Physics Letters B 690 (2010) 316–321

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Dual approaches for defects condensation

L.S. Grigorio [a,c], M.S. Guimaraes [b], R. Rougemont [a], C. Wotzasek [a,*]

[a] Instituto de Física, Universidade Federal do Rio de Janeiro, 21941-972, Rio de Janeiro, Brazil
[b] Departamento de Física Teórica, Instituto de Física, UERJ – Universidade do Estado do Rio de Janeiro, Rua São Francisco Xavier 524, 20550-013 Maracanã, Rio de Janeiro, Brazil
[c] Centro Federal de Educação Tecnológica Celso Suckow da Fonseca 28635-000, Nova Friburgo, Brazil

ARTICLE INFO

Article history:
Received 3 August 2009
Received in revised form 16 April 2010
Accepted 18 May 2010
Available online 25 May 2010
Editor: M. Cvetič

Keywords:
Topological defects
Monopoles
Vortices
Duality
Confinement

ABSTRACT

We review two methods used to approach the condensation of defects phenomenon. Analyzing in details their structure, we show that in the limit where the defects proliferate until occupy the whole space these two methods are dual equivalent prescriptions to obtain an effective theory for the phase where the defects (like monopoles or vortices) are completely condensed, starting from the fundamental theory defined in the normal phase where the defects are diluted.

1. Introduction

The quantum field theory description of a physical system relies on a proper identification of its degrees of freedom which are then interpreted as excited states of the fields defining the theory. However it is sometimes the case that the theory may contain important structures which are not described in this way and cannot be expressed in a simple manner in terms of the fields appearing in the Lagrangian, having a non-local expression in terms of them. These structures appear under certain conditions as *defects*; prescribed singularities of the fields defining the theory. A general conjecture [2] claims that defects are described by a dual formulation in which they appear as excitations of the dual field, but this can be proved only in some particular instances. Nevertheless, much can be gained just with the information that these structures appear as singularities of the fundamental fields even without knowing their precise dynamics. A pressing question is if it is possible to address, with this limited information, the situation in which the collective behavior of defects becomes the dominant feature of a theory. It is one of the purposes of this work to discuss an extreme case of sorts. We want to present a general proposal of how to describe a situation in which the singularities of the fields proliferate defining a new vacuum for the system. In this picture the new degrees of freedom are recognized as excitations of the established condensate of defects.

This view is supported by the fact that if we are interested only in the low lying excitations it is perfectly reasonable to take the condensate as given, not worrying how it was set on, and construct an effective field theory describing the excitations. It is well known for instance that the pions, which can be recognized as excitations of the chiral symmetry breaking condensate composed of quark–antiquark pairs, can be described by an effective field theory without knowing about QCD. Even though we need not know the details of how the condensate is formed it is important to stress that the condensate defines the vacuum and carries vital information about the symmetry content used in the construction of the effective theory. It is in this way also bound to have an effect in all the other fields comprising the system. The example of a superconducting medium also comes to mind, where the condensate vacuum endows the electromagnetic excitations with a mass. This same idea is employed on the electroweak theory where a condensate is the only consistent way to give mass to the force carriers, the W and Z, and in fact to account for all the masses of the standard model. This is an example where the properties of the condensate itself are not completely established and still a matter of debate. The currently accepted view is that its low lying excitations are the Higgs particles, still to be detected, described by a scalar field.

More akin to our take on the condensate concept, as a collective behavior of defects, is the dual superconductor model of confine-

* Corresponding author.
E-mail address: clovis@if.ufrj.br (C. Wotzasek).

0370-2693
doi:10.1016/j.physletb.2010.05.044

ment which is based on the superconductor phenomenology [1]. It is expected that the QCD vacuum at low energies is a chromomagnetic condensate leading to the confinement of color charges immersed in this medium. In dual superconductor models of color confinement, magnetic monopoles appear as topological defects in points of the space where the abelian projection becomes singular [9]. There are in fact many other examples in which the condensation of defects is responsible for drastic changes in the system by defining the new vacuum of the theory. We may mention vortices in superfluids and line-like defects in solids which are responsible for a great variety of phase transitions [6]. All these instances point to the importance of getting a better understanding of the condensation phenomenon.

