The 9th International Conference on Traffic & Transportation Studies (ICTTS’2014)

Time Value Distribution and Multi-modal Intercity Travel Network Shape: Theoretical Analysis for the Typical Setting

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Abstract

In a period of declining demand, should we sustain some intercity transport even if it is unprofitable? And in which situations should we sustain the plurality of modes of transport? In this paper, we analyze the relationship between time-value distribution and multi-modal network shapes. The network shapes considered are dual-mode or single-mode, and a random parameter model is used to describe the route choice of travelers, which takes the time-value distribution into account. First, the relationship between parameters of time-value distribution (i.e., average and dispersion) and the optimal network shape is analyzed; we find that a dual-mode network is optimal when the time-value dispersion is large. Second, as demand declines, the optimal network may change from a dual-mode network to a single-mode network. Third, the conditions in which an equilibrium dual-mode network can be established in a given transport market are described. From a comparison between the equilibrium and optimal conditions for a dual-mode network, we find that dual-mode networks may be in equilibrium even though they are not optimal. It follows that we need not sustain unprofitable mode service in order to realize the optimal network shape. Moreover, in a period of declining demand, it may be beneficial to move from a dual-mode network to a single-mode network from a market driven dual-mode network.

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Keywords: heterogeneity of time value; multi-modal network; intercity transportation; optimal network; equilibrium network

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1. Introduction

The demand for intercity travel in Japan is currently decreasing because of a diminishing population. This decline in demand may result in some public transport operations becoming unprofitable, particularly in rural areas. In Japan, some unprofitable intercity transport services are supported by government subsidies, but it may not be practical to sustain all transport services forever. In particular, with a multi-modal network, some services may no longer be required. The relationships between some modes (e.g., high-speed rail (HSR) and air travel) have become increasingly competitive because of appearance of low-cost carriers and/or the construction of HSR. This means that we can make the intercity transport network more efficient by cancelling some services without seriously affecting the regional economy, because competitive modes can cover the terminated service.

In some cases, however, it may be socially efficient to sustain dual-mode networks, considering the heterogeneity in travelers’ preference (i.e., their time-value) and congestion. First, in the case of larger heterogeneity in travelers’ preference, it may be socially optimal to sustain dual-mode networks, in which they can choose their favorite mode. For example, when travelers have both high and low time-value, the optimal network is often dual-mode, and passengers can choose different modes: a fast but expensive service, or a slow but inexpensive service. Therefore, it is important to determine the relationships between the heterogeneity of travelers’ preferences and optimal network shape for intercity transport. Second, when transport is limited by congestion, we should sustain a dual-mode network. However, in Japan congestion is not a significant problem because of the declining population (with a few notable exceptions, for example, Haneda Airport in Tokyo).

Few studies into intercity public transport have focused on heterogeneity in travelers’ preferences. Many theoretical studies involving airline networks (e.g., Brueckner (2004) and Flores-Fillol (2009)) assume that travelers’ preferences are identical. Adler et al. (2010) considered the heterogeneity of travelers stochastically using a nested multinomial Logit model, but they did not analyze the relationship between variance in travelers’ preferences and the network shape.

Kawasaki (2008) analyzed airline networks to consider two types of travelers: business travelers with high time-value, and leisure travelers with low time-value. He concluded that, in the case of large heterogeneity, the optimal strategy for airlines is to operate two types of service: high-frequency services (i.e., a hub-and-spoke network) for business travelers and low-frequency but direct services (i.e., a point-to-point network) for leisure travelers.

In this paper, we first examine the robustness of Kawasaki’s 2008 analysis by considering a multi-modal network with different cost functions and time-value distributions. The time-value distribution is assumed to be a uniform distribution with a certain range, in contrast to the assumption that there are two types of travelers, each with a single time-value. Second, the optimal network shapes are analyzed in response to a decline in demand. Third, optimal and equilibrium networks in a transport markets with a given set of time-value distributions are compared. As a result of these analyses, we find that 1) a dual-mode network is optimal when the time-value dispersion is large, in agreement with the results of Kawasaki (2008); 2) in a period of declining demand, the optimal network may change from dual-mode to single-mode; and 3) based on a comparison between the optimal and equilibrium networks, we find that a dual-mode network may be in equilibrium even though it is not optimal.

