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Top-pair production at hadron colliders with next-to-next-to-leading logarithmic soft-gluon resummation [☆]

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ABSTRACT

Incorporating all recent theoretical advances, we resum soft-gluon corrections to the total $t\bar{t}$ cross-section at hadron colliders at the next-to-next-to-leading logarithmic (NNLL) order. We perform the resummation in the well established framework of Mellin N -space resummation. We exhaustively study the sources of systematic uncertainty like renormalization and factorization scale variation, power suppressed effects and missing two- and higher-loop corrections. The inclusion of soft-gluon resummation at NNLL brings only a minor decrease in the perturbative uncertainty with respect to the NLL approximation, and a small shift in the central value, consistent with the quoted uncertainties. These numerical predictions agree with the currently available measurements from the Tevatron and LHC and have uncertainty of similar size. We conclude that significant improvements in the $t\bar{t}$ cross-sections can potentially be expected only upon inclusion of the complete NNLO corrections.

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1. Introduction

The production of $t\bar{t}$ pairs at hadron colliders is well understood within next-to-leading-order (NLO) perturbative QCD, where corrections of order $\mathcal{O}(\alpha_s^3)$ are included. Results have been available for a while for the fully inclusive [1–3], one-particle inclusive [4,5], two-particle inclusive production [6], including decay [7], spin correlations [8,9], off-shell effects [10,11] and associated jet production [12–18]. In addition, the resummation of next-to-leading logarithmic (NLL) soft gluon effects has been long established [19], and results beyond the NLO and NLL level of accuracy have recently been published [20–27].

At the current level of precision, the theory agrees [20,23,26,28, 29] with the data from the Tevatron and LHC [30–41]. The large statistics becoming available at the LHC, and the precision of the experiments, will however soon bring the accuracy of the measurements to the level of possibly less than 5 percent, thus challenging the present theoretical systematics.

In this Letter we extend to the next-to-next-to-leading logarithmic accuracy (NNLL) the Mellin N -space soft-gluon resum-

mation of the top-pair production cross-section which was first developed in [42,43] for leading (LL), and in [19] for next-to-leading logarithmic accuracy (NLL). This calculation uses the recently derived two-loop anomalous dimension matrices [44,45], and matches the result to the best known approximation to the NNLO cross-section [22]. The numbers we derive are very robust and show a significant stability – as expected – given that this is the third order at which the soft-gluon resummation is applied to this observable (i.e. LL, NLL and NNLL).

We include in our assessment the latest sets of parton distribution function (PDF) fits, analyze the impact of the emerging contributions at $\mathcal{O}(\alpha_s^4)$ like two-loop Coulomb corrections and, most importantly, missing NNLO corrections. Addressing the full theoretical uncertainty associated with the total $t\bar{t}$ cross-section as currently known, we conclude that we see no evidence for a strong reduction of the theoretical uncertainty compared to the long-ago established NLO + NLL analysis. Based on our comprehensive analysis we, however, speculate on the uncertainty that should be achievable once the full NNLO calculation will be completed, which could be significant.

The Letter is organized as follows: in Section 2 we summarize the key elements of the threshold approximation at NNLO, and develop the Mellin-space resummation formalism to the NNLL level, leaving some technical details to [Appendix A](#). In Section 3 we study the details of the theoretical systematics and the sensitivity of our

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theory prediction to the scale dependence and to a number of currently unknown contributions of $\mathcal{O}(\alpha_s^4)$ and higher. In Section 4 we include the study of the PDF systematics, and present our best predictions for the Tevatron and LHC (7 TeV). We present the dependence of the predicted cross-section on the value of the top mass and compare with the most precise available experimental measurements, finding a very satisfactory agreement. In the concluding section we discuss the comparison of our results with the current literature.

2. Theoretical framework

2.1. NNLO results in the threshold approximation

While the calculation of the full NNLO results for the total cross-section is yet to be completed, recent work has determined its exact behavior near the production threshold, where it can be represented as an expansion in powers of $1/\beta$ and $\log\beta$ ($\beta = \sqrt{1 - 4m^2/\hat{s}}$ being the heavy quark velocity in the $Q\bar{Q}$ rest frame and m the top pole mass). In the limit $\beta \rightarrow 0$ the $\mathcal{O}(\alpha_s^4)$ partonic cross-section can be written as

$$\sigma_{ij,\mathbf{I}}^{(2)}(\beta, \mu, m) = \sigma_{ij,\mathbf{I}}^{(0)}(\beta, \mu, m) \left(\frac{\alpha_s(\mu^2)}{4\pi} \right)^2 \times \left[\sigma_{ij,\mathbf{I}}^{(2,0)}(\beta) + \sigma_{ij,\mathbf{I}}^{(2,1)}(\beta) \log\left(\frac{\mu^2}{m^2}\right) + \sigma_{ij,\mathbf{I}}^{(2,2)}(\beta) \log^2\left(\frac{\mu^2}{m^2}\right) \right], \quad (1)$$

where we defined $\sigma_{ij,\mathbf{I}}^{(0)}$ as the Born-level cross-section. For ease of notation we set the factorization (μ_F) and renormalization (μ_R) scales equal to μ , although in our calculation we allowed them to vary independently.

The index $\mathbf{I} = \mathbf{1}, \mathbf{8}$ corresponds to the color configuration of the heavy quark pair, $(ij) = (q\bar{q}), (gg)$ are the initial-state partons and the functions $\sigma^{(2,n)}$ are expanded as follows:

$$\sigma_{ij,\mathbf{I}}^{(2,n)}(\beta) = \sum_{a=0}^{2-n} \sum_{\substack{b=0 \\ a+b>0}}^{4-2a-n} k_{ij,\mathbf{I}}^{(n;a,b)} \frac{1}{\beta^a} \log^b \beta + C_{ij,\mathbf{I}}^{(2,n)} + \mathcal{O}(\beta). \quad (2)$$

For $n = 1, 2$ the above functions, including the β -independent terms $C^{(2,n)}$, can be derived entirely from the knowledge of the NLO results. In particular, the $C^{(2,n)}$ ($n = 1, 2$) terms can be determined by enforcing the compensation of their scale dependence against that of appropriate terms of $\mathcal{O}(\alpha_s^3)$. For $n = 0$, the functions were extracted in [22] from the threshold behavior of the NNLO result, up to (but excluding) the constant term $C^{(2,0)}$, which will only become available with the completion of a full NNLO calculation.

To the extent that the hadronic production of heavy quark pairs receives an important contribution from the region near threshold, it is meaningful to incorporate the known singular threshold terms into an improved prediction of the total production cross-section, even though they do of course not represent the full NNLO result. We label NNLO_β [46] the approximation that adds to the full NLO all $\mathcal{O}(\alpha_s^4)$ terms that become singular when $\beta \rightarrow 0$. Note that we do not impose scale dependence cancellation at $\mathcal{O}(\alpha_s^4)$, i.e. we exclude all the finite contributions $C^{(2,n)}$: while the $n = 1, 2$ terms are known from scale dependence compensation with $\mathcal{O}(\alpha_s^3)$ terms, the lack of any threshold enhance-

ment prevents us from assuming that their value should be bigger than the unknown $C^{(2,0)}$. (This will be confirmed with direct numerical studies presented in Section 3.2.) Thus we find it more coherent to neglect the non-singular terms. We also remark that if one wants to include these scale dependent terms, one should still consider the ambiguity in the choice of the scale that divides the renormalization and factorization scale in the arguments of the logarithms. These scales are uniquely fixed only if the constant terms are known. Thus, the compensation of the renormalization and factorization scale variations induced by these terms will be counterbalanced by the uncertainties obtained by varying these new scales. It is therefore simpler not to include these terms at all, and let the lack of scale compensation work as an indicator of unknown higher-order terms. Our approach is therefore to account for the ignorance of the $\mathcal{O}(\alpha_s^4)$ constants through the scale variation systematics. In addition, we shall show explicitly in the following that two different choices of these constants, $C^{(2,0)} = 0$ and $C^{(2,0)} = \bar{C}^{(2,0)}$ (see Appendix A), lead to differences consistent with the scale systematics.

2.2. Soft-gluon resummation with NNLL accuracy

We extend here the NLO + NLL formalism for soft-gluon resummation of the total top-pair hadroproduction cross-section, discussed in Ref. [19], to include the resummation of NNLL terms.

For simplicity, we first summarize the qualitative overall structure of our result:

- The $\mathcal{O}(\alpha_s^3)$ contributions correspond to the exact NLO result.
- We perform the resummation in Mellin space and then invert numerically back to x -space using the Minimal Prescription of Ref. [43].
- The truncation of the NNLL result at $\mathcal{O}(\alpha_s^4)$ includes all singular contributions described by NNLO_β , plus non-singular terms that arise from the inverse Mellin transform of the N -space resummation; in particular, terms of $\mathcal{O}(1/N)$ can give non-negligible contributions, which reflect the uncertainty about higher-order non-singular terms. This point in particular underscores a qualitative and quantitative difference with the alternative approach of simply matching at $\mathcal{O}(\alpha_s^4)$ the resummed result to NNLO_β .