In all these examples there are some general features of the condensates which can tell a lot about what to expect of the system when condensation sets in without the precise knowledge of how this happened. These general features are what we intend to explore in this Letter. The main inspiration for this work comes from the study of two particular approaches to this problem: one is the Abelian Lattice Based Approach (ALBA) discussed by Banks, Myerson and Kogut in [3] within the context of relativistic lattice field theories and latter also by Kleinert in [5] in the condensed matter context. The other one is the Julia–Toulouse Approach (JTA) introduced by Julia and Toulouse in [4] within the context of ordered solid-state media and later reformulated by Quevedo and Trugenberger in the relativistic field theory context [7].

The ALBA was used, for example, by Banks, Myerson and Kogut to study phase transitions in abelian lattice gauge theories [3]. A few years latter Kleinert obtained a disorder field theory for the superconductor from which he established the existence of a tricritical point separating the first-order from the second-order superconducting phase transitions [5]. In this Letter we shall be using the notations in the recent book by Kleinert [6].

Developing in the work of Julia and Toulouse, Quevedo and Trugenberger studied the different phases of field theories of compact antisymmetric tensors of rank $h-1$ in arbitrary space–time dimensions $D=d+1$. Starting in a coulombic phase, topological defects of dimension $d-h-1$ ($(d-h-1)$-branes) may condense leading to a confining phase. In that work one of the applications of the JTA was the explanation of the axion mass. It was known that the QCD instantons generate a potential which gives mass to the axion. However, the origin of this mass in a dual description were a puzzle. When the JTA is applied it is clear that the condensation of instantons is responsible for the axion mass.

Recently, some of us and collaborators have made a proposal that the JTA would be able to explain the dual phenomenon to radiative corrections [10] and used this idea to compute the fermionic determinant in the QED_3 case. This result was immediately extended to consider the use of the JTA to study QED_3 with magnetic-like defects. By a careful treatment of the symmetries of the system we suggested a geometrical interpretation for some debatable issues in the Maxwell–Chern–Simons-monopole system, such as the induction of the non-conserved electric current together with the Chern–Simons term, the deconfinement transition and the computation of the fermionic determinant in the presence of Dirac string singularities [11]. It is important to point that the main signature of the JTA is the rank-jumping of the field tensor due to the defects condensation. However, this discontinuous change of the theory still puzzles a few. It is another goal of this investigation to shed some light in this matter.

In the present work we hope to help clarify the above mentioned issues focusing in the analysis of the structure of these two methods, i.e., JTA and ALBA, by working out an explicit example. Introducing a new Generalized Poisson's Identity (GPI) for p-branes in arbitrary space–time dimensions and the novel concept of Poisson-dual branes we show that in the specific limit where the defects proliferate until they occupy the whole space these two approaches are dual equivalent prescriptions to obtain an effective theory for the phase where the defects are completely condensed, starting from the fundamental theory defined in the normal phase where the defects are diluted.

2. Setting the problem

The example we will work here is the Maxwell theory in the presence of monopoles that eventually condense, which serves as an abelian toy model that simulates quark confinement.

The Maxwell field A_μ minimally coupled to electric charges e and non-minimally coupled to magnetic monopoles g is described by the following action:

$$S = S_0^M + S_{int} = -\int d^4x\,\frac{1}{4}\big(F_{\mu\nu} - F_{\mu\nu}^M\big)^2 - \int d^4x\, j_\mu A^\mu, \tag{1}$$

