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Notes on Dependent Attributes in TOPSIS

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Abstract

TOPSIS is a multicriteria decision making technique based on the minimization of geometric distances that allows the ordering of compared alternatives in accordance with their distances from the ideal and anti-ideal solutions. The technique, that usually measures distances in the Euclidean norm, implicitly supposes that the contemplated attributes are independent. However, as this rarely occurs in practice, it is necessary to adapt the technique to the new situation. Using the Mahalanobis distance to incorporate the correlations among the attributes, this paper proposes a TOPSIS extension that captures the dependencies among them, but, in contrast to the Euclidean distance, does not require the normalization of the data. Results obtained by the new proposal have been compared by means of the three Minkowski norms most commonly employed for the calculation of distance: (i) the Manhattan distance (p=1); (ii) the Euclidean distance (p=2); and (iii) the Tchebycheff distance (p=∞). Furthermore, simulation techniques are used to analyse the connection between the TOPSIS results traditionally obtained with the Euclidean distance and those obtained with the Mahalanobis distance.

Keywords: Multicriteria Decision Making; TOPSIS; Dependency; Mahalanobis Distance.

1. Introduction

TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) is one of the most commonly utilized multicriteria techniques for ordering a discreet group of alternatives and selecting the best of them[1]. Implicit in the initial TOPSIS proposal is the consideration that the attributes contemplated for ordering alternatives are independent. Unfortunately, this rarely occurs in the real-life cases to which the technique is applied. The majority of published scientific works concerning TOPSIS do not explicitly deal with problems

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derived from dependence among the attributes[2]. As can be seen in Section 4 of this work, failure to take the question of dependence into account has a decisive influence on results obtained.

After analysing the relevant problems, TOPSIS is adapted to the consideration of dependent attributes by means of the reformulation of a proposal put forward by Hwang and Yoon[3]. Two original modifications are advanced: (i) a new measurement of ideal and anti-ideal distances, based on the Mahalanobis distance[4], that captures the correlation between the attributes and eliminates the common problem of data normalization; (ii) a new method for synthesizing the contribution of the two distances in the final ordering that allows the consideration of both aspects without the problems associated with a quotient (this latter issue will be dealt with in a future work).

The proposal for the incorporation of the dependent attributes has been denominated as *TOPSIS-M* (Mahalanobis distance). It has been applied to a case taken from the published literature and the results are compared with results obtained with traditional TOPSIS, which uses the Euclidean distance.

The structure of the remainder of this paper is as follows: Section 2 outlines the theoretical foundation of the new proposal, that is to say, multicriteria decision making techniques, and a synthesis of the techniques used for the minimization of distances as a methodological support; Section 3 describes the new proposal, based on the Mahalanobis distance; Section 4 applies the proposal to a numerical example taken from the published literature and compares the results with the traditional approach; Section 5 briefly details the most important conclusions of the work and indicates future lines of research.

2. Background

2.1. Multicriteria decision making techniques

Multicriteria Decision Making can be understood as a series of models, methods and techniques that allow a more effective and realistic solution to complex problems that contemplate multiple scenarios, actors and (tangible and intangible) criteria[5]. A variety of multicriteria decision approaches are mentioned in the scientific literature:

(a) Techniques based on the flow of information between the two most important actors in the decision making process[6]: the decision maker and the analyst. These are further classified as: (i) techniques without *a priori* information on the preferences of the decision maker; (ii) techniques with *a priori* information; (iii) Interactive techniques.

(b) Techniques based on whether the set of alternatives is continuous or discreet, often known as Multi-objective (continuous) Programming and Multi-attribute (discrete) Programming.

(c) Techniques based on the different approaches or schools considered for solving multicriteria problems, the most common are: (i) the generation of efficient solutions (without *a priori* information); (ii) the minimization of the distance to a point of reference – the ideal, in the case of Compromise Programming, and the goal, in the case of Goal Programming (with *a priori* information); (iii) the construction of a value function (the American school), either using direct aggregation procedures (Multi-attribute Utility Theory - MAUT) or hierarchical aggregation (Analytical Hierarchy Process - AHP); (iv) methods using binary order relationships (the European school), for example, ELECTRE and PROMETHEE.

