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Homogeneous–heterogeneous reactions in stagnation point flow of Casson fluid due to a stretching/shrinking sheet with uniform suction and slip effects

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Abstract This study deals with the effects of homogeneous–heterogeneous reaction on boundary layer flow of a non-Newtonian fluid near a stagnation point over a porous stretching/shrinking sheet with a constant suction. In this analysis Casson fluid is used to indicate the non-Newtonian fluid behavior by taking diffusion coefficients of both reactant and autocatalysis equal. The basic flow equations in form of partial differential equations are converted into a system of ordinary differential equations and then solved numerically. The influences of physical and fluid parameters on the velocity and concentration profiles are analyzed, presented and discussed through graphs. An increase in fluid velocity slip parameter reduces the magnitude of the velocity as well as increases the concentration in the boundary layer region. Furthermore, a unique solution is possible for all values of the stretching parameter ($\lambda > 0$), while in case of shrinking parameter ($\lambda < 0$), solutions are possible only for its limited ranges.

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1. Introduction

Due to an increasing interest in the flow of fluids, a number of materials are utilized whose flow qualities are not analyzed with the Newtonian fluid model. In this situation non-Newtonian fluid models are very important because of their applications in polymer processing industries, petroleum drilling and biofluids dynamics and many others. The Navier–Stokes theory is inadequate for such type of fluids, and no single constitutive equation is available which covers the properties of all fluids. The most popular subclass of these fluids is Casson [1] fluid which displays yield stress impact. This fluid can be considered as a shear thinning liquid which is supposed to have an infinite viscosity at zero rate of shear, a yield stress below which no flow exists and a zero viscosity at an infinite rate of shear. In other words, this type of fluids acts like a solid, when a shear stress lower than the yield stress

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is applied to it, while it starts to move when a shear stress more than the yield stress is applied. The constitutive equation of Casson fluid is nonlinear in nature and has been defined to describe properly the flow curves of suspensions of pigments in lithographic varnishes utilized for preparing printing inks and silicon suspension [2]. Oka [3] reported the characteristics of Casson fluid model in tubes and considered a generalized model for flow of non-Newtonian fluid in tubes from which the Casson fluid model was constructed as a special case. Eldabe and Salwa [4] discussed the effects of MHD and heat transfer analysis in non-Newtonian Casson fluid flow between two rotating cylinders. Dash et al. [5] investigated the analysis of a Casson fluid in a pipe filled with a homogeneous porous medium by considering the Brinkman model. Recently, Mustafa et al. [6] presented the flow of a Casson fluid near a stagnation-point over a stretching sheet and analytical solution is obtained by means of homotopy analysis method. Due to its wide range of applications, this non-Newtonian fluid model has gained much attention of many authors, see Hayat et al. [7], Bhattacharyya [8,9], Shehzad et al. [10], Bhattacharyya et al. [11], Mukhopadhyay et al. [12], Pramanik [13]. Recently, Rao et al. [14] analyzed the heat transfer analysis in a Casson rheological fluid on a semi-infinite vertical plate with partial slip condition at the wall.

In recent years, boundary layer flow of different fluids in the region of stagnation point on a stretching/shrinking surface has attracted many scientists and engineers due to its real world applications in industry engineering processes. Chiam [15] examined the flow of a viscous fluid near a stagnation-point due to a stretching sheet. Ishak et al. [16] studied the effects of applied magnetic field near a stagnation point flow over a stretching sheet. Wang [17] discussed the flow of viscous fluid toward a stretching sheet with partial slip condition at the wall and obtained exact solution of Navier–Stokes equations. In another paper, Wang [18] studied the stagnation point flow due to a shrinking sheet in the presence of applied magnetic field. A theoretical analysis for different values of power index of the wall velocity using exact and numerical solutions for boundary layer flow of a viscous fluid over a nonlinearly shrinking sheet was investigated by Fang [19]. Javed et al. [20] discussed the numerical solution of heat transfer in the MHD viscous fluid over a porous nonlinearly shrinking sheet using Keller-box method. Rosali et al. [21] contributed to the study of stagnation point and heat transfer over a stretching/shrinking sheet in a porous medium. Bhattacharyya [22] reported the dual solutions in the boundary layer flow and mass transfer near a stagnation point on a stretching/shrinking surface. Muhaimin and Hashim [23] studied the effects of suction, heat and mass transfer on a viscous fluid toward a shrinking sheet with chemical reaction and variable stream condition. Singh and Chamkha [24] investigated the viscous fluid flow and heat transfer with second-order slip at shrinking isothermal sheet in a quiescent medium.