The resummed cross-section in N -space¹ reads:

$$\sigma_N^{(res)}(m^2) = \sum_{ij=q\bar{q}, gg} F_{i,N+1}(\mu^2) F_{j,N+1}(\mu^2) \times \left[\hat{\sigma}_{ij,N}^{(res)}(m^2, \alpha_s(\mu^2), \mu^2) - (\hat{\sigma}_{ij,N}^{(res)}(m^2, \alpha_s(\mu^2), \mu^2))_{\alpha_s^3} \right] + \sigma_N^{(NLO)}(m^2), \quad (3)$$

where $\sigma_N^{(NLO)}$ is the NLO hadronic cross-section [1–3], $\hat{\sigma}_{ij,N}^{(res)}$ is the NNLL resummed partonic cross-section, and $(\hat{\sigma}_{ij,N}^{(res)})_{\alpha_s^3}$ is its perturbative truncation at order α_s^3 . As before, we set here the renormalization (μ_R) and factorization (μ_F) scales equal to μ : they are however kept separate and varied independently in the subsequent studies of the scale systematics.

In the threshold limit $N \rightarrow \infty$ the NNLL resummed partonic cross-section $\hat{\sigma}_{ij,N}^{(res)}$ factorizes:

¹ The Mellin moments N of a function $g(\rho)$, with $\rho = 4m^2/\hat{s}$, are defined by $g_N = \int_0^1 d\rho \rho^{N-1} g(\rho)$.

$$\begin{aligned} & \hat{\sigma}_{ij,N}^{(res)}(m^2, \alpha_s(\mu^2), \mu^2) \\ &= \sum_{\mathbf{I}=1,8} \hat{\sigma}_{ij,\mathbf{I},N}^{(Coul)}(\alpha_s(\mu^2), \mu^2/m^2) \hat{\sigma}_{ij,\mathbf{I}}^{(Hard)} \\ & \quad \times (\alpha_s(\mu^2), \mu^2/m^2) \Delta_{ij,\mathbf{I},N+1} \left(\alpha_s(\mu^2), \frac{\mu^2}{m^2} \right), \end{aligned} \quad (4)$$

in terms of the radiative factors $\Delta_{ij,\mathbf{I},N}$ containing all contributions due to soft-gluon emission, the functions $\hat{\sigma}_{ij,\mathbf{I},N}^{(Coul)}$ containing the threshold-enhanced bound-state contributions (Coulomb effects) and the hard matching functions $\hat{\sigma}_{ij,\mathbf{I}}^{(Hard)}$.

The radiative factors introduced in Eq. (4), $\Delta_{ij,\mathbf{I},N}$, are given by:

$$\begin{aligned} \ln \Delta_{ij,\mathbf{I},N} &= \int_0^1 dz \frac{z^{N-1} - 1}{1-z} \left\{ \int_{\mu^2}^{4m^2(1-z)^2} \frac{dq_{\perp}^2}{q_{\perp}^2} 2\delta_{ij} A_i(\alpha_s(q_{\perp}^2)) \right. \\ & \quad \left. + D_{ij,\mathbf{I}}(\alpha_s(4m^2(1-z)^2)) \right\} + \mathcal{O}(\alpha_s(\alpha_s \ln N)^k). \end{aligned} \quad (5)$$

The process independent anomalous dimensions A_i describe soft-collinear initial state radiation. They are known through three loops [47–49]; the explicit expressions can be found in Ref. [49]. The anomalous dimensions $D_{ij,\mathbf{I}}$ describe wide-angle soft radiation and depend both on the initial and final states [50]. For top-pair production, they have recently been derived through two loops in Refs. [44,45]. Their explicit expressions read:

$$\begin{aligned} D_{q\bar{q},\mathbf{8}} &= -C_A \frac{\alpha_S(\mu^2)}{\pi} + \left(\frac{\alpha_S(\mu^2)}{\pi} \right)^2 \left\{ \left(-\frac{115}{36} + \frac{\pi^2}{12} - \frac{\zeta_3}{2} \right) C_A^2 \right. \\ & \quad \left. + \left(-\frac{101}{27} + \frac{11\pi^2}{18} + \frac{7\zeta_3}{2} \right) C_A C_F \right. \\ & \quad \left. + \frac{11}{18} C_A N_F + \left(\frac{14}{27} - \frac{\pi^2}{9} \right) C_F N_F \right\} + \mathcal{O}(\alpha_S^3), \\ D_{gg,\mathbf{8}} &= -C_A \frac{\alpha_S(\mu^2)}{\pi} + \left(\frac{\alpha_S(\mu^2)}{\pi} \right)^2 \\ & \quad \times \left\{ \left(-\frac{749}{108} + \frac{25\pi^2}{36} + 3\zeta_3 \right) C_A^2 + \left(\frac{61}{54} - \frac{\pi^2}{9} \right) C_A N_F \right\} \\ & \quad + \mathcal{O}(\alpha_S^3), \\ D_{gg,\mathbf{1}} &= \left(\frac{\alpha_S(\mu^2)}{\pi} \right)^2 \left\{ \left(-\frac{101}{27} + \frac{11\pi^2}{18} + \frac{7\zeta_3}{2} \right) C_A^2 \right. \\ & \quad \left. + \left(\frac{14}{27} - \frac{\pi^2}{9} \right) C_A N_F \right\} + \mathcal{O}(\alpha_S^3), \end{aligned} \quad (6)$$

where N_F is the number of light flavors, i.e. lighter than the top quark. The integrations in Eq. (5) can be performed analytically and the resummed NNLL cross-section can be written explicitly. To extend the NLL results of Ref. [19], one can utilize, for example, the results in Ref. [51].

The N -independent, hard matching functions read:

$$\begin{aligned} & \hat{\sigma}_{ij,\mathbf{I}}^{(Hard)}(\alpha_s(\mu^2), \mu^2/m^2) \\ &= (\alpha_s(\mu^2))^2 \left(1 + \frac{\alpha_s(\mu^2)}{\pi} H_{ij,\mathbf{I}}^{(1)}(\mu^2/m^2) \right. \\ & \quad \left. + \left(\frac{\alpha_s(\mu^2)}{\pi} \right)^2 H_{ij,\mathbf{I}}^{(2)}(\mu^2/m^2) + \mathcal{O}(\alpha_s^3) \right). \end{aligned} \quad (7)$$

The one-loop results $H_{ij,\mathbf{I}}^{(1)}(\mu^2/m^2)$ have been calculated in Ref. [52]. The renormalization/factorization scale dependence of the two-loop corrections $H_{ij,\mathbf{I}}^{(2)}(\mu^2/m^2)$ can be obtained, for example, from the results of Ref. [22]:

$$\begin{aligned} H_{q\bar{q},\mathbf{8}}^{(2)}(\mu^2/m^2) &= H_{q\bar{q},\mathbf{8}}^{(2)}(1) + 8.91918 \ln^2 \left(\frac{\mu^2}{m^2} \right) \\ & \quad + 34.7212 \ln \left(\frac{\mu^2}{m^2} \right), \\ H_{gg,\mathbf{8}}^{(2)}(\mu^2/m^2) &= H_{gg,\mathbf{8}}^{(2)}(1) + 9.31619 \ln^2 \left(\frac{\mu^2}{m^2} \right) \\ & \quad + 60.2080 \ln \left(\frac{\mu^2}{m^2} \right), \\ H_{gg,\mathbf{1}}^{(2)}(\mu^2/m^2) &= H_{gg,\mathbf{1}}^{(2)}(1) + 9.31619 \ln^2 \left(\frac{\mu^2}{m^2} \right) \\ & \quad + 38.5239 \ln \left(\frac{\mu^2}{m^2} \right). \end{aligned} \quad (8)$$

The genuine two-loop constants $H_{ij,\mathbf{I}}^{(2)}(1)$ are currently unknown, and are related to the constants $C_{ij,\mathbf{I}}^{(2,0)}$ introduced in (2), as discussed in Appendix A. For simplicity, in Eq. (8) we have given directly the numerical values of the scale dependent term evaluated for $N_F = 5$. In analogy with our discussion of the finite $C^{(2,n)}$ coefficients, which we suppress in the NNLO $_{\beta}$ approximation, we shall set $H_{ij,\mathbf{I}}^{(2)}(\mu^2/m^2) = 0$ in our default NLO + NNLL predictions, as will be motivated by the numerical results of Section 3.2.

To analyze the impact of subleading, power-suppressed corrections we follow Ref. [19] and introduce a power suppressed term controlled by a constant A into the function $H_{ij,\mathbf{I}}^{(1)}$. The choice $A = 0$ sets this additional term to zero, while the default value $A = 2$ was chosen in Ref. [19]. The rationale behind the inclusion of this term was the observation that power suppressed terms $\mathcal{O}(1/N)$ are needed to bring the resummed cross-section closer to the fixed-order NLO result away from the threshold region. We will have more to say about the numerical impact of this term in Section 3.