Despite the diversity of the techniques and the many arguments and discussions that have taken place regarding the different schools and approaches, there is no general agreement that a particular technique is superior to the others[7]. Moreover, in the last decade, debates between the different schools have been replaced by attempts to take advantage of the best elements of each approach with the aim of developing the most effective technique.

2.2. Multicriteria techniques based on distance minimization

The original and most utilized multicriteria technique based on distance minimization is Compromise Programming[8]. This technique, with *a priori* information about the decision maker's preferences (norms and weights), works simultaneously with all the criteria and seeks solution $x^{*}\epsilon X$ that minimizes the distance to the ideal point.

Let (1) be a multi-objective optimization problem where, without losing generality, it is supposed that all the q contemplated criteria are maximized:

$$Max_{x\epsilon X}\, z(x) = \left(z_1(x), \dots, z_q(x)\right) \tag{1}$$

The compromise solution $x^{*}\epsilon X$ is obtained by resolving the optimization problem that minimizes the distance to the ideal point or vector $\left(z^{*} = \left(z_1^{*}(x), \dots, z_q^{*}(x)\right)\right)$, where that distance is usually given by a Minkowski distance expression:

$$Min_{x\epsilon X}\, d(z(x), z^{*}, p) = Min_{x\epsilon X}\left(\sum_{j=1}^{q} w_j^{p}\left|z_j^{*} - z_j(x)\right|^{p}\right)^{1/p} \tag{2}$$

Given that $X = \{x \in \mathbb{R}^n | g_i(x) \leq 0,\ i = 1, \dots, m\}$ and p is the norm considered for distance ($p = 1,2, \dots, \infty$), $w_j > 0$ is the weight of $j - th$ criterion and z^{*} is the ideal vector ideal where each component z_j^{*} of the vector is the individualized optimum of the $j - th$ criteria ($j = 1, \dots, q$), we have:

$$z_j^{*}(x) = Max_{x\epsilon X}\, z_j(x) \tag{3}$$

When p→∞, the expression of the Minkowski distance is known as the Tchebycheff distance; in this case (2) it is:

$$Min_{x\epsilon X}\, d(z(x), z^{*}, p = \infty) = Min_{x\epsilon X} Max_j \left\{w_j\left|z_j^{*} - z_j(x)\right|\right\}, \quad j = 1, \dots, q \tag{4}$$

For reasons of operational functionality, the most commonly used Minkowski norms are: p=1 (Manhattan distance), p=2 (Euclidean distance) and p=∞ (Tchebycheff distance). In the first case, the optimization problem is lineal, in the second it is quadratic and in the third case, the model can be easily transformed to lineal.

Other well-known multicriteria techniques based on minimization of distance which have been widely used in discrete multicriteria decision making are: Goal Programming[9], VIKOR[10] and TOPSIS.

3. Dependent and independent attributes in TOPSIS

TOPSIS is based on the supposition that the contemplated attributes are independent[11,12]. Unfortunately, this is rarely occurs in the real-life cases to which the technique is applied.

3.1. The traditional TOPSIS approach

Given a discrete multicriteria decision problem which considers m alternatives ($A_i,\ i = 1, \dots, m$), evaluated using n criteria ($C_j,\ j = 1, .., n$), traditional TOPSIS contemplates each alternative or object as a point or vector of space $n - dimensional$ and the calculation of the Euclidean distance between the normalized distributive mode data is based on the initial alternatives (A_i) and those of two special alternatives: the ideal (A^{+}) and the anti-ideal (A^{-}), on the understanding that the best alternatives are those which are closest to the ideal and furthest from the anti-ideal[3,13,14]. To apply this technique, the attribute values should be numeric and have commensurable units.

As can be noted in the table of effects related with the problem (Table 1), the value associated with alternative A_i for the attribute or criterion C_j is denoted as x_{ij}, whilst w_j is the weight or importance of the $j-th$ criterion.

