



On the analysis of intermediate–energy Coulomb excitation experiments

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Abstract

In a recent publication [C.A. Bertulani, G. Cardella, M. De Napoli, G. Raciti, E. Rapisarda, Phys. Lett. B 650 (2007) 233] the validity of analysis methods used for intermediate-energy Coulomb excitation experiments was called into question. Applying a refined theory large corrections of results in the literature seemed needed. We show that this is not the case and that the large deviations observed in above mentioned reference are due to the use of the wrong experimental parameters in that publication. We furthermore show that an approximate expression derived in above mentioned reference is in fact equivalent to the theory of Winther and Alder, an analysis method often used in the literature.

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1. Introduction

Coulomb excitation (CE) of radioactive heavy-ion beams on high- Z targets at intermediate to relativistic beam energies ($E/A > 30$ MeV) is an important and now widely used method to study the structure of nuclei furthest from stability and a wealth of experimental information was obtained in laboratories worldwide [1–5] since the method was first applied in 1995 [1]. Even at these high beam energies it is possible to suppress the influence of the strong interaction in the excitation process, by rejecting events with small impact parameter, and to extract largely model independent nuclear structure information.

Experimentally, usually the de-excitation γ ray yields after CE are measured and from these the excitation cross sections are deduced. It is important to note that the deflection angle of the outgoing scattered particle is restricted either by the experimental setup [1,2,6,7] or during the off-line analysis of the data, which are recorded event-by-event [8]. The so imposed maximum scattering angle θ_{\max} corresponds to a certain min-

imum impact parameter b_{\min} which is chosen to exceed the sum of the radii of projectile and target by several femtometer, ensuring the dominance of the Coulomb interaction in the excitation process. The angle integrated CE cross section depends of course strongly on the chosen maximum scattering angle θ_{\max} , which is therefore an important parameter in the analysis. For further details review articles [9–11] or the original references should be consulted.

In order to extract the relevant quantities, namely the reduced transition probabilities $B(\pi\lambda)$ and in particular $B(E2; 0_{\text{gs}}^+ \rightarrow 2_1^+)$ values, various analysis techniques are used:

- distorted-wave theory (using the ECIS code [12]) used, e.g., in [1,13–15],
- the semi-classical theory of Winther and Alder [16] used, e.g., in [2,8,17], and
- the virtual photon method [18] used, e.g., in [19].¹

¹ The virtual photon method is in fact identical to the semi-classical theory of Winther and Alder. Only here the excitation cross section is represented as a product of a virtual photon number and a (virtual) photo absorption cross section.

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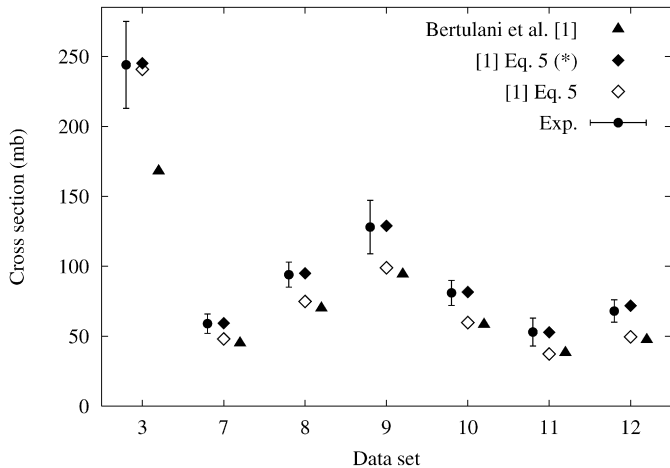


Fig. 1. Experimental CE cross sections as listed in Table 1. The number on the abscissa corresponds to the “Data set” of Table 1 and is identical to the one given in [24]. The circles show the experimental cross sections σ_{exp} and the filled diamonds the cross sections calculated by us according to Eq. (5) of [24] (σ'_{app} in rows with “*” in Table 1). The open diamond shows the result of our own application of Eq. (5) of [24] using the input parameters from the rows without a “*”, i.e., the ones used in [24]. The triangle shows the result published in [24].

More than one method has been applied to certain nuclei and consistent results were obtained, see, e.g., [1,13,20,21]. Recently, a compilation of data obtained on stable nuclides was published and a comparison of the $B(E2; 0_{\text{gs}}^+ \rightarrow 2_1^+)$ values obtained by various methods was given [22]. It was concluded that high-energy CE is at least as precise and accurate as so-called standard methods such as direct lifetime measurements, low-energy “safe” CE, and electron scattering.