In deriving the Coulomb contributions $\hat{\sigma}_{ij,\mathbf{I},N}^{(Coul)}$ we follow the approach of Ref. [19] and absorb in it the exact Born cross-section, except for its overall factor of α_s^2 that we attribute to the hard function – see Eq. (7). The functions $\hat{\sigma}_{ij,\mathbf{I},N}^{(Coul)}(\alpha_s(\mu_R^2), \mu_R^2/m^2)$, which only depend on the renormalization scale, are obtained as the Mellin transform of the following x -space functions:

$$\begin{aligned} & \hat{\sigma}_{ij,\mathbf{I}}^{(Coul)}(\rho, \alpha_s(\mu_R^2), \mu_R^2/m^2) \\ &= \frac{\hat{\sigma}_{ij,\mathbf{I}}^{(Born)}(\rho, \alpha_s(\mu_R^2))}{\alpha_s^2(\mu_R^2)} \left\{ 1 + \frac{\alpha_s(\mu_R^2)}{\pi} C_{ij,\mathbf{I}}^{(1)}(\rho) \right. \\ & \quad \left. + \left(\frac{\alpha_s(\mu_R^2)}{\pi} \right)^2 C_{ij,\mathbf{I}}^{(2)}(\rho, \mu_R^2/m^2) + \mathcal{O}(\alpha_s^3) \right\}. \end{aligned} \quad (9)$$

The Born cross-sections, for all reactions and color configurations, have been given in Ref. [19], together with the one-loop Coulomb functions $C_{ij,\mathbf{1}}^{(1)}(\rho) = C_F \pi^2 / (2\beta)$ and $C_{ij,\mathbf{8}}^{(1)}(\rho) = (C_F - C_A/2) \pi^2 / (2\beta)$. The two-loop functions can be extracted from the results of Ref. [22] by matching them to the Mellin-inverse of Eq. (4):

$$\begin{aligned}
C_{gg,1}^{(2)}(\rho, \mu_R^2/m^2) &= \frac{C_F^2 \pi^4}{12\beta^2} - C_F \left(C_F + \frac{C_A}{2} \right) 2\pi^2 \ln(\beta) \\
&+ \frac{\pi^2}{\beta} \left\{ \left(-\frac{11}{12} C_F C_A + \frac{C_F N_F}{6} \right) \ln\left(\frac{2m\beta}{\mu_R}\right) \right. \\
&\left. + \frac{31}{72} C_F C_A - \frac{5}{36} C_F N_F \right\}, \\
C_{gg,8}^{(2)}(\rho, \mu_R^2/m^2) &= C_{gg,1}^{(2)}(\rho, \mu_R^2/m^2) \left[C_F \rightarrow C_F - \frac{C_A}{2} \right], \\
C_{q\bar{q},8}^{(2)}(\rho, \mu_R^2/m^2) &= C_{gg,8}^{(2)}(\rho, \mu_R^2/m^2) + \left(C_F - \frac{C_A}{2} \right)^2 \frac{4\pi^2}{3} \ln(\beta).
\end{aligned} \tag{10}$$

The Mellin transform of Eq. (9) through NLO has been calculated in Ref. [19]. We do not present here the Mellin transform of the NNLO corrections. They are given by large expressions that are straightforward to calculate following the discussion of Ref. [19] and utilizing the following approximation:

$$\begin{aligned}
\ln\left(\frac{1+\beta}{1-\beta}\right) &\approx -\ln(\rho) + 2(0.9991\beta - 0.4828\beta^2 \\
&+ 0.2477\beta^3 - 0.0712\beta^4).
\end{aligned} \tag{11}$$

Before concluding this section we stress again that in Eq. (3) we use the fixed-order NLO result $\sigma^{(\text{NLO})}(m^2)$ and not $\sigma^{(\text{NNLO}\beta)}(m^2)$ as one might expect. The reason is that all the information to be found in the approximate NNLO cross-section $\sigma^{(\text{NNLO}\beta)}$ is already contained, by *matching*, in the all-order resummed result $\hat{\sigma}_{ij,N}^{(\text{res})}$. In particular, the two-loop anomalous dimensions in Eq. (6) control the single $\ln\beta$ terms in $\sigma^{(\text{NNLO}\beta)}$, and the two-loop Coulomb terms in Eq. (9), including the potentials $\sim \ln\beta$, have been matched to the Coulomb terms in $\sigma^{(\text{NNLO}\beta)}$.

To make this point completely transparent we note that the difference between Eq. (3) and its analogue defined by using $\sigma^{(\text{NNLO}\beta)}$ instead (and, of course, subtracting the terms in $\hat{\sigma}_{ij,N}^{(\text{res})}$ through $\mathcal{O}(\alpha_s^4)$) is given in N -space by the following expression:

$$\sum_{ij=q\bar{q},gg} F_{i,N+1} F_{j,N+1} [\hat{\sigma}_{ij,N}^{(\text{res})} |_{\mathcal{O}(\alpha_s^4)\text{ only}}] - \sigma_N^{\text{NNLO}\beta} |_{\mathcal{O}(\alpha_s^4)\text{ only}}. \tag{12}$$

In other words, Eq. (12) represents the difference between the terms of order α_s^4 derived respectively within the resummed N -space and the fixed-order x -space approaches. Since, as we just explained, the two contain the same input they do cancel each other, at least in the limit $N \rightarrow \infty$. In practice Eq. (12) contains power-suppressed terms that behave as $\mathcal{O}(1/N)$ in the soft limit $N \rightarrow \infty$. These power suppressed terms originate in the lower loop (LO and NLO) terms in Eq. (4) and, as it turns out, are not numerically negligible. Given that both terms in Eq. (12) are the result of an approximation, and as we favor N -space resummation since it's less likely to introduce large terms due to the violation of momentum conservation, we prefer not to introduce the power terms (12) into Eq. (3). Such $\mathcal{O}(1/N)$ ambiguity is inherent in the soft approximation irrespective of the details of its implementation and can only be removed by adding to Eq. (3) the full NNLO result, once it becomes available.

3. Study of the theoretical systematics

In this section we focus on the purely theoretical systematics, arising from the scale dependence of the cross-sections, and from the different possible descriptions of higher-order terms not controlled by resummation, like unknown two-loop (threshold) hard

Table 1

Central values and theoretical systematics for the various approximations to σ_{tot} , in pb, at the Tevatron. $m_{\text{top}} = 173.3$ GeV, PDF_{NLO} = MSTW2008nlo68cl, PDF_{NNLO} = MSTW2008nnlo68cl. Row 9, set in bold, gives our best prediction for central value and scale systematics. The predicted cross-sections are presented, if applicable, depending on the values of the constant A and on whether *pure* two-loop Coulomb corrections (10) are included or not.

Approximation	σ_{tot} [pb]	PDF	A	Pure 2-loop Coulomb
1 NLO	6.681 ^{+0.363 (5.4%)} -0.752 (11.3%)	NLO	-	-
2 NLO + NLL	7.070 ^{+0.212 (3.0%)} -0.432 (6.1%)	NLO	0	-
3 NLO + NNLL	6.930 ^{+0.278 (4.0%)} -0.496 (7.2%)	NLO	2	-
4 NNLO $_{\beta}$, $C_{ij}^{(2,0)} = 0$	7.062 ^{+0.240 (3.4%)} -0.334 (4.7%)	NNLO	-	-
5 NNLO $_{\beta}$, $C_{ij}^{(2,0)} = \bar{C}_{ij}^{(2,0)}$	6.853 ^{+0.268 (3.9%)} -0.386 (5.6%)	NNLO	-	-
6 NLO + NNLL	6.844 ^{+0.197 (2.9%)} -0.353 (5.2%)	NNLO	0	NO
7 NLO + NNLL	6.722 ^{+0.212 (3.2%)} -0.391 (5.8%)	NNLO	2	NO
8 NLO + NNLL	6.844 ^{+0.215 (3.1%)} -0.377 (5.5%)	NNLO	0	YES
9 NLO + NNLL	6.722^{+0.243 (3.6%)} -0.410 (6.1%)	NNLO	2	YES

matching constants $H_{ij,1}^{(2)}(1)$ and terms vanishing at threshold. We shall then complete the study of systematics, including PDF and mass dependence, in Section 4.

3.1. Benchmark results

We shall compute reference values for our $t\bar{t}$ cross-section predictions at $m_t = 173.3$ GeV [37].² The central values of these predictions are obtained for $\mu_R = \mu_F = m_t$. Throughout the Letter we use the strong coupling constant evaluated at scale μ_R as provided by the corresponding PDF set. Our default parton distribution set for NLO (and NLO + NLL) is MSTW2008nlo68cl, whereas for NNLO $_{\beta}$ and NNLL resummed calculations we use the MSTW2008nnlo68cl set [56]. In all cases we include them through the LHAPDF interface [57].

The scale systematics is evaluated by varying the renormalization and factorization scales independently in the range suggested in Ref. [28]:

$$m_t/2 < \mu_R, \mu_F < 2m_t, \quad \text{with } 1/2 < \mu_R/\mu_F < 2, \tag{13}$$

and searching for the minimum/maximum of the resulting cross-section. It is usually sufficient to consider only the endpoints of the range (13), namely the pairs $(\mu_r/m_t, \mu_f/m_t) = (2, 1), (0.5, 1), (1, 2), (1, 0.5), (2, 2)$ and $(0.5, 0.5)$. We have verified that a search over a grid with a few hundred points satisfying Eq. (13) in the (μ_F, μ_R) plane agrees to within few per mille with the minimum and maximum rates found in the scan of the endpoints.