Most chemically reacting system involves both homogeneous and heterogeneous reactions, with examples occurring in combustion, catalysis and biochemical systems. The interaction between the homogeneous reactions in bulk of the fluid and heterogeneous reactions taking place on some catalytic surfaces is usually very complicated, which is included in the generation and consumption of reactant species at different rates both within the fluid and on the catalytic surfaces. Chaudhary and Merkin [25] initially studied simple mathematical model for homogeneous–heterogeneous reactions in boundary layer flow near a stagnation point. They gave the formulation of homogeneous (bulk) reaction by isothermal cubic kinetics and the heterogeneous (surface) reaction by considering the first-order kinetics. Later, Chaudhary and Merkin [26] continued their previous work to include the effect of loss of autocatalyst. Merkin [27] reported the results to study a model for isothermal homogeneous–heterogeneous reactions in boundary layer flow of viscous fluid on a flat plate. Khan and Pop [28] discussed the two-dimensional stagnation-point flow past an infinite permeable wall with homogeneous–heterogeneous reactions and uniform suction/injection numerically. Bachok et al. [29] studied the boundary layer flow with combined effects of stagnation-point and homogeneous–heterogeneous reactions toward a stretching surface. Again, Khan and Pop [30] investigated the effects of homogeneous–heterogeneous reactions on viscoelastic fluid due to a stretching sheet numerically using an implicit finite difference method. Kameswaran et al. [31] presented the effects of homogeneous–heterogeneous reactions in a nanofluid flow over a porous stretching sheet embedded in a porous medium by considering copper–water and silver–water as nanofluids. In another paper, Kameswaran et al. [32] studied the homogeneous–heterogeneous reactions on stagnation-point flow of a nanofluid over a stretching/shrinking sheet and obtained dual solutions for particular values of fluid parameters numerically. Shaw et al. [33] discussed the homogeneous–heterogeneous reactions in micropolar fluid flow over a permeable stretching or shrinking sheet in a porous medium numerically. Most recently, Abbas et al. [34] studied the effect of homogeneous–heterogeneous reactions on MHD viscous fluid near the stagnation-point over stretching/shrinking sheet with generalized slip condition.

The aim of the present study is to investigate the effects of homogeneous–heterogeneous reaction on a non-Newtonian Casson fluid near a stagnation point due to a permeable stretching/shrinking sheet with uniform suction and slip velocity at the wall. In this study, we consider the diffusion coefficients of both reactant and autocatalysis are equal. A numerical solution is constructed from nonlinear similarity equations using a shooting technique with Runge–Kutta method of order 4. The physical significance of the controlling fluid parameters on the flow field and concentration profiles are analyzed, presented and discussed graphically.

2. Formulation of the problem

We consider steady, two-dimensional and incompressible flow of a non-Newtonian fluid near a stagnation point due to a linear stretching/shrinking surface. The flow is confined in the region ($y > 0$) toward a surface coinciding with the plane ($y = 0$) with a fixed stagnation point at $x = 0$, with the x-axis along the surface and the y-axis perpendicular to it. Two equal and opposite forces are applied along the surface so that the wall is stretched/shrunk in the x-direction with linear surface velocity $u_w(x) = mx$, where $m > 0$ and $m < 0$ are for stretching and shrinking sheets, respectively, and the velocity outside the boundary layer is $u_e(x) = cx$, where $c > 0$ is the strength of stagnation flow. Following Mustafa et al. [35] and Subba Rao et al. [36], we assume rheological model of an isotropic flow of a non-Newtonian Casson fluid as follows:

$$\tau^{1/n} = \tau_0^{1/n} + \mu\dot{\gamma}^{1/n} \tag{1}$$

or, see Nakamura and Sawada [37],

$$\tau_{ij} = \left[\mu_{\mathbf{B}} + \left(\frac{p_y}{\sqrt{2\pi}}\right)^{1/n}\right]^n 2e_{ij} \tag{2}$$

where μ is the dynamic viscosity, μ_B is the plastic dynamic viscosity of the non-Newtonian fluid, $\pi = e_{ij}e_{ij}$ and e_{ij} is the (i,j) th component of the deformation rate, π is the product of the component of deformation rate with itself and p_y is the yield stress of the fluid. In several applications $n = 1$, but some researchers considered $n \gg 1$. Here for this study, we assume a simple model for the interaction between a homogeneous (or bulk) and a heterogeneous (on sheet) reactions involving the two chemical species A and B in a boundary layer flow reported by Merkin [27] and Chaudhary and Merkin [26] is given as follows:

$$A + 2B \rightarrow 3B, \quad rate = k_c ab^2, \tag{3}$$

$$A \rightarrow B, \quad rate = k_s a. \tag{4}$$

where a and b are the concentrations of the chemical species A and B, while k_i $(i = c, s)$ are the rate constant. We also consider that both reaction processes are isothermal and far away from the sheet at the ambient fluid, there is a uniform concentration 'a_0' of reactant A and there is no autocatalyst B.