Recently, however, a new approach was proposed [23] and previously published results were called into question in [24] claiming that either only non-relativistic, or only partially relativistic, expressions are used or the effect of the orbit of the nuclei, due to their Coulomb repulsion, is not fully accounted for. Using their new model, changes in the calculated cross sections of up to 30% were reported with respect to results from the literature.

We would like to dispute that assertion and show that the previously published results [2,8,25] are indeed correct and consistent.

2. Cross sections

In [24] Bertulani et al. studied the CE cross sections at intermediate energies (10–500 MeV/ u) using their model, published earlier in [23], which aims to correctly describe the CE process at all beam energies. In addition, since the orbital integrals appearing in the expressions for the excitation amplitudes have to be determined numerically, an approximate expression for the CE cross section was derived, called σ_{app} or σ^{app} in [24], which is a simple expression containing modified Bessel functions. Using these expressions and using the numerically determined orbital integrals it was found that the calculated excitation cross sections for a given excitation strength are smaller by 10–30% in comparison to values from the literature [2,8,25]

and consequently that the published $B(E1)\uparrow$ and $B(E2)\uparrow$ values are in fact too small by 10–30%.

Since this would be a substantial alteration we tried to reproduce (i) our original results in [2,8,25] and (ii) the results of Ref. [24] using Eq. (5) of [24]. Our findings are listed in Table 1 and are displayed in Fig. 1. The number shown in column 1 of the table and plotted on the abscissa in the figure corresponds to the “Data set” as given in [24]. Contrary to Bertulani et al. we essentially found exact agreement between the published cross sections (circles in the figure) and the results of our own calculation using Eq. (5) of [24] (filled diamond in the figure). However, the results reported in [24] (triangles) deviate significantly from those.

In order to understand the differences we performed a calculation using the beam energies and scattering angles listed in Table 1 of [24]. We would like to point out that at least for data sets number 7–12 (original publications [2,8]) the given values correspond to the *incident* beam energy and maximum *laboratory* scattering angle. If we assume, however, that the listed beam energy is the average beam energy in the mid-plane of the CE target² and the angle is the scattering angle in the center-of-mass system, we can in all cases, except for ¹¹Be, reproduce the values given in [24] (see open diamonds in Fig. 1).

We therefore conclude that mentioned discrepancies are due to an interchange of center-of-mass and laboratory angles and the use of the incident beam energy instead of the mid-target beam energy.

We furthermore found that the expressions given for σ_{app} in [24] are in fact analytically identical to the one of Winther and Alder [16] (see Appendix A). Since for all experimental cross sections listed in Table 1 and shown in Fig. 1 the semi-classical theory of Winther and Alder was employed during the analysis of the data, the σ_{app} results *must* be identical to the original results σ_{exp} provided the proper input parameters are used.

This conclusion is in fact supported by an earlier work of Bertulani et al. [23] where it is shown in Fig. 3 (curve RR) that for the case of an E2 excitation the cross section determined within the Winther and Alder theory deviates by less than 5% from the exact treatment for beam energies above 30 MeV/ u .

We would also like to note that the analysis of CE data using a simple maximum scattering angle is not applicable for all experiments in the literature. For instance for the analysis of the CE of ³²Mg [1] the use of a sharp cutoff θ_{max} is meaningless (“Data set” 6 in [24]). Here the particle detection efficiency depended (smoothly) on the scattering angle with maximum values of about 80%. In this case the differential cross section

² To determine the effective (theoretical) cross section σ the beam energy dependent cross section $\sigma(E_b)$ should be averaged over the target thickness d_t

$$\sigma = \frac{1}{d_t} \int_{x=0}^{d_t} \sigma(E_b(x)) dx \approx \sigma(E_b(d_t/2)),$$

where $E_b(x)$ is the beam energy at target depth x . Due to the almost linear dependence of the cross section on beam energy within the small energy loss in the target it is sufficient to use the beam energy in the center of the target in the calculation.