Different power-suppressed terms are probed by varying the parameter A over the two values $A = 0$ and $A = 2$, as discussed in [19].

Our numerical results are summarized in Tables 1 and 2. As a benchmark, in rows 1–3 of these tables we present the well-understood NLO and NLO + NLL ($A = 0$ and $A = 2$) results. They are an update of Ref. [19].

² The best measured value for the top mass has been recently updated in Ref. [53] to 173.2 ± 0.9 GeV (for a recent review see [54]). However, for use here the previously published value of 173.3 to facilitate the comparison with other recent theoretical analyses that used this m_t , such as [55]. We estimate (see Section 4) that the change of 0.1 GeV, from 173.3 to 173.2 GeV, would lead to an increase of about 0.3% in the cross-section values, well within the overall theoretical uncertainties.

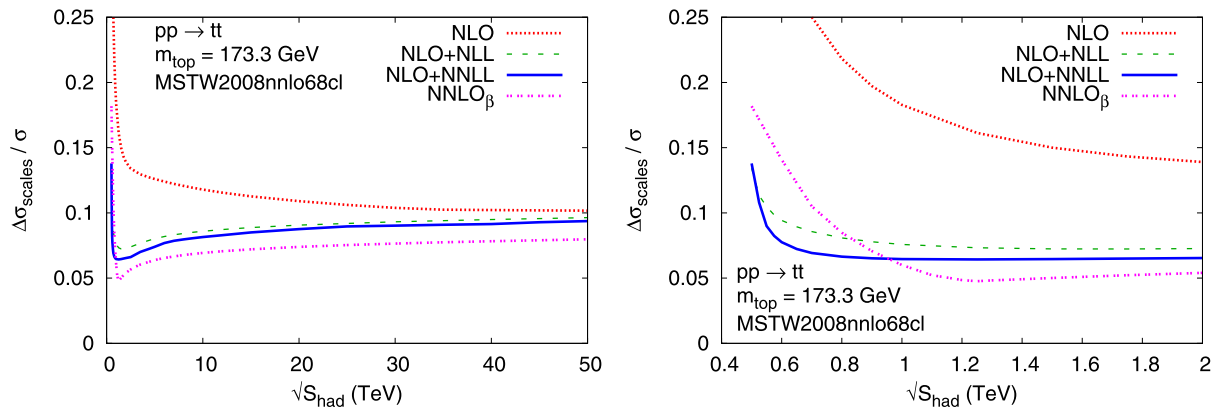


Fig. 1. Relative half uncertainty in the total inclusive $t\bar{t}$ cross-section as a function of the center-of-mass energy $\sqrt{S_{\text{had}}}$ of a proton–proton collider. Left plot: whole energy range up to $\sqrt{S_{\text{had}}} = 50$ TeV. Right plot: blow-up of the threshold region, up to $\sqrt{S_{\text{had}}} = 2$ TeV.

Table 2
As in Table 1 but for the LHC at 7 TeV.

	Approximation	σ_{tot} [pb]	PDF	A	Pure 2-loop Coulomb
1	NLO	$158.1^{+19.5(12.3\%)}_{-21.2(13.4\%)}$	NLO	–	–
2	NLO + NLL	$174.8^{+17.6(10.1\%)}_{-15.3(8.8\%)}$	NLO	0	–
3	NLO + NLL	$167.1^{+14.3(8.6\%)}_{-15.4(9.2\%)}$	NLO	2	–
4	NNLO $_{\beta}$, $C_{ij}^{(2,0)} = 0$	$161.2^{+11.3(7.0\%)}_{-10.8(6.7\%)}$	NNLO	–	–
5	NNLO $_{\beta}$, $C_{ij}^{(2,0)} = \bar{C}_{ij}^{(2,0)}$	$154.0^{+12.0(7.8\%)}_{-8.6(5.6\%)}$	NNLO	–	–
6	NLO + NNLL	$161.5^{+14.5(9.0\%)}_{-12.3(7.6\%)}$	NNLO	0	NO
7	NLO + NNLL	$155.9^{+11.5(7.4\%)}_{-13.0(8.3\%)}$	NNLO	2	NO
8	NLO + NNLL	$164.7^{+15.0(9.1\%)}_{-12.8(7.8\%)}$	NNLO	0	YES
9	NLO + NNLL	$158.7^{+12.2(7.7\%)}_{-13.5(8.5\%)}$	NNLO	2	YES

In row 4 we present the NNLO $_{\beta}$ approximation as defined in Section 2, and in row 5 we show the effect of including non-zero values for the constants $C_{ij}^{(2,0)}$, setting them to the value $\bar{C}_{ij}^{(2,0)}$ defined in Appendix A in Eq. (A.2). We notice that, while the numerical impact of including these constants is noticeable, it is smaller than the overall scale uncertainty. Therefore we argue that the scale variation can largely account for the uncertainty stemming from unknown part of the higher-order terms.

At order α_s^4 the fixed-order NNLO approximation contains terms that are not predicted by the NLO + NLL result, like NNLL soft-enhanced terms and pure two-loop Coulomb terms, i.e. α_s^4 Coulomb terms that do not arise in the expansion of Eq. (4) from the product of one-loop contributions to $\hat{\sigma}_N^{(\text{Coul})}$ with $\hat{\sigma}^{(\text{Hard})} \Delta_{N+1}$. It lacks however terms at $\mathcal{O}(\alpha_s^5)$ and beyond that are contained in the resummed results. The NLO + NNLL approximation combines both these ingredients, and is superior to the NNLO $_{\beta}$ one, since it contains all the information to be found in NNLO $_{\beta}$, plus the towers of soft LL, NLL and NNLL logs beyond order $\mathcal{O}(\alpha_s^4)$. The NLO + NNLL rates are given in rows 6–9, where we also describe the impact of subleading $1/N$ terms (through the constant A), and of the two-loop Coulomb effects that were absent in the NLO + NLL results of Ref. [19].

We observe that these two-loop Coulomb terms (10) have a sub-per-mille effect on the central values for the Tevatron. At the LHC the effect is larger, of order 2%. Their effect on the scale uncertainty is at most at the few-per-mille level and is thus negligible (for both Tevatron and LHC). We do not resum the Coulombic cor-

rections beyond order α_s^4 . This resummation has been performed in Ref. [55] and the effect was found to be negligible.

From Tables 1 and 2 we also observe a dependence of the predicted cross-section on the value of the constant A. As we emphasized before, we consider the inclusion of non-zero A as a model for the power suppressed terms $\sim 1/N$ that are not controlled by the threshold approximation. We observe a modest 0.5% decrease in scale dependence from including $A \neq 0$ and a 2% shift of the central value, which is consistent with the overall scale systematics (the corresponding changes for the LHC are 0.5% and 3% respectively). We conclude that, while our estimate of the size of the power suppressed terms is not comprehensive, it clearly shows that power suppressed terms can be a significant, few-percent effect on the central value both at the Tevatron and LHC.

Before closing this section we offer an alternative graphical representation of the scale uncertainty of the various scenarios given in Tables 1 and 2. In Fig. 1 we plot the relative half uncertainty due to scale variations (defined as $\Delta\sigma_{\text{scales}}/\sigma \equiv (\sigma_{\text{max}} - \sigma_{\text{min}})/(2\sigma_{\text{central}})$), so that one can quote for the cross-section $\sigma_{\text{central}} \pm \Delta\sigma_{\text{scales}}$ as a function of the hadronic center of mass energy for pp collisions and for a top mass of 173.3 GeV. As expected, the uncertainty of the NLO calculation is largest when the production is closest to threshold, and decreases for larger $\sqrt{S_{\text{had}}}$. Upon resummation of the threshold logarithms, the biggest improvement (i.e. reduction in uncertainty) can likewise be obtained close to the threshold. One can easily observe the better stability of the NLO + NLL and NLO + NNLL results when the threshold is approached. As the $t\bar{t}$ production takes place at larger $\sqrt{S_{\text{had}}}$ the effect of resummation is reduced, as expected, since the resummed logarithms become smaller. When $S_{\text{had}} \gg 4m^2$ one does not expect any significant improvement from a resummation of logarithms that are strongly suppressed. Indeed, one observes from the plot that the uncertainty of the resummed results is practically identical to that of the fixed-order calculation in this limit.³

3.2. Impact of scale dependent finite terms at $\mathcal{O}(\alpha_s^4)$

We discuss here the effect of the $\mathcal{O}(\alpha_s^4)$ constant terms $C_{ij}^{(2,n)}$, which we introduced in (2) and which we suggested should not be incorporated in either NNLO $_{\beta}$ or in the NNLL resummed results. Their main impact is the large reduction in the scale variation of