where $j^\mu = e\delta^\mu(x;L')$ ($\tilde{j}^\mu = g\delta^\mu(x;L)$) is the electric (magnetic) current, being $\delta^\mu(x;L')$ ($\delta^\mu(x;L)$) a δ-distribution that localizes the world line L' (L) of the electric (magnetic) charge e (g). $F_M^{\mu\nu} = g\tilde{\delta}^{\mu\nu}(x;S) := \frac{g}{2}\epsilon^{\mu\nu\alpha\beta}\delta_{\alpha\beta}(x;S)$ is the magnetic Dirac brane, with $\delta_{\mu\nu}(x;S)$ a δ-distribution that localizes the world surface S of the Dirac string coupled to the monopole [8] and has the current $\tilde{j}_\mu$ in its border. The field A_μ experiences a jump of discontinuity as it crosses S, hence $F_{\mu\nu}$ has a δ-singularity over S [13] that exactly cancels the one in $F_{\mu\nu}^M$ such that $F_{\mu\nu} - F_{\mu\nu}^M := F_{\mu\nu}^{obs}$ is the regular combination which expresses the observable fields $\vec{E}$ and $\vec{B}$. As we shall see, the quantum field theory associated to this action has two different kinds of local symmetries: the first one is the usual electromagnetic gauge symmetry, $A_\mu(x) \to A'_\mu(x) = A_\mu(x) + \partial_\mu\Lambda(x)$, with integrable Λ, i.e., $[\partial_\mu, \partial_\nu]\Lambda = 0$. The second one corresponds to the freedom of moving the unphysical surface S over the space:

$$F_{\mu\nu}^M \to F_{\mu\nu}^{M\,\prime} = F_{\mu\nu}^M + \partial_\mu\Lambda_\nu^M - \partial_\nu\Lambda_\mu^M,$$
$$A_\mu \to A'_\mu = A_\mu + \Lambda_\mu^M, \tag{2}$$

where $\Lambda_\mu^M = g\tilde{\delta}_\mu(x;V) := \frac{g}{3!}\epsilon_{\mu\nu\alpha\beta}\delta^{\nu\alpha\beta}(x;V)$, being $\delta_{\mu\nu\rho}(x;V)$ a δ-distribution that localizes the volume V spanned by the deformation $S \to S'$ (the boundary ∂S of S is physical and is kept fixed in the transformation such that $\partial S = \partial S'$). We name here this second kind of local symmetry as brane symmetry. Taking into account the current conservation we see that the action (1) is invariant under gauge transformations. But (1) changes under brane transformations as $\Delta S = S'_{int} - S_{int} = -\int d^4x\, j_\mu \Lambda_M^\mu = -egn$, $n \in \mathbb{Z}$ so that (2) is not a symmetry of (1). But being the Dirac string unphysical we should not be able to detect it experimentally. So we need to impose some consistency condition to make the Dirac string physically undetectable within the present formalism. We can do it only by means of a quantum argument: the phase factor appearing in the partition function associated to (1) changes under brane symmetry as $e^{iS} \to e^{iS'} = e^{i(S+\Delta S)} = e^{iS}e^{-iegn}$, $n \in \mathbb{Z}$. It should be clear now that to keep the physics unchanged under brane transformations the consistency condition needed to impose is $e^{-iegn} \equiv 1$, $n \in \mathbb{Z} \Rightarrow eg \equiv 2\pi N$, $N \in \mathbb{Z}$, which is the famous Dirac quantization condition [8], a possible explanation for the charge quantization.

Now in order to consider the monopole condensation (which will induce the electric charge confinement) it is best to go to the dual picture. To obtain the dual action to (1) we introduce an auxiliary field $f_{\mu\nu}$ and define the master action by lowering the order of the derivatives appearing in (1) via Legendre transformation:

$$\tilde{S} := \int d^4x\left[-\frac{1}{2}f_{\mu\nu}F^{\mu\nu}_{obs} + \frac{1}{4}f^2_{\mu\nu} - j_\mu A^\mu\right]. \tag{3}$$