Table 1

Coulomb excitation cross sections are shown in the three columns on the right-hand side with σ_{exp} being the experimental CE cross section, σ_{app} the cross section from reference [24] and σ'_{app} the cross section calculated by us using Eq. (5) of [24]. The $B(E\lambda)\uparrow$ values are listed in units of $e^2 \text{fm}^{2\lambda}$. Values in bold were taken verbatim from [24] and the “Data set” number # corresponds to the one used in [24]. In the rows starting with “**” the correct center-of-mass scattering angles $\theta_{\text{max}}^{\text{cm}}$ and mid-target (m-t) beam energies $E_{\text{beam}}^{\text{m-t}}$ are listed; an agreement of σ_{exp} and σ'_{app} is evident for all cases. The rows not starting with “**” list the values as given in [24] and σ'_{app} is calculated by us using these values. In most cases σ_{app} and σ'_{app} agree, but σ_{exp} and σ_{app} do not

#	Iso.	Ref.	$E_{\text{beam}}^{\text{m-t}}$	$\theta_{\text{max}}^{\text{cm}}$	E_{γ}	$B(E\lambda)$	σ_{exp}	σ_{app}	σ'_{app}
3	¹¹ Be	[25]	57.6	3.80	0.320	0.079	244	168.0	240.7
3	¹¹ Be	[25]	57.6	4.02	0.320	0.079	244	–	245.1
7	³⁸ S	[2]	39.2	4.10	1.292	235	59	45.0	48.0
7	³⁸ S	[2]	34.6	4.92	1.292	235	59	–	59.3
8	⁴⁰ S	[2]	39.5	4.10	0.891	334	94	70.0	74.8
8	⁴⁰ S	[2]	35.3	4.96	0.891	334	94	–	95.0
9	⁴² S	[2]	40.6	4.10	0.890	397	128	94.3	98.9
9	⁴² S	[2]	36.6	5.00	0.890	397	128	–	128.9
10	⁴⁴ Ar	[2]	33.5	4.10	1.144	345	81	58.3	59.7
10	⁴⁴ Ar	[2]	30.9	5.04	1.144	345	81	–	81.6
11	⁴⁶ Ar	[2]	35.2	4.10	1.554	196	53	38.2	37.3
11	⁴⁶ Ar	[2]	32.8	5.08	1.554	196	53	–	52.8
12	⁴⁶ Ar	[8]	76.4	2.90	1.554	212	68	47.4	49.6
12	⁴⁶ Ar	[8]	73.2	3.62	1.554	212	68	–	71.8

must first be multiplied by the experimental particle detection efficiency before integrating over the scattering angle. This can only be done when the efficiency curve is known, which is shown in Fig. 2 (lower panel) in [1].

We furthermore found the following errors in [24]:

- There is a misprint in Eq. (5). For the E2 excitation cross section the power of E_{γ} should be 2 (E_{γ}^2) and not 3 (E_{γ}^3).
- The wrong citation was used for ^{38,40,42}S and ^{44,46}Ar. It should be Scheit et al. [2] and not Chromik et al. [19].
- An M1 excitation was considered for ¹⁷Ne, but the excitation process proceeds through an E2 transition with only a very weak M1 component. The decay, though, is an almost pure M1 transition.
- The deviation of the cross sections published in [24] (triangle in Fig. 1) for ¹¹Be (“Data set” 3) seems to be due to a forgotten factor of $e^2 = \alpha\hbar c \approx 197/137 \approx 1.44$ in [24].

3. Conclusion

We conclude that the theory presented in [23,24] does not represent a significant improvement over analysis methods used so far. In fact, a good approximation [24] to the full theory is equivalent to the theory of Winther and Alder [16], which is widely used in the analysis of CE data at high beam energies. We furthermore conclude that the large differences of up to 30% reported in [24] are likely solely due to an interchange of center-of-mass and laboratory angles and the use of incident beam energy instead of the mid-target beam energy in the calculation of the CE cross sections.

We nevertheless agree with the authors of [24] that a refined theory which includes relativistic effects and properly treats the

orbit of the projectile is needed and should be applied in the future, when high-precision results become available at next-generation radioactive nuclear beam facilities. The currently available data will be hardly affected due to rather large statistical and systematical uncertainties.