³ The NNLO $_{\beta}$ result seems to display a slightly smaller uncertainty than the resummed ones. The difference is however likely not significant and, in particular, it should not be considered as suggestive that this approximation constitutes a better prediction.

the cross-section, due to the scale logarithms present for $n = 1, 2$, whose coefficients compensate by construction the scale dependence of $\mathcal{O}(\alpha_s^3)$ terms. If we focus on the NNLO $_{\beta}$ case, the addition of the known $C_{ij}^{(2,n)}$ contributions ($n = 1, 2$), for different choices of the unknown $n = 0$ terms, leads to the following results:

$$\text{Tevatron: } \sigma^{\text{NNLO}_{\beta}^{(2)}} = 6.853_{-0.408}^{+0.092} (1.3\%) \text{ pb} \\ \text{for } C_{ij}^{(2,0)} = \bar{C}_{ij}^{(2,0)}, \quad (14)$$

$$\sigma^{\text{NNLO}_{\beta}^{(2)}} = 7.062_{-0.262}^{+0.064} (0.9\%) \text{ pb} \\ \text{for } C_{ij}^{(2,0)} = 0, \quad (15)$$

$$\text{LHC: } \sigma^{\text{NNLO}_{\beta}^{(2)}} = 154.0_{-3.7}^{+2.8} (1.8\%) \text{ pb} \text{ for } C_{ij}^{(2,0)} = \bar{C}_{ij}^{(2,0)}, \quad (16)$$

$$\sigma^{\text{NNLO}_{\beta}^{(2)}} = 161.2_{-4.7}^{+2.1} (1.3\%) \text{ pb} \text{ for } C_{ij}^{(2,0)} = 0. \quad (17)$$

The central values coincide with those obtained in absence of the $C_{ij}^{(2,n)}$ ($n = 1, 2$) terms, since the logarithms vanish at the central value $\mu = m$. However, the scale dependence is much smaller than in rows 4 and 5 of our previous tables. At the Tevatron (LHC), the scale dependence drops from about $\pm 5\%$ ($\pm 7\%$) to about $\pm 3\%$ ($\pm 2\%$). This significant reduction clashes however with the comparable or larger cross-section variations (3% at the Tevatron and 4% at the LHC) induced by the variation of $C_{ij}^{(2,0)}$ within an a priori reasonable range. We conclude that the significant reduction in scale dependence in presence of $C_{ij}^{(2,0)}$ ($n = 1, 2$) terms cannot be interpreted as a genuine reduction in the total theoretical uncertainty, unless it is combined with the systematics emerging from the unknown value of the two-loop constants $C_{ij}^{(2,0)}$.

Similar conclusions can be drawn from the study of the NLO + NNLL results. From the viewpoint of threshold expansion, in the limit $N \rightarrow \infty$ one has to neglect the unknown constants $H_{ij,1}^{(2)}(1)$ and the N -independent, μ_F, μ_R -dependent logarithmic terms, i.e. the whole two-loop hard function $H_{ij,1}^{(2)}(\mu^2/m^2)$. This is completely analogous to what happens in the NNLO $_{\beta}$ approximation. In case the scale dependent terms in $H_{ij,1}^{(2)}(\mu^2/m^2)$ are retained, the theoretical uncertainty should include the variation of the unknown two-loop constant $H_{ij,1}^{(2)}(1)$, again as in the NNLO $_{\beta}$ approximation. For the sake of documentation, we quote here the relevant results for the scale dependence obtained after inclusion of the hard function $H_{ij,1}^{(2)}(\mu^2/m^2)$, exploring as an example the two cases of $H_{ij,1}^{(2)}(1) = 0$ and $H_{ij,1}^{(2)}(1) = \bar{H}_{ij,1}^{(2)}(1)$ introduced in Appendix A:

$$\text{Tevatron: } \sigma^{\text{NLO+NNLL}^{H^{(2)}}} = 6.722_{-0.320}^{+0.017} (0.3\%) \text{ pb} \\ \text{for } H_{ij,1}^{(2)}(1) = 0, \quad (18)$$

$$\sigma^{\text{NLO+NNLL}^{H^{(2)}}} = 6.968_{-0.224}^{+0.009} (0.1\%) \text{ pb} \\ \text{for } H_{ij,1}^{(2)}(1) = \bar{H}_{ij,1}^{(2)}(1), \quad (19)$$

$$\text{LHC: } \sigma^{\text{NLO+NNLL}^{H^{(2)}}} = 158.7_{-6.9}^{+5.6} (3.6\%) \text{ pb} \\ \text{for } H_{ij,1}^{(2)}(1) = 0, \quad (20)$$

$$\sigma^{\text{NLO+NNLL}^{H^{(2)}}} = 167.9_{-7.5}^{+5.2} (3.1\%) \text{ pb} \\ \text{for } H_{ij,1}^{(2)}(1) = \bar{H}_{ij,1}^{(2)}(1), \quad (21)$$

where $A = 2$ throughout. These scale uncertainties are slightly larger than those found for NNLO $_{\beta}$ in (14)–(17), but they are still small compared to the impact of the unknown finite contributions

of $H_{ij,1}^{(2)}(1)$, as suggested by the comparison between the $H = 0$ and $H = \bar{H}$ results above.

These remarks justify our choice not to include the $H_{ij,1}^{(2)}$ function in our benchmark predictions for the central value and the theoretical systematics. We notice nevertheless that the significant reduction in scale variation obtained with the inclusion of the known, finite scale dependent terms, is indicative of the uncertainty of the full NNLO result, when it will become available.

4. Phenomenology

It is well known that the purely theoretical uncertainty related to the lack of high-order corrections is only a fraction of the overall systematics. Recent studies of the $t\bar{t}$ cross-section uncertainty due to the PDF parameterization, including the latest fits by several groups, have been reported in [58,59]. These studies considered the fixed-order NLO results and the approximate NNLO calculation of Ref. [20], as implemented in the program HATHOR [60]. The main conclusion of those studies was the consistency, at the LHC energy, between the central values and the uncertainty bands obtained using most PDF fits (MSTW08 [56], NNPDF2.1 [59], GJR [61], CT10 [62]), both at NLO and NNLO, with some differences, incompatible with the estimates of the systematics, with respect to other sets such as ABKM09 [63] and HERAPDF [64].

Given the minor changes in fixed-order versus resummed predictions, we expect that the PDF uncertainties estimated in [58,59] should not be affected by resummation. We verify this result explicitly here, considering our default PDF set MSTW2008nnlo68cl [56], and the PDF set NNPDF21_nnlo_nf5_100 [59].

Our results for the total $t\bar{t}$ cross-section using MSTW2008nnlo-68cl are:

$$\sigma_{\text{tot}}^{\text{NLO+NNLL}}(\text{Tevatron}; m_t = 173.3 \text{ GeV}) \\ = 6.722_{-0.410}^{+0.238} (3.5\%) [\text{scales}]_{-0.115}^{+0.160} (2.4\%) [\text{PDF}] \text{ pb}, \\ \sigma_{\text{tot}}^{\text{NLO+NNLL}}(\text{LHC}_7 \text{ TeV}; m_t = 173.3 \text{ GeV}) \\ = 158.7_{-13.5}^{+12.2} (7.7\%) [\text{scales}]_{-4.4}^{+4.3} (2.7\%) [\text{PDF}] \text{ pb}, \quad (22)$$

where we defined the upper and lower limit of the scale variation using the endpoint scan defined after Eq. (13). With NNPDF21_nnlo_nf5_100 we obtain instead⁴:

$$\sigma_{\text{tot}}^{\text{NLO+NNLL}}(\text{Tevatron}; m_t = 173.3 \text{ GeV}) \\ = 7.021_{-0.436}^{+0.250} (3.6\%) [\text{scales}]_{-0.119}^{+0.126} (1.8\%) [\text{PDF}] \text{ pb}, \\ \sigma_{\text{tot}}^{\text{NLO+NNLL}}(\text{LHC}_7 \text{ TeV}; m_t = 173.3 \text{ GeV}) \\ = 163.1_{-14.2}^{+12.9} (7.9\%) [\text{scales}]_{-4.9}^{+4.9} (3.0\%) [\text{PDF}] \text{ pb}. \quad (23)$$

The PDF uncertainties in both these sets of predictions should be considered to be at the $1 - \sigma$ level.

A few comments are in order. To start with, we confirm that the relative PDF uncertainty at NLO + NNLL is similar to that derived from a fixed-order calculation, as in [58,59]. We also notice that the scale uncertainty is rather independent of the PDF set. This is consistent with the fact that the relative contribution of $gg, q\bar{q}$ and qg initial states does not change significantly when changing PDFs.

We also confirm the consistency of the central value and of the PDF uncertainty estimated, for the LHC at 7 TeV, using the MSTW and the default NNPDF2.1 sets. We note, on the other hand,

⁴ As a central value we take the number derived with the central NNPDF set, not the mean over the whole set of PDFs. The difference is at the per-mille level and thus completely negligible.

