_\parallel - t_\perp = 0. \tag{8}$$

The Kennedy–Thorndike experiment

Let us now consider the Kennedy–Thorndike experiment. The arms of the interferometer in this experiment do not have the same length, so let us call them L and L_1 (Fig. 3b). The times of the light propagation along the two arms will not be the same. The result of the original experiment [2] is the same, as if Earth had not moved at all:

$$t_\parallel - t_\perp = \frac{2(L_1 - L)}{c} \tag{9}$$

As with the Michelson experiment, with the application of Eq. (5) we go on to derive Eq. (9). From Eq. (6) we derive the following for the time of the propagation of light along the long arm of the interferometer:

$$n_1 T_s = n_1\frac{\lambda_o(0^\circ)}{c+v} = n_1\frac{\lambda_o(180^\circ)}{c-v} \tag{10}$$

while for the time of the propagation of light along the short arm of the interferometer we derive:

$$nT_s = n\frac{\lambda_o(90^\circ)}{\sqrt{c^2-v^2}} \tag{11}$$

Using the same notation as in the Michelson experiment, let us use the notation $t_\parallel$ for the time within which the light propagates back and forth on the horizontal arm of the interferometer and $t_\perp$ for the time of propagation up and down on the transverse arm of the interferometer. For these times the following equation system is thus derived for the Kennedy–Thorndike experiment from Eqs. (10) and (11):

$$\left.\begin{array}{l} t_\parallel = n_1\frac{\lambda_o(0^\circ)}{c+v} + n_1\frac{\lambda_o(180^\circ)}{c-v} \\ t_\perp = 2n\frac{\lambda_o(90^\circ)}{\sqrt{c^2-v^2}} \end{array}\right\}. \tag{12}$$

The following fact must now be borne in mind: the velocities c and v, the periods of emission and reception and the wavelengths (or frequencies) of emission and reception do not change in these two experiments if they are carried out with the same light source (and of course in the two experiments the length of the transverse arm of the interferometer must be the same). Given this, the general solution of the Michelson experiment (Eq. (6)) is exactly the same as for the Kennedy–Thorndike experiment; this can also be seen from Eqs. (10) and (11). But the thing that changes in these two experiments is the time of the propagation of light along the

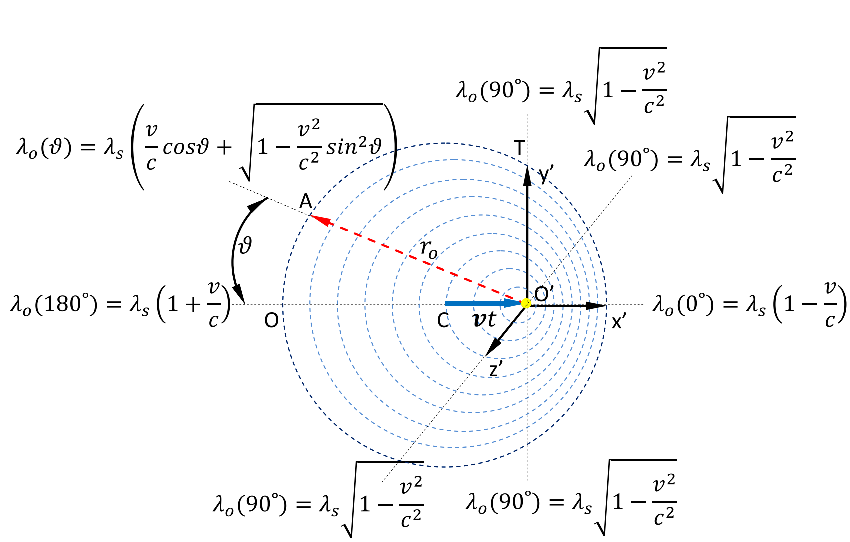

Fig. 2. Doppler effect through wavelengths.

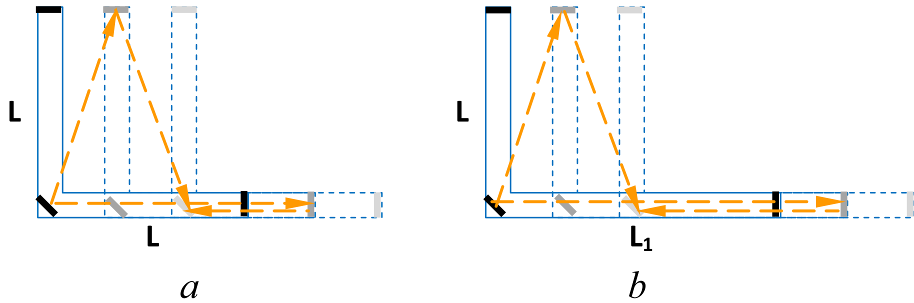

Fig. 3. Michelson experiment (a) and Kennedy–Thorndike experiment (b).

arms of the interferometer, because of their different lengths – i.e. $nT_s \neq n_1 T_s$. Thus, with the subtraction of these two times we derive the following from equation system (12):

$$t_{\parallel} - t_{\perp} = \frac{2\lambda_s(n_1 - n)}{c}. \tag{13}$$

Since $L = n\lambda_s$ and $L_1 = n_1\lambda_s$ it can be seen that Eq. (13) is identical to Eq. (9).

The Ives–Stilwell experiment

As is seen from Eq. (3) and Fig. 2, for $\vartheta = 0^{\circ}$ and $\vartheta = 180^{\circ}$ the longitudinal Doppler effect (LDE) is derived; while for $\vartheta = 90^{\circ}$ the transverse Doppler effect (TDE) is derived:

$$\lambda_o(90^{\circ}) = \lambda_s\sqrt{1 - \frac{v^2}{c^2}} \tag{14}$$

It has been, and continues to be, easy to prove LDE. The challenge for physics is to prove TDE, i.e. Eq. (14). This equation expressed with periods and frequencies looks like this:

$$T_o(90)^{\circ} = T_s\sqrt{1 - \frac{v^2}{c^2}} \tag{15}$$

$$v_o(90^{\circ}) = \frac{v_s}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{16}$$

In 1938 TDE was proved through the Ives–Stilwell experiment [3], i.e. the Eq. (15) which is the same as saying that Eqs. (14) and (16) were proved. Later these equations were proved in other experiments [4,5]. Thus, to the question of C.I. Christov: Time dilation or frequency change? [6] I would reply in accordance with Eqs. (14), (15) and (16) – i.e. that there is no time dilation, but only frequency change (specifically, different emitting and receiving periods)!

Conclusions

Using the general formula for the Doppler effect at any arbitrary angle the three famous experiments for special theory of relativity are examined. The Michelson, Kennedy–Thorndike and Ives–Stilwell experiments are explained very precisely and elegantly, without postulates, arbitrary assumptions or approximations.

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