With all these assumptions and usual boundary layer approximation, the governing flow equations of this problem can be written in dimensional form:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{5}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e\frac{du_e}{dx} + \nu\left(1 + \frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2}, \tag{6}$$

$$u\frac{\partial a}{\partial x} + v\frac{\partial a}{\partial y} = D_A\frac{\partial^2 a}{\partial y^2} - k_c ab^2, \tag{7}$$

$$u\frac{\partial b}{\partial x} + v\frac{\partial b}{\partial y} = D_B\frac{\partial^2 b}{\partial y^2} + k_c ab^2, \tag{8}$$

in which u and v denote the velocity components in the x- and y-directions, ν is the kinematic viscosity, $\beta = \mu_B\sqrt{2\pi_c}/p_y$ is the non-Newtonian (Casson) parameter and D_A and D_B are the respective diffusion coefficients.

The corresponding boundary conditions of the present problem are

$$\begin{aligned} &u(0) = u_w(x) = mx + L\left(1 + \frac{1}{\beta}\right)\frac{\partial u}{\partial y}, \quad v(0) = -v_w, \\ &D_A\left.\frac{\partial a}{\partial y}\right|_{y=0} = k_s a(0), \quad D_B\left.\frac{\partial b}{\partial y}\right|_{y=0} = -k_s a(0), \\ &u(\infty) = u_e(x) = cx, \quad v(\infty) = 0, \quad a(\infty) = a_0, \quad b(\infty) = 0. \end{aligned} \tag{9}$$

where 'm' and 'c' are constants having dimension $(\text{time})^{-1}$, $v_w(>0)$ is the constant mass transfer (suction) velocity, L is the velocity slip length parameter and has dimension $(\text{velocity})^{-1}$ and a_0 is a constant. The no-slip condition can be obtained for $L = 0$ in this case. The flow equations can be reduced into a set of ordinary differential equations with the following non-dimensional variables

$$\eta = \sqrt{\frac{c}{\nu}}y, \quad u = cxf'(\eta), \quad v = -\sqrt{c\nu}f(\eta), \quad g(\eta) = \frac{a}{a_0}, \quad h(\eta) = \frac{b}{a_0}, \tag{10}$$

with the help of Eq. (10), the continuity Eq. (5) is identically satisfied and Eqs. 6–8 reduce to

$$\left(1 + \frac{1}{\beta}\right)f''' + ff'' - f'^2 + 1 = 0, \tag{11}$$

$$\frac{1}{Sc}g'' + fg' - Kgh^2 = 0, \tag{12}$$

$$\frac{\delta}{Sc}h'' + fh' + Kgh^2 = 0, \tag{13}$$

subject to the boundary conditions

$$f(0) = S, \quad f'(0) = \lambda + \gamma\left(1 + \frac{1}{\beta}\right)f''(0), \quad f'(\infty) = 1, \tag{14}$$

$$g'(0) = K_s g(0), \quad g(\infty) = 1, \tag{15}$$

$$\delta h'(0) = -K_s g(0), \quad h(\infty) = 0. \tag{16}$$

Here a prime denotes differentiation with respect to η, $S = v_w/(c\nu)^{1/2}(>0)$ the mass suction parameter, $\lambda = m/c$ is the ratio of stretching rate to external flow rate with $\lambda > 0$ for a stretching sheet, $\lambda < 0$ for a shrinking sheet and $\lambda = 0$ for a static sheet, $\gamma = L\sqrt{c/\nu}$ is the velocity slip parameter, $Sc = \nu/D_A$ is the Schmidt number, $\delta = D_B/D_A$ is the ratio of the diffusion coefficients, $K = k_c a_0^2/c$ gives a measure of the strength of the homogeneous reaction and $K_s = k_s Re^{-1/2}/D_A$ measures the strength of heterogeneous reaction and $Re = c/\nu$ is the Reynolds number.

In many applications, it is expected that the diffusion coefficients of chemical species A and B to be of a comparable size, which leads us to further assumption that the diffusion coefficients D_A and D_B are equal. Following Chaudhary and Merkin [25], we take $\delta = 1$ and this assumption leads to the relation

$$g(\eta) + h(\eta) = 1. \tag{17}$$

Thus, Eqs. (12) and (13) finally reduce to

$$\frac{1}{Sc}g'' + fg' - Kg(1-g)^2 = 0 \tag{18}$$

with boundary conditions

$$g'(0) = K_s g(0), \quad g(\infty) = 1. \tag{19}$$

The physical quantity of interest is the skin friction coefficient C_f, which can be written as

$$C_f = \frac{\tau_w}{\rho u_w^2}, \tag{20}$$

in which τ_w is the shear stress at the surface of the wall and is given by

$$\tau_w = \left(\mu_B + \frac{p_y}{\sqrt{2\pi_c}}\right)\left(\frac{\partial u}{\partial y}\right)_{y=0}. \tag{20}$$

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In view of Eqs. (9), (19) and (20), it reduces to

$$Re_x^{1/2} C_f = \left(1 + \frac{1}{\beta}\right) f''(0), \tag{21}$$

where $Re_x = xu_w/v$ is the local Reynolds number.