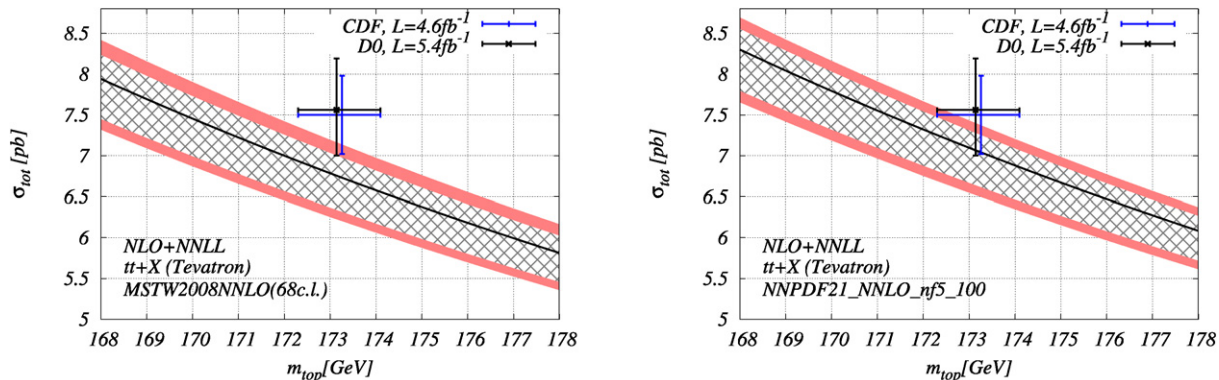


Fig. 2. Theoretical prediction for the total inclusive $t\bar{t}$ cross-section at the Tevatron, as a function of the top mass, versus the measurements of Refs. [30,39]. The left plot is for the MSTW2008nnlo68cl PDF set, the right plot for NNPDF21_nnlo_nf5_100. The uncertainty is a linear sum of scale uncertainty (the white central band) and PDF uncertainty (red bands). The central value is shown with a black line. The theoretical predictions in this figure correspond to row 9 in Table 1. The horizontal bars on the measurements reflect the uncertainty in the measured top mass (see footnote 2). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this Letter.)

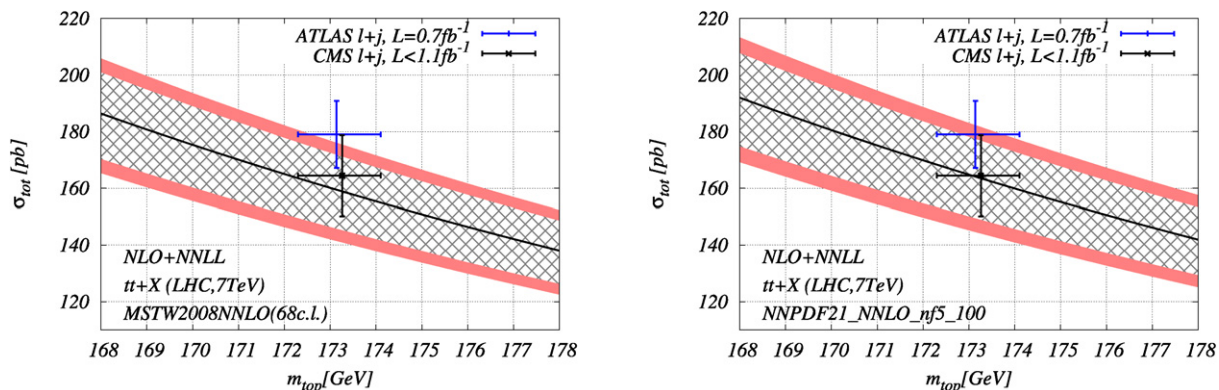


Fig. 3. Theoretical prediction for the total inclusive $t\bar{t}$ cross-section at the LHC at 7 TeV as a function of the top mass versus the measurements of Refs. [40,41]. The left plot is for the MSTW2008nnlo68cl PDF set, the right plot for NNPDF21_nnlo_nf5_100. The uncertainty is a linear sum of scale uncertainty (the white central band) and PDF uncertainty (red bands). The central value is shown with a black line. The theoretical predictions in this figure correspond to row 9 in Table 2. The horizontal bars on the measurements reflect the uncertainty in the measured top mass (see footnote 2). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this Letter.)

that at the Tevatron the NNPDF prediction is larger than MSTW by about 5%, compared to the individual estimates of PDF systematics, which are of the order of $\pm 2\%$. This difference can be understood in terms of the different values of the strong coupling constant associated with the MSTW2008nnlo68cl ($\alpha_s(M_Z) = 0.117$) and the NNPDF21_nnlo_nf5_100 ($\alpha_s(M_Z) = 0.119$) sets, and the fact that the cross-section scales like α_s^2 . To better quantify this effect we have also used the set NNPDF21_nnlo_as_0117_100 [59] to compute the central values corresponding to Eq. (23):

$$\begin{aligned} \sigma_{\text{tot}}^{\text{NLO+NNLL}}(\text{Tevatron}; m_t = 173.3 \text{ GeV}) &= 6.742 \text{ pb}, \\ \sigma_{\text{tot}}^{\text{NLO+NNLL}}(\text{LHC}_7 \text{ TeV}; m_t = 173.3 \text{ GeV}) &= 156.8 \text{ pb}. \end{aligned} \quad (24)$$

These results confirm that the apparent inconsistency between the Tevatron predictions of the NNPDF and MSTW NNLO sets disappears when they both use the same coupling constant ($\alpha_s(M_Z) = 0.117$ in this case). This also suggests that an additional uncertainty of the order of $\pm 1\text{--}2\%$ should likely be added to any $t\bar{t}$ total cross-section evaluation as a result of the uncertainty with which α_s is known, if not already included with the PDF uncertainty.

We plot in Figs. 2 and 3 our predictions for the total $t\bar{t}$ cross-section as a function of the top mass in the range 168–178 GeV for both Tevatron and LHC ($\sqrt{s} = 7 \text{ TeV}$). In view of the difference between the MSTW and NNPDF results for the Tevatron, we present the results for the two PDF sets on different plots. The un-

certainties from scales and parton distributions quoted in Eq. (22) are added linearly. On the same figures we compare our theoretical predictions with the most accurate available experimental measurements from the Tevatron [30,39] and LHC [40,41]. We display the experimental points at the current world-average value of $m_t = 173.2 \pm 0.9 \text{ GeV}$, without applying any correction factor to account for the difference in experimental acceptance with respect to the m_t values used in the measurements. These amount to a reduction in rate at the sub-percent level.

We observe that the uncertainties of the current theoretical predictions and experimental measurements are comparable in size. The predictions agree with all measurements within the uncertainties, although at the Tevatron the data tend to be on high side of the theoretical band, particularly in the case of the MSTW cross-sections.

For ease of use, we have fitted the mass dependence of the $\sigma^{\text{NLO+NNLL}}$ predictions relative to the MSTW sets using the functional form

$$\sigma(m) = \sigma(m_{\text{ref}}) \left(\frac{m_{\text{ref}}}{m} \right)^4 \left(1 + a_1 \frac{m - m_{\text{ref}}}{m_{\text{ref}}} + a_2 \left(\frac{m - m_{\text{ref}}}{m_{\text{ref}}} \right)^2 \right). \quad (25)$$

The resulting parameters, for the central curve as well as the scales and PDF uncertainties separately, are collected in Table 3. They provide fits that are accurate to within about one per mille in the

Table 3

Parameters resulting from the fit of the functional form in Eq. (25) to our best prediction, NLO + NNLL, for the top cross-section at the Tevatron and the LHC (7 and 8 TeV) as a function of the top mass, including the uncertainties from scale variations and PDFs (the MSTW2008nnlo68cl set). These parameters provide fits that are accurate to within about one per mille in the 150–200 GeV mass range, but should not be used indiscriminately much beyond this region.

$m_{ref} = 173 \text{ GeV}$		$\sigma(m_{ref}) \text{ (pb)}$	a_1	a_2
Tevatron, $p\bar{p}$ at $\sqrt{s} = 1.96 \text{ TeV}$	Central	6.785	-1.394	7.451×10^{-1}
	Scales +	7.030	-1.409	8.047×10^{-1}
	Scales -	6.370	-1.379	6.919×10^{-1}
	PDFs +	6.946	-1.373	7.106×10^{-1}
	PDFs -	6.669	-1.408	7.527×10^{-1}
LHC, pp at $\sqrt{s} = 7 \text{ TeV}$	Central	160.1	-1.191	8.042×10^{-1}
	Scales +	172.4	-1.224	9.096×10^{-1}
	Scales -	146.5	-1.162	7.957×10^{-1}
	PDFs +	164.4	-1.175	7.867×10^{-1}
	PDFs -	155.7	-1.205	8.416×10^{-1}
LHC, pp at $\sqrt{s} = 8 \text{ TeV}$	Central	228.6	-1.069	6.798×10^{-1}
	Scales +	246.8	-1.104	7.335×10^{-1}
	Scales -	208.8	-1.042	6.299×10^{-1}
	PDFs +	234.2	-1.054	6.533×10^{-1}
	PDFs -	222.7	-1.083	6.964×10^{-1}

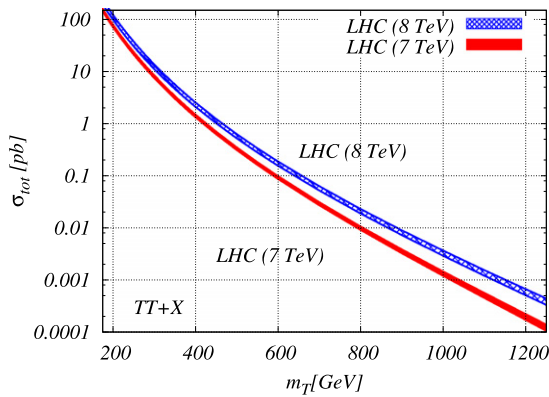


Fig. 4. Theoretical prediction for the total inclusive production cross-section at the LHC (7 and 8 TeV) of a $T\bar{T}$ pair of hypothetical heavy fourth-generation quarks. Effects from the running of top quarks in loops are expected to be very small and are neglected in these numerical predictions.