Table 1. TOPSIS decision matrix

	w1	w2	…	wj		wn
	C1	**C2**	**…**	**Cj**	**…**	**Cn**
A1	x_{11}	x_{12}	…	x_{1j}	…	x_{1n}
…	…	…	…	…	…	…
Ai	x_{i1}	x_{i2}	…	x_{ij}	…	x_{in}
…	…	…	…	…	…	…
Am	x_{m1}	x_{m2}	…	x_{mj}	…	x_{mn}

The procedure is better described in the following steps, as suggested by Hwang and Yoon[3] in their original proposal (traditional TOPSIS):

Step 1. Calculate the normalized decision matrix

As TOPSIS allows the evaluated criteria to be expressed in different measurement units, it is necessary to convert them into normalized values. The normalization process, like the metric used to calculate the ideal and anti-ideal distances, is Euclidean. In this case, the normalization of element x_{ij} of the decision matrix (Euclidean normalization mode) is calculated as:

$$n_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} {x_{ij}}^2}},\ i = 1, \dots, m,\ j = 1, \dots, n \tag{5}$$

Step 2. Calculate the weighted normalized decision matrix

The weighted normalized value v_{ij} of a weighted normalized decision matrix is calculated as:

$$v_{ij} = w_j \cdot n_{ij},\ i = 1, \dots, m,\ j = 1, \dots, n \tag{6}$$

Where $w_j > 0$ represents the weight or importance of the $j-th$ attribute, given that $\sum_{j=1}^{n} w_j = 1$. The weights are usually obtained[8] from different modes: direct assignation, AHP, etc.

Step 3. Determine the "positive ideal" and "negative ideal" alternatives

Without losing generality and supposing that all the criteria are maximized, the ideal positive solution is given by $\mathrm{A}^+ = \{\mathrm{v}_1^+, \dots, \mathrm{v}_\mathrm{n}^+\}$, where $\mathrm{v}_\mathrm{j}^+ = \max_\mathrm{i} \mathrm{v}_\mathrm{ij}, \mathrm{i} = 1, \dots, \mathrm{m}, \mathrm{j} = 1, \dots, \mathrm{n}$, and the ideal negative or anti-ideal solution is given by $\{v_1^-, \dots, v_n^-\}$, where $\mathrm{v}_\mathrm{j}^- = \min_\mathrm{i} \mathrm{v}_\mathrm{ij}, \mathrm{i} = 1, \dots, \mathrm{m}, \mathrm{j} = 1, \dots, \mathrm{n}$.

Step 4. Calculate the distances

The separation of each alternative A_i from the ideal solution A^+ is calculated as:

$$d_i^+ = \left(\sum_{j=1}^{n} \left|v_j^+ - v_{ij}\right|^2\right)^{1/2},\ i = 1, \dots, m \tag{7}$$

The separation of each alternative A_i from the ant-ideal solution A^- is calculated as:

$$d_i^- = \left(\sum_{j=1}^{n} \left|v_j^- - v_{ij}\right|^2\right)^{1/2},\ i = 1, \dots, m \tag{8}$$

Step 5. Calculate the relative proximity to the ideal solution

The relative proximity of A_i with regards to $A^+$and A^- is given by R_i and can be expressed as:

$$R_i = \frac{d_i^+}{d_i^+ + d_i^-},\ i = 1, \dots, m \tag{9}$$

where R_i is the best while it is most proximate to 0 ($0 \le R_i \le 1$).

Step 6. Preference order

Finally, R_i is used to order the alternatives; the greater the proximity the value of R_i is to 0, the greater is its proximity to the ideal and, therefore, it has a higher priority than the $i - th$ alternative.

3.2 TOPSIS-M and dependent attributes.

As previously mentioned, the traditional TOPSIS approach does not consider the dependency between the attributes. This means that the calculation of distance using a Minkowski metric incorporates redundant information. A possible solution to this problem is to use the Mahalanobis distance[15,16] that determines the similarity between two multi-dimensional random variables as well as considering the existent correlation between them ($m > n$ is required to obtain Σ^{-1}). The Mahalanobis distance between two random variables with the same x and y probability distribution and with Σ variance-covariance matrix is formally defined as:

$$d_m(x, y) = \left((x - y)^T \Sigma^{-1} (x - y)\right)^{1/2} \tag{10}$$

where:

$$\Sigma = \frac{1}{n-1} (X_c)^T (X_c) \tag{11}$$

X is the data matrix with *m* objects in rows by *n* columns, X_c is the centered matrix, $X_c = (X - \bar{x})$, and $\bar{x}$ the arithmetic mean.