3. Results and discussion

The dual solutions of nonlinear boundary layer Eqs. (11) and (18) together with boundary conditions (14) and (19) are obtained numerically by using fourth-order Runge–Kutta–Fehlberg method with shooting technique. After applying the shooting technique, the resultant initial value problem is solved by using fourth-order Runge–Kutta–Fehlberg method with step – size $\Delta\eta = 0.01$. We set various values for $f''(0)$ and $\theta'(0)$, where all the velocity and concentration profiles satisfy the infinity boundary condition asymptotically for the values of all physical parameters and choosing a suitable finite values of η_∞ (where η_∞ corresponds to $\eta \to \infty$). The procedure of obtaining numerical solution is repeated until we get the results up to the desired degree of accuracy 10^{-6}. The computations have been done for different values of the controlling governing parameters such as Casson parameter β, stretching/shrinking parameter λ, slip parameter γ, suction parameter S, strength of the homogeneous parameter K, strength of the heterogeneous parameter K_s and Schmidt number Sc. For physical interpretations of the results the numerical values are plotted in Figs. 1–13 for dimensionless velocity field and concentration profiles.

Fig. 1 illustrates that the effects of the stretching/shrinking parameter λ on the dimensionless velocity of the fluid $f'(\eta)$ with other parameters are fixed. From this figure, it is evident that in case of stretching parameter ($\lambda > 0$) the solutions can be obtained for all values and the fluid velocity $f'(\eta)$ increases greater than the free stream velocity, but the fluid velocity $f'(\eta)$ decreases as the value of η increases. It is further noted that for shrinking parameters ($\lambda < 0$) the dual solutions exist at a certain values of λ such as $\lambda = -1.32$, -1.42 and -1.52, and the fluid velocity is initially negative but it is increased with increasing η and becomes positive. Fig. 2 shows the dual solutions of fluid velocity $f'(\eta)$ for several values of Casson parameter β with shrinking parameter $\lambda = -1.52$ keeping other governing parameters fixed. It is interesting to note that the

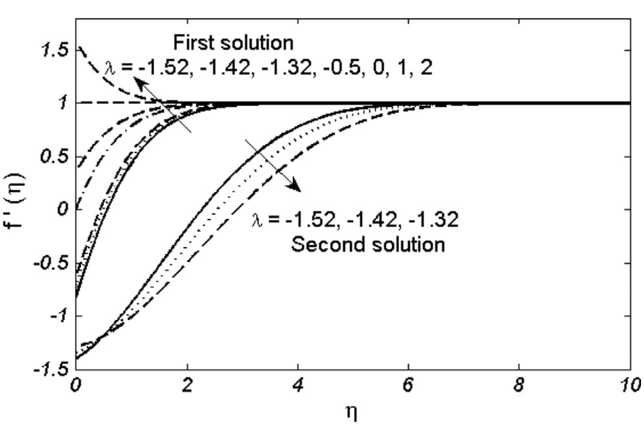

Figure 1 Velocity profiles for various values of the stretching/shrinking parameter λ when $S = 0.5$, $\beta = 3$ and $\gamma = 0.3$.

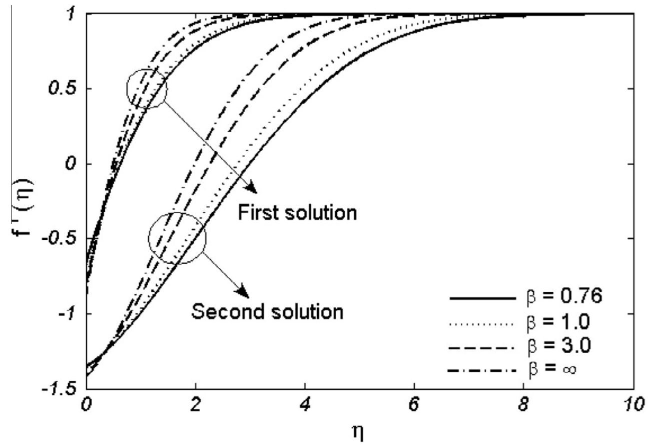

Figure 2 Velocity profiles for various values of the non-Newtonian (Casson) parameter β when $S = 0.5$, $\lambda = -1.52$ and $\gamma = 0.3$.