150–200 GeV mass range, but should not be used indiscriminately much beyond this region.

Finally, due to the interest in current experimental searches of possible [65] fourth-generation quarks, both at the Tevatron [66–68] and at the LHC [69], we extend our results to the production of a hypothetical very heavy quark, T . Its production cross-section at the LHC (for $\sqrt{s} = 7$ and 8 TeV) up to $m_T = 1200$ GeV is shown in Fig. 4, and the detailed breakdown of the systematics is given, for a set of mass values, in Table 4. The cross-sections are calculated by simply changing the mass parameter value in the top cross-section calculation, thus neglecting the small corrections due to the top quark in the evolution of α_s and of PDFs.

The overall uncertainty (scale + PDF) is roughly uniform at the ± 10 –15% level in the range of the plot. This is the result of a decreasing scale uncertainty, which is more than compensated by an increasing PDF uncertainty, due to the large partonic x values probed by the production of a large mass object, and PDFs being generally less well known in this region.

Finally, we present our best predictions for the LHC configuration foreseen for the 2012 data taking ($\sqrt{s} = 8$ TeV), with the PDF sets MSTW2008nnlo68cl [56]:

$$\begin{aligned} \sigma_{\text{tot}}^{\text{NLO+NNLL}}(\text{LHC}_8 \text{ TeV}; m_t = 173.3 \text{ GeV}) \\ = 226.6_{-19.4(8.6\%)}^{+17.8(7.8\%)} [\text{scales}]_{-5.8(2.6\%)}^{+5.6(2.5\%)} [\text{PDF}] \text{ pb}, \end{aligned} \quad (26)$$

and NNPDF21_nnlo_nf5_100 [59]:

$$\begin{aligned} \sigma_{\text{tot}}^{\text{NLO+NNLL}}(\text{LHC}_8 \text{ TeV}; m_t = 173.3 \text{ GeV}) \\ = 233.5_{-20.5(8.8\%)}^{+18.9(8.1\%)} [\text{scales}]_{-6.5(2.8\%)}^{+6.5(2.8\%)} [\text{PDF}] \text{ pb}. \end{aligned} \quad (27)$$

All of the numerical results presented in this section, and their extrapolation to different values of the heavy quark mass or of the scale parameters, can be obtained through a simple web interface [70]. These, and more general results obtained under the various approximation scenarios outlined in Section 3, can furthermore be computed with the help of the program TOP++ [71].

5. Concluding remarks

Using recent theoretical developments, we extend in this Letter the soft-gluon resummation of the total $t\bar{t}$ cross-section to the NNLL order, using the Mellin N -space formalism. The result includes all known NNLO terms that are singular at the production threshold. The current work represents the third-order logarithmic improvement for this important collider observable that has been instrumental in developments in precision collider physics.

We explored the implications of the NNLL approximation in a comprehensive phenomenological study of the total $t\bar{t}$ cross-section, to quantify the full theoretical uncertainty currently associated with this observable.

In fixed-order calculations the theoretical uncertainty is typically identified with the residual scale sensitivity of an observable. While not perfect, such a procedure is well understood and gives a meaningful way of comparing theoretical predictions across different observables and levels of precision. The procedure relies on the following considerations: since the exact result must be scale independent, and the scale dependence of each fully calculated order must be of higher order, one assumes that the residual scale dependence of a calculation is numerically comparable to the higher-order scale independent terms, which can only be obtained via the complete calculation, and whose size the theoretical systematics attempt to estimate.⁵ When dealing with an approximate NNLO calculation for $t\bar{t}$ hadroproduction, and trying to assess its uncertainty via the residual scale dependence, one must keep in mind that known terms of $\mathcal{O}(\alpha_s^4)$ include: (a) terms singular at the production threshold, both scale dependent and independent, whose behavior at higher orders is determined by general dynamical considerations, and which can therefore be included and resummed, with a genuine improvement of the accuracy; (b) finite, but scale dependent, terms, whose value can be fixed by imposing full $\mathcal{O}(\alpha_s^4)$ scale independence. Inclusion of such terms will lead, by construction, to a reduction of the scale dependence, but this reduction does not reflect the real size of the theoretical uncertainty, which is rather governed by the unknown constant terms of $\mathcal{O}(\alpha_s^4)$.

In this Letter we assessed the possible size of several unknown higher-order contributions, and studied their contribution to the theoretical uncertainty. In particular, we demonstrated that the reduced scale sensitivity, obtained by using the exact $\mathcal{O}(\alpha_s^4)$ scale dependence, leads, once the uncertainty of the unknown terms is

⁵ A detailed presentation of these well-known assumptions is given in [72], where they are then used to argue for an alternative way of characterizing the perturbative theoretical uncertainty. The method proposed in this Letter has however so far only been detailed for e^+e^- collisions, and cannot therefore be applied to hadronic top production.

Table 4
Total cross-sections, at NLO + NNLL level, for the production of a heavy quark at the LHC ($\sqrt{s} = 7$ and 8 TeV), including the uncertainties from scale variations and PDFs (using the MSTW2008nnlo68cl set).

m_T (GeV)	LHC, $\sqrt{s} = 7$ TeV			LHC, $\sqrt{s} = 8$ TeV		
	σ_{tot} [pb]	Scale	PDF	σ_{tot} [pb]	Scale	PDF
200	74.71	+5.498 (7.4%) -6.165 (8.3%)	+2.189 (2.9%) -2.205 (3.0%)	108.59	+8.100 (7.5%) -9.022 (8.3%)	+2.940 (2.7%) -2.980 (2.7%)
300	7.83	+0.469 (6.0%) -0.563 (7.2%)	+0.275 (3.5%) -0.272 (3.5%)	12.09	+0.750 (6.2%) -0.882 (7.3%)	+0.399 (3.3%) -0.397 (3.3%)
400	1.38	+0.070 (5.1%) -0.089 (6.5%)	+0.053 (3.9%) -0.052 (3.8%)	2.25	+0.119 (5.3%) -0.148 (6.6%)	+0.082 (3.7%) -0.081 (3.6%)
500	0.32	+0.015 (4.6%) -0.019 (6.0%)	+0.014 (4.3%) -0.013 (4.1%)	0.56	+0.026 (4.6%) -0.034 (6.1%)	+0.022 (4.0%) -0.022 (3.9%)
	σ_{tot} [fb]	Scale	PDF	σ_{tot} [fb]	Scale	PDF
600	90.80	+3.945 (4.3%) -5.059 (5.6%)	+4.380 (4.8%) -4.031 (4.4%)	166.74	+7.290 (4.4%) -9.480 (5.7%)	+7.240 (4.3%) -6.860 (4.1%)
700	28.60	+1.191 (4.2%) -1.512 (5.3%)	+1.613 (5.6%) -1.406 (4.9%)	56.04	+2.349 (4.2%) -3.021 (5.4%)	+2.747 (4.9%) -2.475 (4.4%)
800	9.76	+0.391 (4.0%) -0.494 (5.1%)	+0.652 (6.7%) -0.546 (5.6%)	20.50	+0.827 (4.0%) -1.058 (5.2%)	+1.153 (5.6%) -0.995 (4.9%)
900	3.52	+0.135 (3.8%) -0.172 (4.9%)	+0.280 (8.0%) -0.229 (6.5%)	7.97	+0.310 (3.9%) -0.397 (5.0%)	+0.518 (6.5%) -0.436 (5.5%)
1000	1.32	+0.049 (3.7%) -0.063 (4.8%)	+0.125 (9.5%) -0.101 (7.7%)	3.24	+0.122 (3.8%) -0.157 (4.8%)	+0.246 (7.6%) -0.202 (6.2%)
1100	0.51	+0.018 (3.6%) -0.023 (4.6%)	+0.057 (11.2%) -0.046 (9.1%)	1.36	+0.049 (3.6%) -0.064 (4.7%)	+0.120 (8.9%) -0.098 (7.2%)
1200	0.20	+0.007 (3.5%) -0.009 (4.6%)	+0.026 (13.1%) -0.021 (10.7%)	0.58	+0.021 (3.5%) -0.027 (4.6%)	+0.060 (10.3%) -0.049 (8.3%)

accounted for, to a larger overall systematics, comparable to that of the NLO + NLL cross-section.