Extremizing $\tilde{S}$ with respect to $f_{\mu\nu}$ we get $f_{\mu\nu} = F^{obs}_{\mu\nu}$ and substituting that in (3) we reobtain the original action (1) while extremizing $\tilde{S}$ with respect to A_μ we get the condition $\partial_\mu f^{\mu\nu} = j^\nu$, which can be solved by $f_{\mu\nu} \equiv \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}\tilde{F}^{\alpha\beta}_{obs} := \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}(\tilde{F}^{\alpha\beta} - \tilde{F}^{\alpha\beta}_E)$. We introduced the dual vector potential $\tilde{A}_\mu$ in $\tilde{F}_{\mu\nu} := \partial_\mu\tilde{A}_\nu - \partial_\nu\tilde{A}_\mu$ and the electric Dirac brane $\tilde{F}^E_{\mu\nu}$ that localizes the world surface of the electric Dirac string coupled to the electric charge. Substituting this result in (3) and discarding an electric brane–magnetic brane contact term that does not contribute to the partition function due to the Dirac quantization condition, we obtain the dual action:

$$\tilde{S} = \tilde{S}^E_0 + \tilde{S}_{int} = \int d^4x\left[-\frac{1}{4}\tilde{F}^{obs\,2}_{\mu\nu} - \tilde{A}^\mu\tilde{j}_\mu\right], \tag{4}$$

where the couplings are inverted relatively to the ones in the original action (1): here the dual vector potential $\tilde{A}_\mu$ couples minimally with the monopole and non-minimally with the electric charge.

3. Abelian lattice based approach

We are now in position to consider monopole condensation by applying the ALBA to the dual Maxwell action (4). The main goal of this approach is to obtain an effective action for the condensed phase in the dual picture. The ALBA is based on the observation that upon condensation, the magnetic defects initially described by δ-distributions are elevated to the field category describing the long-wavelength fluctuations of the magnetic condensate. The condition triggering the complete condensation of the defects is given by the disappearance of the Poisson-dual brane (defined below) coming from a Generalized Poisson's Identity (see the discussion in Appendix A).

We suppose that for the electric charges there are only a few fixed (external) worldlines L' while for the monopoles we suppose that there is a fluctuating ensemble of closed worldlines L that can eventually proliferate (the details of how such a proliferation takes place is a dynamical issue not addressed neither by the ALBA nor by the JTA). The magnetic current is written in terms of the magnetic Dirac brane as $\tilde{j}^\sigma = \frac{1}{2}\epsilon^{\sigma\rho\mu\nu}\partial_\rho F^M_{\mu\nu}$. In order to allow the monopoles to proliferate we must give dynamics to their magnetic Dirac branes since the proliferation of them is directly related to the proliferation of the monopoles and their worldlines. Thus we supplement the dual action (4) with a kinetic term for the magnetic Dirac branes of the form $-\frac{\epsilon_c}{2}\tilde{j}^2_\mu$, which preserves the local gauge and brane symmetries of the system. This is an activation term for the magnetic loops. Hence, the complete partition function associated to the extended dual action reads:

$$Z^c := \int \mathcal{D}\tilde{A}_\mu\,\delta\left[\partial_\mu\tilde{A}^\mu\right]e^{i\tilde{S}^E_0}Z^c[\tilde{A}_\mu], \tag{5}$$

where the Lorentz gauge has been adopted for the dual gauge field $\tilde{A}_\mu$ and the partition function for the brane sector $Z^c[\tilde{A}_\mu]$ is given by,

$$Z^c[\tilde{A}_\mu] := \sum_{\{L\}}\delta\left[\partial_\mu\tilde{j}^\mu\right]\exp\left\{i\int d^4x\left[-\frac{\epsilon_c}{2}\tilde{j}^2_\mu + \tilde{j}_\mu\tilde{A}^\mu\right]\right\}, \tag{6}$$

where the functional δ-distribution enforces the closeness of the monopole worldlines.