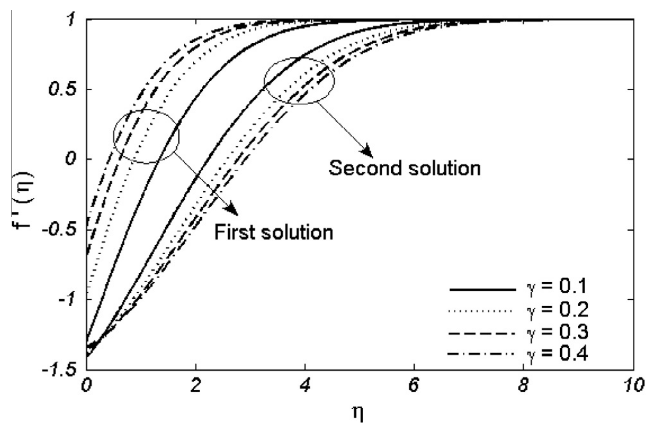

Figure 3 Velocity profiles for various values of the slip parameter γ when $S = 0.5$, $\beta = 1$ and $\lambda = -1.52$.

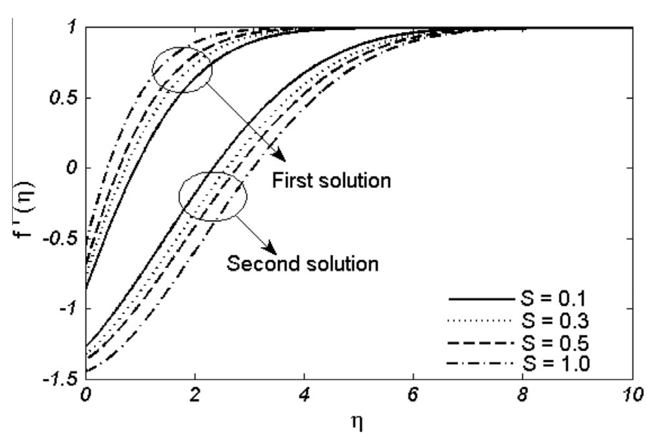

Figure 4 Velocity profiles for various values of the suction parameter S when $\gamma = 0.3$, $\beta = 1$ and $\lambda = -1.52$.

fluid velocity $f'(\eta)$ increases with an increase in Casson parameter β, i.e. the fluid behaves as Newtonian fluid as Casson parameter β becomes large. It is also noticed that the increase in Casson parameter is to decrease the yield stress and

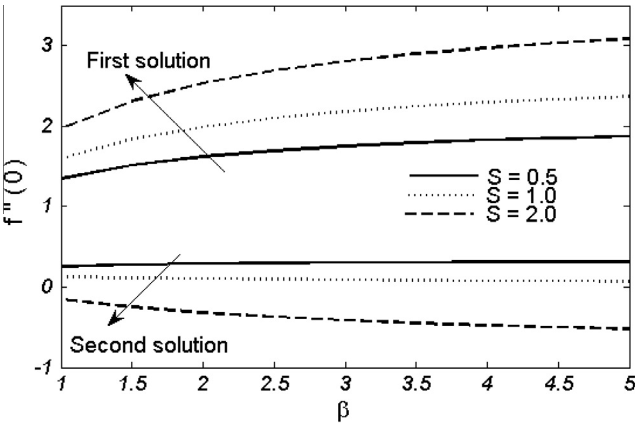

Figure 5 Effects of suction parameter S on $f''(0)$ with Casson parameter β when $\gamma = 0.3$ and $\lambda = -1.52$.

consequently the momentum boundary layer thickness decreases. This contraction in boundary layer thickness is caused by the introduction of tensile stress due to elasticity. The effects of the velocity slip parameter γ on the dimensionless fluid velocity $f'(\eta)$ for shrinking parameter $\lambda = -1.52$ are presented in Fig. 3. It can be seen from this figure that for first solution (which is stable and physically realizable in practice) both the absolute values of fluid velocity and the momentum boundary layer thickness decrease as γ increases. Physically, with shrinking of the sheet, the retracting forces reduce momentum development and cause deceleration in the boundary layer flow, whereas reverse trend is noted in the case of second solution. Fig. 4 shows the influence of fluid velocity $f'(\eta)$ for different values of mass suction parameter S with $\gamma = 0.3$, $\beta = 1$ and $\lambda = -1.52$ are fixed. It is worthwhile to note that the effects of suction parameter S on the fluid velocity and momentum boundary layer thickness are same as of velocity slip parameter due to the fact that suction is an agent causing resistance to the fluid flow. Fig. 5 gives the effect of the suction parameter S on the shear stress at the surface $f''(0)$ with Casson parameter β with $\gamma = 0.3$ and $\lambda = -1.32$ are fixed. We can see from figure that the absolute values of $f''(0)$ increase with an increase in S and β in case of first solution, while an opposite trend is noted for second solution.