The reminder of the paper is organized as follows. In Chapter 2 we describe the model used to analyze the relationship between the time-value distribution and the network shape. In Chapter 3 we present the results of our analyses of a typical setting: the network shape is either single-mode or dual-mode, and the time-value distribution is assumed to be uniform and limited to a given range. Chapter 4 concludes the paper.

2. Model

In this chapter, we describe the models used to analyze the relationship between the time-value distribution, and the optimal and equilibrium network shapes. The model consists of a description of the travelers’ route choice and a description of the network, in which public transport operators determine link fares and the network shape. In section 2.1 we describe the travelers’ route choice model; in Section 2.2 we describe the network model to analyze the optimal network shape; and in Section 2.3 we describe the network model for analysis of the equilibrium network shape.
2.1. Travelers’ route choice model

In the travelers’ route choice model, we assume that a traveler chooses a route $k$ with the lowest generalized cost, $GC_{i,k}$. We assumed that $GC_{i,k}$ is the sum of the route fare $f_k$ and route time $T_k$ multiplied by the time value $v_i$ of traveler $i$ where all link services consisting of route $k$ are operated. This may be formulated as follows:

$$\min_{k \in K_i} GC_{i,k},$$  \hspace{1cm} (1)

subject to:

$$GC_{i,k} = \begin{cases} f_k + v_i T_k & \text{if } \prod_{l \in Q_k} \delta_l = 1, \\ \infty & \text{if } \prod_{l \in Q_k} \delta_l = 0, \end{cases}$$ \hspace{1cm} (2)

where $K_i$ is the set of routes, $Q_k$ is the link set included in route $k$, and $\delta_l \in \{0,1\}$ is the existence of link services, where $\delta_l = 1$ corresponds to a link that operates and $\delta_l = 0$ corresponds to a link that does not. From the distribution density of the time-value relation $g(v)$, and from Equations (1) and (2), the number of travelers on route $k$ is given by:

$$p_k = \int_0^\infty h_k(v)dv,$$ \hspace{1cm} (3)

where:

$$h_k(v) = \begin{cases} g(v) & \text{if } GC_k(v) = \min_{k \in K_i} GC_k(v), \\ 0 & \text{if } GC_k(v) > \min_{k \in K_i} GC_k(v), \end{cases}$$ \hspace{1cm} (4)

is the traveler density of route $k$ for a given time-value $v$.

This model is similar to the random parameter model proposed by Walker and Ben-Akiva (2002) in the way it treats the time-value distribution, but our model does not include random disturbances for each traveler (which are usually assumed to follows a Weibull distribution). As a result of the absence of such a random disturbance, travelers with the same time-value relation choose the same route.

2.2. Optimal network model

In the optimal network model, we assume that a public transport operator manages all links, determines the link fares $f_i$, and determines the existence of link services, $\delta_l$. The optimal network is one that minimizes the total generalized cost $TC_{\delta}$. To minimize $TC_{\delta}$, the network operator should maximize the total benefit for travelers under the conditions of fixed demand. This can be formulated as follows:

$$\min_{\delta} TC_{\delta} = \sum_k \int_0^\infty h_k(v)GC_k(v)dv,$$ \hspace{1cm} (5)

subject to:

$$f_k = \sum_{l \in Q_k} f_l,$$ \hspace{1cm} (6)

$$p_i = \sum_{l \in K_i} a_{il} p_l,$$ \hspace{1cm} (7)

and:

$$\sum_k f_k p_k \geq \sum_l \delta_l (E_l + C_l p_l),$$ \hspace{1cm} (8)
Here, Equation (6) is a non-negative constraint of the public transport operator’s profit. The left-hand side of Equation (6) is the total fare receipts, and the right-hand side is the total operating cost. The cost of link \( l \) is assumed to be the sum of the fixed-cost \( E_l \) and unit variable cost \( C_l \) multiplied by the number of travelers \( p_l \). Equations (7)-(9) are constraints that describe the relationship between the route variables and the link variables.