Next, use is made of the Generalized Poisson's Identity (GPI) [12] (see Eq. (A.6) in Appendix A) in $d = 4$

$$\sum_{\{L\}}\delta\left[\eta_\mu(x) - \delta_\mu(x;L)\right] = \sum_{\{\tilde{V}\}}e^{2\pi i\int d^4x\,\tilde{\delta}_\mu(x;\tilde{V})\eta^\mu(x)}, \tag{7}$$

where L is a 1-brane and $\tilde{V}$ is the 3-brane of complementary dimension. The GPI works as an analogue of the Fourier transform: when the lines L in the left-hand side of (7) proliferate, the volumes $\tilde{V}$ in the right-hand side become diluted and vice versa (see the discussion in Appendix A). We shall say that the branes L and $\tilde{V}$ (or the associated currents $\delta_\mu(x;L)$ or $\tilde{\delta}_\mu(x;\tilde{V})$) are Poisson-dual to each other. Using (7) we can rewrite (6) as:

$$\begin{aligned}
Z^c[\tilde{A}_\mu] &= \int \mathcal{D}\eta_\mu\sum_{\{L\}}\delta\left[g\left(\frac{\eta_\mu}{g} - \delta_\mu(x;L)\right)\right] \\
&\quad\times\delta\left[g\left(\partial_\mu\frac{\eta_\mu}{g}\right)\right]\exp\left\{i\int d^4x\left[-\frac{\epsilon_c}{2}\eta^2_\mu + \eta_\mu\tilde{A}^\mu\right]\right\} \\
&= \int \mathcal{D}\eta_\mu\sum_{\{\tilde{V}\}}e^{2\pi i\int d^4x\,\tilde{\delta}_\mu(x;\tilde{V})\frac{\eta^\mu}{g}}\int \mathcal{D}\tilde{\theta} \\
&\quad\times e^{i\int d^4x\,\tilde{\theta}\partial_\mu\frac{\eta^\mu}{g}}\exp\left\{i\int d^4x\left[-\frac{\epsilon_c}{2}\eta^2_\mu + \eta_\mu\tilde{A}^\mu\right]\right\} \\
&= \sum_{\{\tilde{V}\}}\int \mathcal{D}\tilde{\theta}\int \mathcal{D}\eta_\mu\exp\left\{i\int d^4x\left[-\frac{\epsilon_c}{2}\eta^2_\mu + \right.\right. \\
&\quad\left.\left. -\,\eta^\mu\frac{1}{g}\left(\partial_\mu\tilde{\theta} - \tilde{\theta}^V_\mu - g\tilde{A}_\mu\right)\right]\right\}.
\end{aligned} \tag{8}$$

In the first line we introduced the auxiliary field η_μ which will replace the δ-distribution current in the condensed phase as discussed above. In the second line we exponentiated the current conservation condition through use of the $\tilde{\theta}$ field and also made use of the GPI to bring into the game the Poisson-dual current $\tilde{\theta}^V_\mu = 2\pi\tilde{\delta}_\mu(x;\tilde{V})$. We also made an integration by parts and discarded a constant multiplicative factor since it drops out in the calculation of correlation functions.

Integrating the auxiliary field η_μ in the partial partition function (8) and substituting the result back in the complete partition function (5) we obtain, as the effective total action for the condensed phase in the dual picture, the London limit of the Dual Abelian Higgs Model (DAHM):

$$\tilde{S}^L_{DAHM} = \int d^4x\left[-\frac{1}{4}\tilde{F}^{obs\,2}_{\mu\nu} + \frac{m^2_{\tilde{A}}}{2g^2}\left(\partial_\mu\tilde{\theta} - \tilde{\theta}^V_\mu - g\tilde{A}_\mu\right)^2\right], \tag{9}$$

where we defined $m^2_{\tilde{A}} := \frac{1}{\epsilon_c}$. This effective action is the main result of this approach. In the next section we shall dualize this result and one could be concerned with the fact that (9) constitutes a nonrenormalizable theory, thus requiring a cutoff in order to be well defined as an effective quantum theory. However, one can always think of its UV completion, in this case the complete DAHM, which is renormalizable, and then take its dual, taking the London limit afterwards [9]. At least in the case considered here, the result is exactly the same one obtains by directly dualizing the London limit (9) of the DAHM, thus justifying the procedure we shall adopt in the next section.