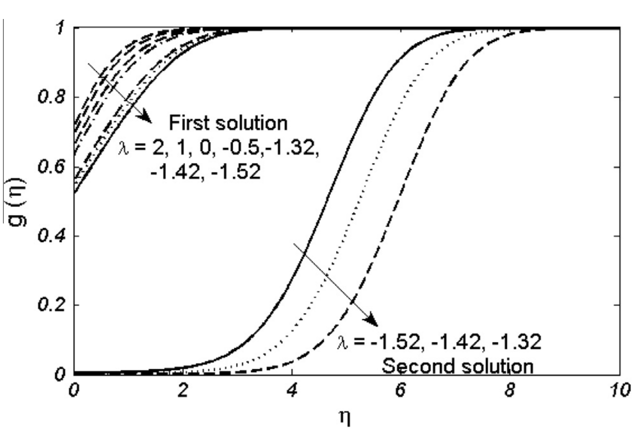

Figure 6 Concentration profiles for various values of the velocity parameter λ when $S = 0.5$, $\beta = 3$, $\gamma = 0.3$, $K_s = 0.5$, $K = 0.5$ and $Sc = 1$.

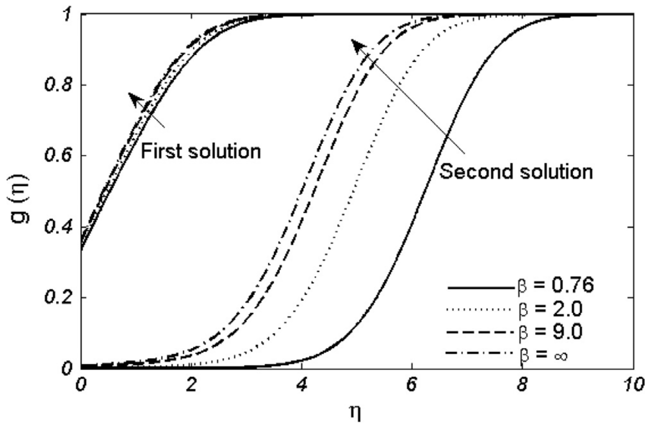

Figure 7 Concentration profiles for various values of the non-Newtonian (Casson) parameter β when $S = 0.5$, $\gamma = 0.3$, $\lambda = -1.52$, $K_s = 1$, $K = 0.5$ and $Sc = 1$.

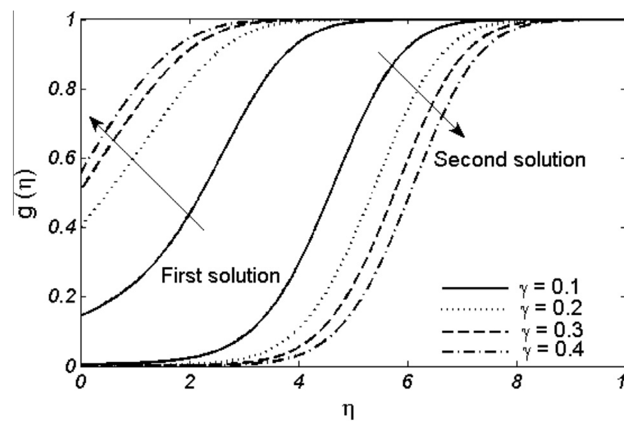

Figure 8 Concentration profiles for various values of the slip parameter γ when $S = 0.5$, $\beta = 1$, $\lambda = -1.52$, $K_s = 0.5$, $K = 0.5$ and $Sc = 1$.

The variation of concentration profile $g(\eta)$ versus η is exhibited for several values of stretching/shrinking parameter λ in Fig. 6. From this figure, it is evident that for the concentration profile $g(\eta)$, the graphs contain a dual solution at $\lambda = -1.32$, -1.42 and -1.52, and all the curves are started from the origin and they are increased nonlinearly with η to follow 'S' shape and consequently approach unity according to the given condition which agrees with the results reported by Shaw et al. [33]. It is also observed that for the first solution, the concentration boundary thickness is increased with an increase in λ, while an opposite phenomenon appears for the second solution. Fig. 7 exhibits the effects of Casson fluid parameter β on the concentration profile $g(\eta)$ with other physical parameters are fixed. From this figure, it is noticed that the effect of increasing Casson parameter β leads to enhance the concentration profile and the concentration boundary layer thickness decreases, this is due to increase in the elasticity stress parameter. Fig. 8 shows the variation of the concentration profile $g(\eta)$ for different values of velocity slip parameter γ with shrinking parameter $\lambda = -1.52$. It is noticed from figure that the increases in velocity slip parameter γ are to increase the concentration profile, but concentration boundary layer thickness