2.3. Equilibrium network model

The equilibrium network model assumes that each public transport operator for each link determines the link fares \( f_l \) and existence of link services \( \delta_l = \{0, 1\} \) in order to minimize their profit \( \pi_l \) independently. This is formulated as follows:

\[
\min_{f_l, \delta_l} \pi_l = \delta_l \left( f_l p_l - (E_l + C_l p_l) \right)
\]

s.t. Equations (2)-(4), (7)-(9).

In section 3.5, Nash Equilibrium of non-cooperative game theory is analyzed to determine the network shape that is established in the transport market without government subsidies.

3. Theoretical Results for Typical Setting

3.1. Settings

In this section, the relationship between the time-value distribution and the network shape is analyzed in single- and dual-mode cases, which are shown in Fig. 1. There are three possible network shape: an air-only (AO) network, as shown in Fig. 2(a); a rail-only (RO) network, as shown in Fig. 2(b); and a dual-mode or hybrid (HB) network as shown in Fig. 2(c).

![Network setting](image)

Fig. 1. Network setting

![All network shapes](image)

Fig. 2. All network shapes

As shown in Fig. 1, we can treat the two modes, air and rail, by considering some characteristics of each mode. We make the following three assumptions: 1) the air travel time \( T_A \) is shorter than the rail travel time \( T_R \); 2) the fixed cost of rail travel \( E_R \) is greater than that of air travel \( E_A \), and 3) the unit variable cost (per seat) of the railway \( C_R \) is less than that of the airline \( C_A \). We then have:

\[
T_A < T_R, \quad E_A < E_R, \quad C_A > C_R.
\]
The time-value distribution is assumed to be uniform within a limited range. This can be expressed as follows:

\[ g(v) = \begin{cases} x & \text{if } d_a - d_v < v < d_a + d_v \\
0 & \text{if } v < d_a - d_v, d_a + d_v < v \end{cases} \]  

(14)

where \( x \) is the distribution density of the time-value, \( d_a \) is the average of time-value distribution, and \( d_v \) indicates the dispersion of the time-value distribution, which is subject to the following constraints:

\[ 0 < d_a - d_v < d_a + d_v \Rightarrow 0 < d_v < d_a, \]  

(15)

i.e., the time-value should be non-negative. This is time-value distribution is plotted in Fig. 3.

Here, we assumed a fixed demand case, whereby the total number of travelers \( D \) is constraint. It follows that \( x \) may be found from:

\[ D = \int_0^x g(v)dv = 2d_v x \iff x = \frac{D}{2d_v}. \]  

(16)

3.2. Route choice results in HB network

In this section we analyze the number of travelers on routes \( A \) and \( R \), i.e., \( p_A \) and \( p_R \) in HB network. As described by Equation (4), all travelers with same time-value choose the same route. Therefore, in an HB network, travelers with a time-value greater than \( v' \) use air travel, where \( v' \) is the time-value at which the generalized cost of the railway is equal to that of the airline, i.e.:

\[ GC_R(v') = GC_A(v') \iff v' = \frac{f_A - f_R}{T_R - T_A}. \]  

(17)

These route choice data are plotted in Fig. 4.

The numbers of travelers on each route can be described by a linear function of the fares of the two modes, \( f_A \) and \( f_R \), i.e.:
Because the number of travelers on each route is given by a linear function of the fare, the total generalized cost \( TC_f \) and profit \( \pi \) for the operators are quadratic functions of the fares. We can therefore solve the fares for the optimal and equilibrium networks analytically.

### 3.3. Optimal network results

In this section, the optimal network shape for a pair of average time-value \( d_a \) and time-value dispersion \( d_v \) is determined by comparing the minimum total generalized costs for each network shape (i.e., \( TC_{f, AO} \), \( TC_{f, RO} \) and \( TC_{f, HB} \)). From Equations (2)-(6), (14), and (16), the minimum total generalized cost for an AO network is:

\[
TC_{f, AO} = (E_a + C_a D) + d_a T_a D,
\]

and for an RO network we have:

\[
TC_{f, RO} = (E_R + C_R D) + d_a T_R D.
\]