This distance coincides with the Euclidean distance if the covariance matrix is the identity matrix, i.e. if all bivariate correlations between variables are zero. This statement is easily verified by applying both measures to a data matrix that has independent columns. An example is the matrix data that appear in Table 3, where the data are derived from the application of a principal component analysis to the example taken from the literature[12] (“Profiles of Graduate Fellowship Applicants”).

Table 2. Results obtained by Principal Component Analysis

Alternatives	F1	F2	F3	F4	F5
A_1	1,0151	-0,8463	0,001	1,4672	0,5175
A_2	-0,8201	1,5214	-0,0746	1,0127	-0,3852
A_3	-0,2968	0,0543	-1,385	-0,7713	1,25
A_4	-0,5684	-1,0871	-0,5394	-0,2665	-1,5165
A_5	1,4887	0,7791	0,3708	-0,962	-0,5298
A_6	-0,8185	-0,4214	1,6272	-0,4801	0,664

Table 3. R_i for traditional TOPSIS and TOPSIS-M

	TOPSIS		TOPSIS-M	
	R_i	Rank	R_i	Rank
A_1	0,4335	1	0,4335	1
A_2	0,4738	2	0,4738	2
A_3	0,5915	5	0,5915	5
A_4	0,8206	6	0,8206	6
A_5	0,4826	3	0,4826	3
A_6	0,4885	4	0,4885	4

The values of R_i (measured with both Euclidean and Mahalanobis distances), and the rankings obtained, are shown in Table 3. It is clear that the preference order is the same because the attributes are independent and the correlation matrix is an identity matrix; in addition, the R_i indexes are also the same.

4. Numerical example

This section compares the procedure and results obtained by traditional TOPSIS and TOPSIS-M using the variants described above. A numerical example was taken from the literature and the relative proximity of R_i to each A_i with respect to A^+ and A^- was calculated as follows: (i) the data were normalized; (ii) results were compared using Minkowski metric distances; (iii) results were compared using the Mahalanobis distance.

4.1 Example: TOPSIS and TOPSIS-M

The example ("Profiles of Graduate Fellowship Applicants") is a problem in which six alternatives are evaluated with respect to five criteria[12]. The data are shown in Table 4.

Table 5 shows the relative proximity R_i calculated with the Manhattan distance (MD), using a variety of normalization modes (the distributive mode-DM; the Euclidean mode-EM; the ideal mode-IM; the utility mode-UM); the statistical standardization mode-SS; non-normalized data-NN) and the ranking of the alternatives (Rank).

Table 4. Data for "Profiles of Graduate Fellowship Applicants"

Alternatives	C1	C2	C3	C4	C5
A_1	690	3.1	9	7	4
A_2	590	3.9	7	6	10
A_3	600	3.6	8	8	7
A_4	620	3.8	7	10	6
A_5	700	2.8	10	4	6
A_6	650	4	6	9	8

Table 5. R_i for the Manhattan distance with various normalization modes (DM, EM, IM, UM, EE, NN)

Alternatives	DM	Rank	EM	Rank	IM	Rank	UM	Rank	SS	Rank	NN	Rank
A1	0,6200	5	0,6178	5	0,6024	5	0,5182	5	0,5332	5	0,1643	2
A2	0,4151	2	0,4166	2	0,4288	3	0,5000	3	0,4866	3	0,9206	6
A3	0,4548	4	0,4555	4	0,4583	4	0,5152	4	0,5071	4	0,8443	5
A4	0,4230	3	0,4239	3	0,4201	2	0,4621	2	0,4616	2	0,6855	4
A5	0,6414	6	0,6388	6	0,6319	6	0,5333	6	0,5435	6	0,0881	1
A6	0,3744	1	0,3757	1	0,3750	1	0,3909	1	0,3938	1	0,4481	3

The resulting rankings for the different normalization modes are:
A6≻A2≻A4≻A3≻A1≻A5 for DM and EM
A6≻A4≻A2≻A3≻A1≻A5 for IM, UM and SS
A5≻A1≻A6≻A4≻A3≻A2 for non-normalized data

The relative proximity R_i calculated with the Euclidean distance (ED) using normalizations DM, EM, IM, UM, SS and NN and the ranking for the alternatives can be seen in Table 6.