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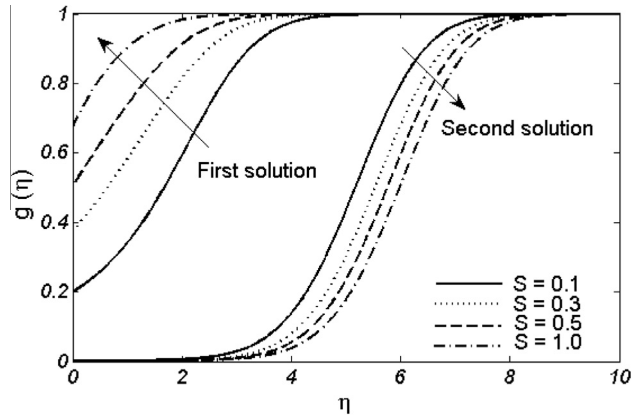

Figure 9 Concentration profiles for various values of the suction parameter S when $\gamma = 0.3$, $\beta = 1$, $\lambda = -1.52$, $K_s = 0.5$, $K = 0.5$ and $Sc = 1$.

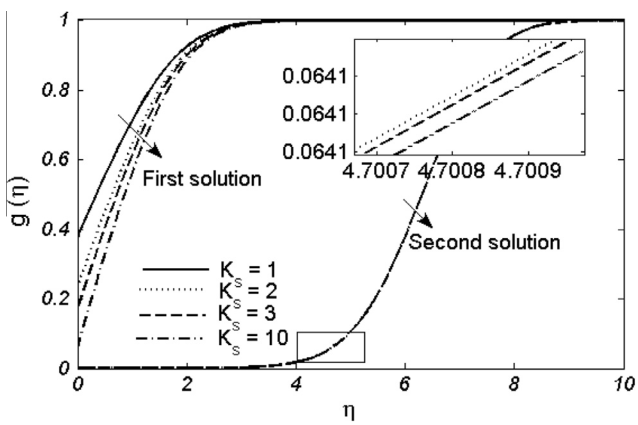

Figure 10 Concentration profiles for various values of K_s (strength of the heterogeneous) when $\gamma = 0.3$, $\beta = 2$, $\lambda = -1.32$, $S = 0.5$, $K = 0.5$ and $Sc = 1$.

decreases as γ increases (first solution). It is further noted that the fluid velocity is reduced due to slip condition at the wall, and this leads to an increase in the solute concentration, and an opposite phenomenon is observed for second solution. Also, the increase in concentration with increasing slip parameter in the case of shrinking sheet is caused by the decreased area available in the boundary layer when the sheet shrinks. Fig. 9 presents the change in concentration profiles $g(\eta)$ in concentration boundary thickness for several values of mass suction parameter S with all parameters keeping fixed. From this figure, we can see the similar effects of the suction parameter S on the concentration profile as of the velocity slip parameter γ. This behavior is noted because fluid experiences a resistance on increasing the friction among its layers. Figs. 10 and 11 show the effect of heterogeneous and homogeneous reactions (K_s, K) on the concentration profile $g(\eta)$ with shrinking parameter $\lambda = -1.32$. In these figures, we consider $\lambda = -1.32$ and so a dual solution exists. We infer from these figures that the concentration boundary layer of the reactants is increasing with η in both cases and after a certain value of η, the homogeneous and heterogeneous reactions have no effects on the concentration of the reactants i.e. they all coincide. Fig. 12 gives the variation of $g(0)$ versus homogeneous reaction

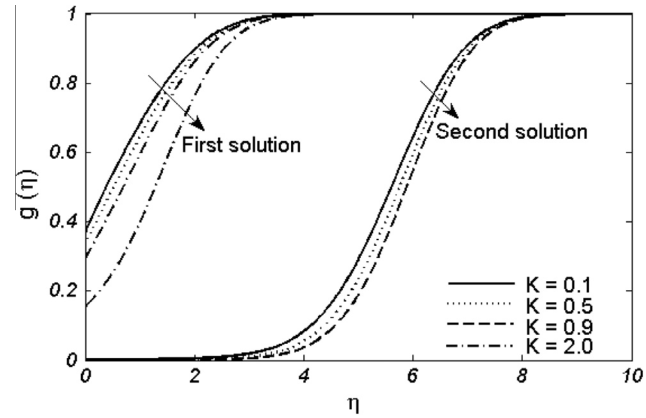

Figure 11 Concentration profiles for various values of K (strength of the homogeneous reaction) when $\gamma = 0.3$, $\beta = 1$, $\lambda = -1.52$, $S = 0.5$, $K_s = 1$ and $Sc = 1$.