From Equations (2)-(6), (14), and (16)-(19), optimal fares \( f_a^* \), \( f_R^* \) that minimizing the total generalized cost are as follows:

\[
f_a^* = C_a + \frac{E_a + E_R}{D},
\]

and:

\[
f_R^* = C_R + \frac{E_a + E_R}{D},
\]

which leads to the following total generalized cost for a hybrid network:

\[
TC_{f, HB} = E_a + E_R + p_a^* (C_a - C_R) + DC_R + \int_{-d_v}^{d_v} vT_a xd\nu + \int_{-d_v}^{d_v} vT_R xd\nu.
\]

By comparing \( TC_{f, AO} \), \( TC_{f, RO} \) and \( TC_{f, HB} \), we can obtain the optimal network shape for a pair of average time-value \( d_a \) and dispersion \( d_v \) as shown in Fig. 5.
The boundary lines of Fig. 5 are as follows:

\[ TC_{t, AO} = TC_{t, RO} \iff d_a = \frac{(C_a - C_R)D - (E_R - E_A)}{(T_R - T_A)D} \]  
\[ (25) \]

\[ TC_{t, AO} = TC_{t, HB} \iff d_a = d_v - 2\sqrt{\frac{E_A}{D(T_R - T_A)}}\sqrt{d_v} + \frac{C_a - C_R}{T_R - T_A} \]  
\[ (26) \]

and:

\[ TC_{t, RO} = TC_{t, HB} \iff d_a = -d_v + 2\sqrt{\frac{E_A}{D(T_R - T_A)}}\sqrt{d_v} + \frac{C_a - C_R}{T_R - T_A} \]  
\[ (27) \]

First, it is evident from Fig. 5 that, with a larger average time-value, an AO network is optimal. Second, with lower average time-value, an RO network is optimal. The HB network is only optimal with a relatively large time-value dispersion. In other words, an HB network is effective in reducing the time lost for high time-value travelers and in reducing the variable cost for low time-value travelers.

We can determine the total operational cost of the optimal network using Equations (22) and (23) which is equal to total fare revenue, i.e.:

\[ f_t^* p_A^* + f_t^* p_R^* = (C_A p_A^* + E_A) + (C_R p_R^* + E_R) \]  
\[ (28) \]

It follows that the profits from one of these modes will be negative, unless \( d_a \) and \( d_v \) satisfy the relation:

\[ f_t^* p_A^* - (C_A p_A^* + E_A) = 0 \iff d_a = -\frac{E_R - E_A}{E_R + E_A} d_v + \frac{C_a - C_R}{T_R - T_A} \]  
\[ (29) \]

i.e., the profits of both modes are zero. In other words, in the optimal HB network, the deficit of unprofitable mode is just subsidized by the other mode’s profit.

### 3.4. Optimal network after demand decline

Fig. 6(a) shows the shift in the boundary lines of shown in Fig. 5 (which were determined from Equations (25)-(27)) due to a decline in the total demand. As a result of these shifts in the boundary lines, the optimal network shape changes in some areas, as shown in Fig. 6(b).
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It is evident from the data plotted in Fig. 6(b) that the optimal network shape changes from an RO network to an AO network in area (A). Because of the declining demand, the generalized cost per traveler increases, and the AO network, with its lower fixed cost, becomes optimal. In area (B), the optimal network changed from an HB network to an AO network, and in area (C) it changed to an RO network. As a consequence, the area corresponding to an HB network being optimal becomes smaller, due to the larger fixed cost.

We validate these analytical results by numerical tests of 3 cases which are plotted in Fig. 6(a). The results are shown in Table 1. In this test, parameters (i.e., the number of travelers, fixed cost, and unit variable cost) are set as follows:

- total number of traveler before the declining demand \( D = 1000 \) (person); total number of travelers after the declining demand \( D' = 600 \) (person); \( E_d = 100,000 \) (yen); \( E_R = 200,000 \) (yen); \( C_A = 6,000 \) (yen per person); \( C_R = 3,000 \) (yen per person); \( T_d = 60 \) (min); \( T_R = 120 \) (min).

From Table 1, it can be observed that the generalized cost per traveler increase because of the declining demand. In the case [2] which is plotted in area (B) (Fig. 6), optimal network shape changes from an HB network to AO network.