Considering now that a complete condensation of monopoles takes place we let their worldlines L proliferate and occupy the whole space, implying that $\tilde{\theta}^V_\mu \to 0$ as seen from (7) and the discussion afterwards (notice that $\tilde{\theta}^V_\mu$ appears as a vortex-like defect for the scalar field $\tilde{\theta}$ describing the magnetic condensate, being a parameter that controls the monopole condensation). Integrating the Higgs field $\tilde{\theta}$ we get a transverse mass term for $\tilde{A}_\mu$ (Higgs mechanism) such that the electric field has a finite penetration depth $\lambda = \frac{1}{m_{\tilde{A}}} = \sqrt{\epsilon_c}$ in the DSC: this is the dual Meissner effect. Integrating now the field $\tilde{A}_\mu$ we obtain after some algebra the effective action:

$$\tilde{S}_{eff} = \int d^4x \left[-\frac{m^2_{\tilde{A}}}{4} \tilde{F}^E_{\mu\nu} \frac{1}{\partial^2 + m^2_{\tilde{A}}} \tilde{F}^{\mu\nu}_E - \frac{1}{2} j_\mu \frac{1}{\partial^2 + m^2_{\tilde{A}}} j^\mu \right]. \quad (10)$$

The first term in (10) is responsible for the charge confinement: it spontaneously breaks the electric brane symmetry such that the electric Dirac string $\tilde{F}^E_{\mu\nu}$ acquires energy becoming physical and constitutes now the electric flux tube connecting two charges of opposite sign immersed in the DSC. The flux tube has a thickness equal to the penetration depth of the electric field in the DSC: $\lambda = \frac{1}{m_{\tilde{A}}} = \sqrt{\epsilon_c}$. The shape of the Dirac string is no longer irrelevant: the stable configuration that minimizes the energy is that of a straight tube (minimal space). Substituting in the first term of (10) such a solution for the string term, $\tilde{F}^E_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}\frac{1}{n\cdot\partial}(n^\alpha j^\beta - n^\beta j^\alpha)$, where $n^\mu := (0, \vec{R} := \vec{R}_1 - \vec{R}_2)$ is a straight line connecting $+e$ in $\vec{R}_1$ and $-e$ in $\vec{R}_2$, and taking the static limit we obtain a linear confining potential between the electric charges [9]. We also note that eliminating the magnetic condensate (i.e., taking the limit $m_{\tilde{A}} \to 0$) we recover the diluted phase with no confinement: the interaction between the electric currents in (10) becomes of the long-range (Coulomb) type and the confining term goes to zero (in terms of the flux tube we see that it acquires an infinite thickness such that the electric field is no longer confined and occupies the whole space).

In summary, the supplementing of the dual action with a kinetic term for the magnetic Dirac branes which respects the local symmetries of the system, the subsequent use of the GPI (A.6) and the consideration of the limit where the Poisson-dual current $\tilde{\theta}^V_\mu$ goes to zero gives us a proper condition for the complete condensation of monopoles, leading to confinement, as viewed from the dual picture.

4. Julia–Toulouse approach

Now we want to analyze the monopole condensation within the direct picture, where the defects couple non-minimally with the gauge field A_μ.

Using the Dirac quantization condition we can rewrite (1) as:

$$S = \int d^4x \left[-\frac{1}{4} F^{obs\,2}_{\mu\nu} - \frac{1}{4} F^{\mu\nu}_{obs} \epsilon_{\mu\nu\alpha\beta} \tilde{F}^{\alpha\beta}_E \right]. \quad (11)$$

Julia and Toulouse made the crucial observation that if the monopoles completely condense we have a complete proliferation of the magnetic strings associated to them, hence the field A_μ can not be defined anywhere in the space. This implies that $F^{obs}_{\mu\nu}$ can no longer be written in terms of A_μ. The JTA consists in the rank-jump *ansatz* of taking the object $F^{obs}_{\mu\nu}$ as being the fundamental field describing the condensed phase. Hence $F^{obs}_{\mu\nu}$ acquires a new meaning and becomes the field describing the magnetic condensate. Defining $F^{obs}_{\mu\nu} := -m_\Lambda \Lambda_{\mu\nu}$ and supplementing (11) with a kinetic term of the form $\frac{1}{12}(\partial_\mu \Lambda_{\alpha\beta} + \partial_\beta \Lambda_{\mu\alpha} + \partial_\alpha \Lambda_{\beta\mu})^2$ for the new field $\Lambda_{\mu\nu}$, we obtain as the effective action for the condensed phase, in the direct picture, the massive Kalb–Ramond action:

$$S_{K-R} = \int d^4x \left[-\frac{1}{2}(\partial_\mu \tilde{\Lambda}^{\mu\nu})^2 + \frac{m^2_\Lambda}{4} \tilde{\Lambda}^2_{\mu\nu} + \frac{m_\Lambda}{2} \tilde{\Lambda}^{\mu\nu} \tilde{F}^E_{\mu\nu} \right], \quad (12)$$

where $\tilde{\Lambda}^{\mu\nu} := \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}\Lambda_{\alpha\beta}$.

Notice that in implementing the JTA the fundamental field of the theory experiences a rank-jump through the phase transition: we started with a 1-form in the normal phase and finished with a 2-form in the completely condensed phase. The rank-jump is a general feature of the JTA since in implementing this prescription we always use the *ansatz* of reinterpretating the kinetic term with non-minimal coupling for the field describing the diluted phase as being a mass term for the new field describing the condensate formed in the phase where the defects proliferate until occupy the whole space.

Let us now apply the duality transformation in (9). For this we introduce an auxiliary field $f_{\mu\nu}$ such that the master action reads:

$$S_{Master} := \int d^4x \left[-\frac{1}{2} f_{\mu\nu} (\tilde{F}^{\mu\nu} - \tilde{F}^{\mu\nu}_E) + \frac{1}{4} f^2_{\mu\nu} + \frac{m^2_{\tilde{A}}}{2g^2} (\partial_\mu \tilde{\theta} - \tilde{\theta}^V_\mu - g\tilde{A}_\mu)^2 \right]. \quad (13)$$

Extremizing (13) with respect to $f_{\mu\nu}$ we get $f_{\mu\nu} = \tilde{F}^{obs}_{\mu\nu}$ and substituting this result back in the master action we recover (9). On the other hand, extremizing (13) with respect to $\tilde{A}_\mu$ we obtain:

$$\tilde{A}^\nu = -\frac{1}{m^2_{\tilde{A}}} \partial_\mu f^{\mu\nu} + \frac{1}{g}(\partial^\nu \tilde{\theta} - \tilde{\theta}^\nu_V). \quad (14)$$

Substituting (14) in (13), it follows that:

$$S_{Master} = \int d^4x \left[-\frac{1}{2m^2_{\tilde{A}}} (\partial_\mu f^{\mu\nu})^2 + \frac{1}{4} f^2_{\mu\nu} + \frac{1}{2} f_{\mu\nu} \tilde{F}^{\mu\nu}_E + \frac{1}{g} \partial_\mu \tilde{\theta}^V_\nu f^{\mu\nu} \right], \quad (15)$$

where we integrated by parts and considered the antisymmetry of $f^{\mu\nu}$ in order to use $\partial_\mu \partial_\nu f^{\mu\nu} = 0$.

Defining now $f_{\mu\nu} := m_{\tilde{A}} \tilde{\Lambda}_{\mu\nu}$ and making the identification $m_{\tilde{A}} \equiv m_\Lambda$, we get as the dual action to (9) the massive Kalb–Ramond action in the presence of vortices, a generalization of the result obtained by Quevedo and Trugenberger in [7]:

$$S^V_{KR} = \int d^4x \left[-\frac{1}{2}(\partial_\mu \tilde{\Lambda}^{\mu\nu})^2 + \frac{m^2_\Lambda}{4} \tilde{\Lambda}^2_{\mu\nu} + \frac{m_\Lambda}{2} \tilde{\Lambda}_{\mu\nu} \tilde{F}^{\mu\nu}_E + \frac{m_\Lambda}{2g} (\partial_\mu \tilde{\theta}^V_\nu - \partial_\nu \tilde{\theta}^V_\mu) \tilde{\Lambda}^{\mu\nu} \right]. \quad (16)$$