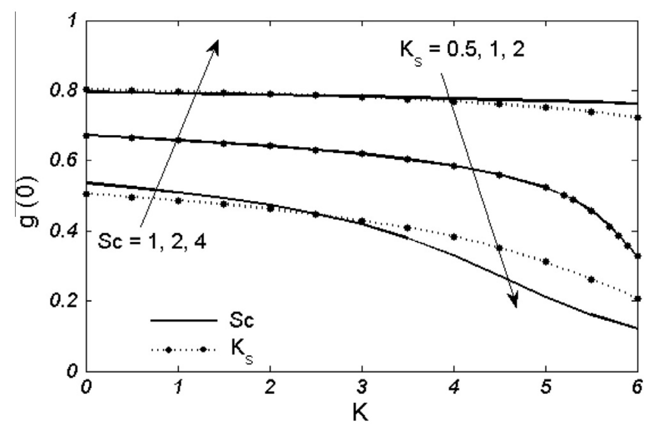

Figure 12 Effects of various values of Schmidt number Sc and K_s versus K on concentration when $S = 1$, $\beta = 2, \gamma = 0.3$ and $\lambda = -1.32$.

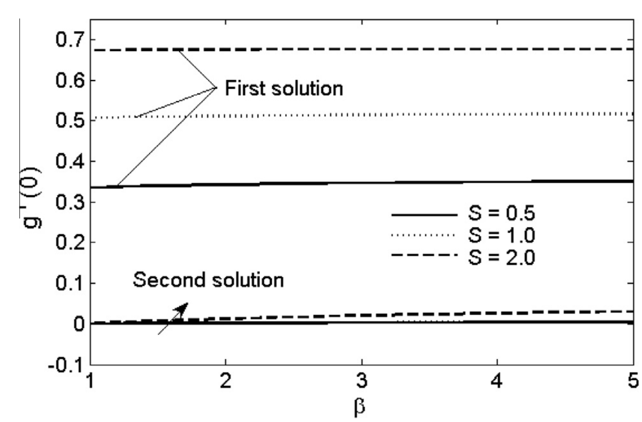

Figure 13 Effects of suction parameter S on $g'(0)$ with Casson parameter β when $\gamma = 0.3$, $\lambda = -1.52$, $K_s = 1, K = 0.5$ and $Sc = 1$.

parameter K for several values of the Schmidt number Sc and K_s (heterogeneous reaction parameter), because the concentration of the reactant depends on the Schmidt number Sc and

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heterogeneous reaction parameter K_s and the Schmidt number Sc is the ratio of a viscous diffusion rate to a molecular diffusion rate. From this figure it is noticed that for a fixed molecular diffusion rate, the increase in Schmidt number is to increase the viscous diffusion of the reactants rate, which reduces the fluid velocity and leads to increase the concentration of reactants. It is further noted that both homogeneous and heterogeneous reaction parameters influence the reaction rate of the solute and lead to reduce the concentration of the reactants, which agrees with the results reported by Khan and Pop [30] and Shaw et al. [33]. Fig. 13 presents the variation of the gradient of concentration at the surface $g'(0)$ with Casson parameter β for different values of mass suction parameter S with $\lambda = -1.52$. From this figure, it is evident that the absolute values of $g'(0)$ increase as S and β are increased (first solution) and the graphs for second solution with $S = 1$ and 2 coincide.

4. Concluding remarks

In the present analysis, the stagnation point flow of Casson fluid due to a permeable stretching/shrinking sheet with homo geneous–heterogeneous reactions is studied. Using similarity transformations, the momentum and concentration equations are converted to coupled ordinary differential equation which are then solved using shooting method with Runge–Kutta algorithm. It is found that a unique solution exists for all values of stretching parameters, whereas a dual solution exists for the shrinking sheet case for specific values of shrinking parameter λ. Variations of the fluid velocity and concentration profile with dual solutions are shown by graphs. In addition to this, from this study, the following conclusion can be made:

- The fluid velocity increases in the boundary layer, while the momentum boundary layer thickness decreases by increasing β, γ and S for first solution but it has opposite trend in case of γ and S for second solution.
- The concentration of the species is increased with an increase in λ, β, γ and S, whereas the concentration boundary layer thickness decreases by increasing λ, β, γ and S for first solution. For second solution, the concentration field increases with η by increasing λ, γ and S, but it decreases with η for larger values of β.
- The concentration boundary layer of reactants is increased with η for both homogeneous and heterogeneous reactions.
- The concentration at the surface increases with Schmidt number and it is reduced by increasing homogeneous and heterogeneous reactions.

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