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Appendix A. Coulomb excitation cross sections

In the following σ_B denotes the expression for σ_{app} from Bertulani et al. [24, Eq. (5)] and $\sigma_{W/A}$ the result given by Winther and Alder [16]. For a given multipolarity λ these two expressions for the CE cross section differ explicitly only in the expressions containing the modified Bessel functions K_n and we will consider the ratio of the two expressions leaving out explicit common pre-factors. In particular for the E2 case:

$$\frac{\sigma_B^{(E2)}}{\sigma_{W/A}^{(E2)}} = \frac{8\pi^2 \beta^{-4} (2\gamma^{-2} K_1^2 + \xi(1 + \gamma^{-2})^2 K_0 K_1 - \frac{\beta^4 \xi^2}{2} (K_1^2 - K_0^2))}{\sum_{\mu=-2, \dots, 2} |G_{E2\mu}(\beta^{-1})|^2 g_{\mu}(\xi)}, \quad (\text{A.1})$$

where the functions g_{μ} and $G_{\pi\lambda\mu}$ are given in [16, Eqs. (2.16) and (3.4)]. The former contain the Bessel functions $K_{\mu, \mu+1}$ and the latter is a polynomial function of β^{-1} . After a lengthy, but

straight-forward, calculation one obtains:

$$\begin{aligned} & \sum_{\mu} |G_{E2\mu}(\beta^{-1})|^2 g_{\mu}(\xi) \\ &= \frac{4\pi^2 \xi}{75\beta^4} (-3\xi\gamma^{-2}K_0^2 - \xi(\beta^4 + \gamma^{-2})K_1^2 \\ & \quad + \xi(\beta^4 + 3\gamma^{-2})K_2^2 - (8\gamma^{-2} + 2\beta^4)K_1K_2 \\ & \quad + \xi\gamma^{-2}K_3^2 - 4\gamma^{-2}K_2K_3). \end{aligned} \quad (\text{A.2})$$

Using the recurrence relations for the Bessel functions K_n [26]

$$K_0 - K_2 = -\frac{2}{\xi}K_1 \quad \text{and} \quad -K_1 + K_3 = \frac{4}{\xi}K_2, \quad (\text{A.3})$$

the Bessel functions K_{μ} with $\mu > 1$ can be expressed in terms of K_0 and K_1

$$K_2 = \frac{2}{\xi}K_1 + K_0, \quad (\text{A.4})$$

$$K_3 = \frac{4}{\xi}K_2 + K_1 = \left(\frac{8}{\xi^2} + 1\right)K_1 + \frac{4}{\xi}K_0. \quad (\text{A.5})$$

The result is

$$\begin{aligned} & \sum_{\mu} |G_{E2\mu}(\beta^{-1})|^2 g_{\mu}(\xi) \\ &= \frac{4\pi^2}{75\beta^4} (\beta^4 \xi^2 K_0^2 + (-\xi^2 \beta^4 + 4\gamma^{-2})K_1^2 \\ & \quad + 2\xi(1 + \gamma^{-2})^2 K_0 K_1), \end{aligned} \quad (\text{A.6})$$

where $\beta^4 + 4\gamma^{-2} = (1 + \gamma^{-2})^2$ was used. The last result (A.6) is identical to the numerator of (A.1) and therefore

$$\sigma_B^{(E2)} = \sigma_{W/A}^{(E2)}. \quad (\text{A.7})$$

Similarly for the E1 case:

$$\frac{\sigma_B^{(E1)}}{\sigma_{W/A}^{(E1)}} = \frac{32\pi^2 \beta^{-2} (\xi K_0 K_1 - \frac{\beta^2 \xi^2}{2} (K_1^2 - K_0^2))}{\sum_{\mu=-1,0,1} |G_{E1\mu}(\beta^{-1})|^2 g_{\mu}(\xi)}, \quad (\text{A.8})$$

and

$$\begin{aligned} & \sum_{\mu} |G_{E1\mu}(\beta^{-1})|^2 g_{\mu}(\xi) \\ &= \frac{16\pi^2 \xi^2}{9} \left((\beta^{-2} - 1)(K_1^2 - K_0^2) \right. \\ & \quad \left. + \beta^{-2} \left(-K_1^2 + K_2^2 - \frac{2}{\xi} K_1 K_2 \right) \right). \end{aligned} \quad (\text{A.9})$$

Using the above recurrence relations for the Bessel functions we obtain

$$\begin{aligned} & \sum_{\mu} |G_{E1\mu}(\beta^{-1})|^2 g_{\mu}(\xi) \\ &= \frac{16\pi^2 \xi^2}{9} \left(K_0^2 - K_1^2 + \frac{2}{\xi} \beta^{-2} K_0 K_1 \right). \end{aligned} \quad (\text{A.10})$$

The last result (A.10) is identical to the numerator of (A.8) and therefore

$$\sigma_B^{(E1)} = \sigma_{W/A}^{(E1)} \quad (\text{A.11})$$

also for the E1 case.

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