### Table 1. Numerical tests of the changes in the optimal network shape due to declining demand.

<table>
<thead>
<tr>
<th>Parameters of Time Value Distribution</th>
<th>The number of demand</th>
<th>( TC_{C,RO}/D ) (yen per person)</th>
<th>( TC_{C,HO}/D ) (yen per person)</th>
<th>( TC_{C,HB}/D ) (yen per person)</th>
<th>Optimal network shape</th>
</tr>
</thead>
<tbody>
<tr>
<td>( d_a ) (yen/min)</td>
<td>( d_v ) (yen/min)</td>
<td>Before the decline</td>
<td>After the decline</td>
<td>Before the decline</td>
<td>After the decline</td>
</tr>
<tr>
<td>[1]</td>
<td>50</td>
<td>40</td>
<td>9,100</td>
<td>9,200</td>
<td>8,700</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>9,200</td>
<td>9,400</td>
<td>9,000</td>
</tr>
<tr>
<td>[2]</td>
<td>65</td>
<td>40</td>
<td>10,000</td>
<td>11,000</td>
<td>9,966</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>11,000</td>
<td>11,200</td>
<td>10,266</td>
</tr>
<tr>
<td>[3]</td>
<td>65</td>
<td>20</td>
<td>10,000</td>
<td>11,000</td>
<td>10,181</td>
</tr>
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<td></td>
<td></td>
<td>11,000</td>
<td>11,200</td>
<td>10,481</td>
</tr>
</tbody>
</table>

### 3.5. Equilibrium network results

In this section, we describe the conditions in which an HB network can be established in the transport market, i.e., is in equilibrium, and compare this with the conditions whereby an HB network is optimal. An HB network can exhibit Nash equilibrium under three conditions. First, each of the transport operators cannot increase their profit by changing their fare, i.e.:
Second, the profits of all transport operators are positive, i.e.:

\[ \pi_A \geq 0, \quad \pi_R \geq 0, \]  

(31)

Third, the number of travelers in each mode is positive, i.e.:

\[ p_A > 0, \quad p_R > 0 \Leftrightarrow d_a - d_v < \frac{f_A - f_R}{T_R - T_A} < d_a + d_v. \]  

(32)

From Equations (30)-(32), (2)-(4), (10), and (14)-(19), we can determine two conditions whereby a Nash equilibrium is achieved. These are as follows:

\[ \pi_A \geq 0 \Leftrightarrow d_a > -3d_v + 3 \sqrt{\frac{2E_A}{D(T_R - T_A)}} \sqrt{d_v} + \frac{C_A - C_R}{T_R - T_A} \]  

and:

\[ \pi_R \geq 0 \Leftrightarrow d_a > 3d_v - 3 \sqrt{\frac{2E_R}{D(T_R - T_A)}} \sqrt{d_v} + \frac{C_A - C_R}{T_R - T_A}. \]  

(34)

Fig. 7(a) shows these conditions as a function of the average time-value and the time-value dispersion, illustrating the region whereby an HB network is in Nash equilibrium. From Fig. 7(a), we can see that when the time-value dispersion is large, an HB network is in Nash equilibrium. Fig. 7(b) shows that an HB network that is optimal is also realized as Nash equilibrium. Furthermore, an HB network may be in Nash equilibrium, but not always optimal. It follows that there are some cases whereby we can make the intercity transport network more efficient by cancelling one mode, thereby reducing the fixed cost.

4. Conclusion

We investigated the relationships between time-value distribution and multi-modal intercity transport networks, where the random parameter model was used for route choice. From an analysis of the relationships between time-value distribution and the optimal network shape, we find that a hybrid (i.e., dual mode) network is optimal when time-value dispersion is large. Following a decline in demand, in many cases the optimal network shape changed
from a hybrid to a single-mode network, due to the reduced fixed cost of a single-mode network. The conditions whereby a dual-mode network can be established in Nash equilibrium were investigated. We find that a dual-mode network can exist in Nash equilibrium even when it is not optimal.

From these results, we can conclude that dual-mode networks should be sustained when the time-value dispersion is large. However, as demand declines, the optimal network may change to one with lower fixed costs. Furthermore, because a dual-mode network may be in Nash equilibrium even though it is not optimal, in some cases it may be beneficial to change to a single-mode network.

References