Table 6. R_i for the Euclidean distance (ED) with various normalizations (DM, EM, IM, UM, EE, NN)

	DM	Rank	**EM**†	**Rank**	IM	Rank	UM	Rank	SS	Rank	NN	Rank
A1	0.6321	5	**0.6297**	**5**	0.6116	5	0.5127	4	0.5335	6	0.1080	2
A2	0.4130	2	**0.4144**	**2**	0.4295	3	0.5000	3	0.4832	3	0.9443	6
A3	0.4382	4	**0.4388**	**4**	0.4379	4	0.5129	5	0.4998	4	0.8979	5
A4	0.4355	3	**0.4364**	**3**	0.4280	2	0.4729	2	0.4727	2	0.7231	4
A5	0.6340	6	**0.6307**	**6**	0.6223	6	0.5183	6	0.5299	5	0.0623	1
A6	0.3869	1	**0.3896**	**1**	0.3944	1	0.4263	1	0.4298	1	0.4541	3

The resulting rankings for the different normalization modes are:
A6≻A2≻A4≻A3≻A1≻A5 for DM and EM
A6≻A4≻A2≻A3≻A1≻A5 for IM

† Results associated with traditional TOPSIS are in **bold.**

A6≻A4≻A2≻A1≻A3≻A5 for UM
A6≻A4≻A2≻A3≻A5≻A1 for SS
A5≻A1≻A6≻A4≻A3≻A2 for non-normalized data

The relative proximity R_i calculated with the Tchebycheff distance (TD) using the normalizations DM, EM, IM, UM, SS and NN and the ranking for the alternatives can be seen in Table 7.

Table 7. R_i for the Tchebycheff distance (TD) with various normalizations (DM, EM, IM, UM, EE, NN)

	DM	Rank	EM	Rank	IM	Rank	UM	Rank	SS	Rank	NN	Rank
A1	0.6822	6	0.6820	6	0.6667	6	0.5238	5	0.5760	6	0.0909	2
A2	0.3832	1	0.3834	1	0.4000	1	0.5000	2	0.4474	2	0.9483	6
A3	0.4459	4	0.4457	4	0.4286	3	0.5769	6	0.5388	5	0.9091	5
A4	0.4171	2	0.4168	2	0.4000	1	0.4286	1	0.4232	1	0.7273	4
A5	0.6157	5	0.6109	5	0.6000	5	0.5000	2	0.5055	3	0.0517	1
A6	0.4282	3	0.4332	3	0.4444	4	0.5000	2	0.5210	4	0.4545	3

The resulting rankings for the different normalization modes are:
A2≻A4≻A6≻A3≻A5≻A1 for DM and EM
A2 =A4≻A3≻A6≻A5≻A1 for IM
A4≻A2 =A5 =A6≻A1≻A3 for UM
A4≻A2≻A5≻A6≻A3≻A1 for SS
A5≻A1≻A6≻A4≻A3≻A2 for non-normalized data

The relative proximity R_i calculated with the Mahalanobis distance (MD) using the normalizations DM, EM, IM, UM, SS and NN and the ranking for the alternatives can be seen in Table 8.