We also demonstrated that the predicted cross-section has a few-percent sensitivity to currently unknown $1/N$ suppressed terms that are beyond any of the approximations available in the literature. Summarizing these observations we conclude that at present the total uncertainty of the total $t\bar{t}$ cross-section at the NLO + NNLL order is only modestly lower compared to the long-established NLO + NLL result. Guided by the small scale dependence of the results obtained imposing the exact $\mathcal{O}(\alpha_s^4)$ scale dependence, we nevertheless speculate a significant decrease of the theoretical uncertainty in the total $t\bar{t}$ cross-section once the full NNLO result becomes available.

Finally we would like to briefly compare our work with theoretical works that have appeared in the recent past and that make, to various extent, use of NNLO approximations.

Ref. [20] uses a fixed-order approximate NNLO approach to the total inclusive cross-section, including the $C^{(2,n)}$ terms that implement the exact $\mathcal{O}(\alpha_s^4)$ scale dependence. The overall uncertainty is estimated by just varying the scale in this framework, without accounting for the uncertainty of the finite $C^{(2,0)}$ pieces, leading, as we argued above, to a much reduced and in our view optimistic systematics. This calculation has been implemented in the program HATHOR [60].

Ref. [55] pursues a resummation approach that shares many similarities with our work. Its authors resum directly the total inclusive cross-section by implementing the same anomalous dimensions and 2-loop Coulomb terms used here, and do not impose exact $\mathcal{O}(\alpha_s^4)$ scale dependence. The resummation method instead differs. In Ref. [55] the so-called momentum space approach of Ref. [73] is used, which is an x -space approach, while we use an N -space resummation, followed by a Mellin inversion. In the approach of Ref. [73], the x -space perturbative expansion of the resummed cross-section is convergent, while in our approach the perturbative expansion of our N -space result is convergent, and its Mellin inversion to x space is asymptotic. This feature has been criticized as a drawback of the Mellin space approach⁶ in Ref. [73]. However, we remind the reader that the ambiguity associated with

the asymptotic nature of the Mellin inversion is very weak, corresponding to an effect that is suppressed more strongly than any inverse power of the process scale, and that in practice has totally negligible effects. Furthermore, factorization in N guarantees naturally momentum conservation. Although momentum conservation can be abandoned in the soft approximation, it was shown in [42] that it can lead to large subleading effects. We thus believe that the Mellin space approach is worth pursuing for this positive feature.⁷

At the approximate NNLO order, our NNLO $_{\beta}$ rates and scale systematics for $C^{(2,0)} = 0$ (Tables 1 and 2) agree precisely with the equivalent results, labeled NNLO $_{\text{app}}$, in Tables 8 and 10 of [55]. After resummation, the differences with our work in the final predictions and theoretical systematics must be attributed to the different formalism. One such source of difference, for example, is that in the x -space resummation approach additional scales are present (in our case these are only μ_F and μ_R). We also note the different default values used for the unknown two-loop constants, which is also a reflection of the different formalisms used, since in the N -space approach the resummed terms vanish for $N = 1$ by construction.⁸ It is perhaps surprising that the largest difference among central values is observed for the Tevatron, while at the LHC central values are very close. This could be related to the observation made in [55], namely that the contribution of the $q\bar{q}$ channel is poorly approximated by the threshold expansion. Due to the dominance of this channel at the Tevatron, Ref. [55] argues that this could also explain why the Tevatron uncertainty does not improve after NNLL resummation. In all cases, the numerical differences are nevertheless consistent with the overall uncertainties quoted both in our work and in [55].

An alternative approach to the total $t\bar{t}$ cross-section has been pursued in Refs. [23,26,27]. Like Ref. [55], Refs. [23,26,27] are based on the momentum space approach. There are a number of additional differences between our work and these papers. They

this pole is irrelevant for the derivation of the N space formula, that in fact has a convergent perturbative expansion.

⁷ A critical comparison of the x - and N -space method at the analytic level has been presented in Ref. [74] for the case of Drell-Yan pair production.

⁸ This is to avoid introducing corrections at $N = 1$ from a formalism that is valid at large N .

⁶ In Ref. [73] it is claimed that integration over the Landau pole also arise in the computation of the N -space resummation formula. We remark, however, that

resum not the total but the differential cross-section for $t\bar{t}$ production. Once resummation is performed at the differential level, the differential distribution is integrated over phase space to obtain the total inclusive rate. In such an approach the leading terms at absolute threshold are correctly reproduced (see Ref. [45]) but one introduces a different set of unknown power corrections (as also pointed out in Ref. [55]). Various choices for the hard scales, which become available when calculating differential quantities, have been explored in Refs. [23,26,27]. While the central value of the total cross-section in Ref. [27] is slightly different from ours and from the results of Ref. [55], it is reassuring that they all still fit within the quoted uncertainty bands (the minor difference in the reference top mass in [27], $m_{\text{top}} = 173.1$ GeV, has no impact in this comparison).

Overall, the non-PDF related uncertainties in the x -space resummation approaches [27,55] tend to be smaller than ours (by between about 25% to 40%), with the exception of the Tevatron prediction of Ref. [55], which has a corresponding uncertainty about 20% larger than ours.

An approach that shares similarities with [23,26,27] has been pursued by Ref. [25]. In that reference an approximate, fixed-order truncation of the differential cross-section is derived and integrated over phase space to obtain the fully inclusive cross-section. For the LHC the scale variation and central values derived in Ref. [25] are similar to those of Ref. [60] (which is about 50% smaller than our benchmark result). For the Tevatron the central values of these two references are also rather close, while the uncertainty of Ref. [25] is much smaller than that of all other groups (it is about 60% smaller than ours); see also Ref. [75].

Similarities and differences between some of the approaches above have already been addressed in Refs. [75,55]. Such significant differences can be partially understood with the help of the discussion in Section 3. Overall, the large differences between central values and systematics reported in the various papers discussed in this section appear to be another confirmation of our conclusions about the precision with which the total $t\bar{t}$ cross-section is presently calculated.

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Appendix A. Properties of the finite threshold terms

The presently unknown constants $C_{ij,1}^{(2,0)}$ introduced in Eq. (2) are related to the also unknown constants $H_{ij,1}^{(2)}(1)$ appearing in the two-loop hard matching function (7). With a direct calculation, and presenting directly numerical values, we obtain:

$$\begin{aligned} C_{q\bar{q},8}^{(2,0)} &= -489.168 + 16H_{q\bar{q},8}^{(2)}(1), \\ C_{gg,8}^{(2,0)} &= -1334.18 + 16H_{gg,8}^{(2)}(1), \\ C_{gg,1}^{(2,0)} &= -643.397 + 16H_{gg,1}^{(2)}(1), \end{aligned} \quad (\text{A.1})$$

Table 5

Values of $\Delta_{ij,1}$ in pb for $\mu_R = \mu_F = m = 173.3$ GeV and the MSTW2008nnlo68cl PDF set.

Collider	$\Delta_{qq,8}$	$\Delta_{gg,8}$	$\Delta_{gg,1}$
Tevatron	0.3452	0.0241	0.0079
LHC7	1.698	4.313	1.305
LHC14	5.338	27.14	7.967

which, for the case $H = 0$, results in the following combinations of the constants $C_{ij,1}^{(2,0)}$ that enter the color-averaged cross-section:

$$\begin{aligned} \bar{C}_{gg}^{(2,0)} &= -1136.81, \\ \bar{C}_{q\bar{q}}^{(2,0)} &= -489.168. \end{aligned} \quad (\text{A.2})$$

In an analogous way we shall define as $\bar{H}^{(2)}(1)$ the values of the $H^{(2)}(1)$ constants obtained when setting $C_{ij,1}^{(2,0)} = 0$. We note that the constants $C_{ij,1}^{(2,0)}$ and $H_{ij,1}^{(2)}(1)$ are defined in different normalizations ($\alpha_s/(4\pi)$ in Eq. (1) and α_s/π in Eq. (7)).

The dependence of the cross-section on the constants $C_{ij,1}^{(2,0)}$ can be estimated from:

$$\begin{aligned} \sigma_{\text{tot}} &= \sigma_{\text{tot}}(C_{ij,1}^{(2,0)} = 0) + \left(\frac{C_{qq,8}^{(2,0)}}{1000}\right)\Delta_{qq,8} + \left(\frac{C_{gg,1}^{(2,0)}}{1000}\right)\Delta_{gg,1} \\ &\quad + \left(\frac{C_{gg,8}^{(2,0)}}{1000}\right)\Delta_{gg,8}. \end{aligned} \quad (\text{A.3})$$

For $\mu_R = \mu_F = m = 173.3$ GeV [37], and with PDF set MSTW2008-nnlo68cl [56], the values of $\Delta_{ij,1}$ are provided in Table 5. Clearly, reasonable variation of the unknown constants results in variation of the predicted cross-section by a few percent, setting an intrinsic limit to the precision in absence of the full knowledge of the NNLO result.

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