Table 8. R_i for the Mahalanobis distance (ED) with various normalizations (DM, EM, IM, UM, EE, NN)

	DN	Or	EN	Or	IM	Or	UM	Or	EE	Or	NN	Or
A1	0.4713	3	0.4713	3	0.4713	3	0.4713	3	0.4713	3	0.4713	3
A2	0.4726	4	0.4726	4	0.4726	4	0.4726	4	0.4726	4	0.4726	4
A3	0.4699	1	0.4699	1	0.4699	1	0.4699	1	0.4699	1	0.4699	1
A4	0.4743	6	0.4743	6	0.4743	6	0.4743	6	0.4743	6	0.4743	6
A5	0.4727	5	0.4727	5	0.4727	5	0.4727	5	0.4727	5	0.4727	5
A6	0.4709	2	0.4709	2	0.4709	2	0.4709	2	0.4709	2	0.4709	2

The resulting ranking for the different normalization modes is:
A3≻A6≻A1≻A2≻A5≻A4 for any normalization type and non-standardized data

4.2 Analysis of the results

Firstly, it can be observed that the rankings obtained using traditional TOPSIS (distance and Euclidean normalization) and TOPSIS-M (Mahalanobis distance and non-normalized data) are clearly different when there is certain dependence with regards to the data. Even in the case of attributes with close to null dependence (Gleason-Staelin's ϕ < 0.0230), traditional TOPSIS rankings and those of TOPSIS-M do not coincide. In the example used in Section 4.1, the value of the indicator of the Gleason-Staelin redundancy measure (Phi) is ϕ= 0.6736, which is higher than the 0.5 threshold considered as necessary for the contemplation of the existence of redundancy and dependency. This result can be verified with any of the other Minkowski distances that were used (Manhattan - $p = 1$ - or Tchebycheff - $p = \infty$ -). If the attributes are independent, as previously mentioned, the values obtained with traditional TOPSIS and TOPSIS-M are exactly the same.

Secondly, it should be noted that the data normalization mode that is followed for the Minkowski distances conditions the results obtained. This does not occur with the Mahalanobis distance as the results are the same if the initial data are normalized or not, irrespective of the type of normalization that is employed.

If the Manhattan distance ($p = 1$) is used, the denominator of expression (9) is constant ($d_i^+ + d_i^- = K, i = 1, \dots, m$), so the ranking given by the measurement of relative proximity (R_i) is the same as that given by the ideal distance ($Min_i\ d_i^+, i = 1, \dots, m$) and the anti-ideal distance ($Max_i\ d_i^-, i = 1, \dots, m$).

Furthermore, it can be seen, as is well known, that the Minkowski distances diminish as the p order of the augmented norm increases ($\|.\|_1 \geq \|.\|_2 \geq \cdots \geq \|.\|_\infty$). The example demonstrates that with a fixed norm order (p), $p = 1,2,\dots,\infty$, the Minkowski distances to the ideal and anti-ideal increase with the different normalization modes, in the following manner‡: $||\cdot||_p^{DM} \leq ||\cdot||_p^{EM} \leq ||\cdot||_p^{IM} \leq ||\cdot||_p^{UM} \leq ||\cdot||_p^{SS} \leq ||\cdot||_p^{NN}$. This latter result is not verified by relative proximity (R_i) for the problem that presents the synthesis of the ideal and anti-ideal distances as a quotient.

5. Conclusions

TOPSIS, in its traditional form, is one of the multicriteria techniques used for decision making in the world of business. However, the method by which the technique is applied (considering the independence between the evaluated attributes), does not occur in real-life cases and it is therefore necessary to adapt it in order to consider dependent attributes.

This paper suggests replacing the Euclidean distance with the Mahalanobis distance to capture the effect of the correlation between the attributes in construction. Despite its greater complexity, TOPSIS-M is recommended for calculating the distances to the ideal and the anti-ideal points when conducting evaluation processes in which the attributes are dependent.

With the aim of analysing the significant differences between the rankings obtained with traditional TOPSIS and the new proposal (TOPSIS-M) when the attributes are dependent, a simulation study is being developed that will reflect the evolution of the rankings in accordance with the level of dependence (Gleason-Staelin's ϕ); the study will provide a set of rules for deciding which type of distance is the most appropriate for each situation.

Another future work will present a new method for the synthesis of the two distances (ideal and anti-ideal) in the final arrangement that allows the combination of both aspects without the problems associated with a quotient.

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‡ Limitations of space do not allow us to include justification tables.

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