More precisely, this extension consists in the construction of an action for the case with an incomplete condensate that is however already described by a rank-jumped tensor. If we now take the limit $\tilde{\theta}^V_\mu \to 0$ in (16) we recover exactly the massive Kalb–Ramond action (12) obtained in [7] through the application of the JTA to (1). That establishes the duality between the JTA and the ALBA in the limit where the Poisson-dual current goes to zero, which physically corresponds to the limit of complete condensation of the defects. However, (16) with $\tilde{\theta}^V_\mu \neq 0$ displays a new and important result, which is a consequence of this formalism, showing that the rank-jump which is the signature of the JTA also occurs in the partial condensation process with the presence of vortex-like defects.

5. Conclusion

We established the equivalence through duality of two different approaches developed to handle defects, represented by magnetic monopoles in the example worked here, in the physically interesting context where the defects dominate the dynamics of the system. It was clearly shown that the two approaches are complementary, being different descriptions of the same phenomenon in the limit where the Poisson-dual current vanishes which characterizes the complete condensation of the defects. Indeed, within the formalism here called as ALBA the transition becomes smoother since the Poisson-dual current $\tilde{\theta}^V_\mu$ appears as a parameter that controls the proliferation of the magnetic defects. On the other hand, within the formalism referred to as JTA the phase transition is signalized by a rank-jump of the tensor field and seems to be a discontinuous phenomenon. However, the duality JTA–ALBA brings a new possibility.

It is important to say that this dual equivalence was possible due to a suitable interpretation of the generalization of the Poisson identity developed here. We clearly showed that this identity is an essential tool to use in the context of defects condensation: the proliferation of the branes in one of the sides of the identity is accompanied by the dilution of the branes of complementary dimension in the other side of the identity. Due to this observation we were able to identify the signature of the complete condensation of defects in the dual picture (ALBA) with the vanishing of the Poisson-dual current. As the main result, we showed that in this specific limit, when the Poisson-dual current is set to zero, the ALBA and the JTA are two dual equivalent prescriptions for describing condensation of defects.

As the final remark we point out the fact that when we consider nonzero configurations of the Poisson-dual current $\tilde{\theta}^V_\mu$ we allow the description of an intermediary region interpolating between the diluted and the completely condensed phases. As discussed, this corresponds to the presence of vortex-like defects in the condensate. It is possible to see that this new phase with the presence of vortices ($\tilde{\theta}^V_\mu \neq 0$), just like in the extreme case where the complete monopole condensation sets in, is also described within the direct picture by a rank-jumped action. The JTA as originally described by Quevedo and Trugenberger, therefore, will describe the physically interesting extreme case where all defects are condensed.

Acknowledgements

We thank Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq), Coordenação de Aperfeiçoamento de Pessoal de Nível Superior (CAPES) and Fundação de Amparo à Pesquisa do Estado do Rio de Janeiro (FAPERJ) for financial support.

Appendix A. Generalized Poisson's Identity (GPI)

In this appendix we generalize the reasoning used in [6] in order to account for an ensemble of p-branes in arbitrary space–time dimensions.

Let us consider a d-dimensional hypercubical lattice with spacing a. Attribute to each site $x = (x_1, x_2, \dots, x_d)$, $x_1, x_2, \dots, x_d \in a\mathbb{Z}$ of the lattice a configuration

$$\vartheta^V_i(x) := 2\pi \frac{n_i(x)}{a^p}, \tag{A.1}$$

where $p \leqslant d$, $p, d \in \mathbb{N}$ and i is a set of $k \leqslant d$, $k \in \mathbb{N}$ indices each one of them running from 1 to d and $n_i(x) \in \mathbb{Z}$.

The Poisson's summation formula is given by

$$\sum_{n\in\mathbb{Z}} e^{2\pi inf} = \sum_{m\in\mathbb{Z}} \delta(f - m), \tag{A.2}$$

where f is a integrable function.

Using (A.2) for each pair (x, i) it follows that

$$\sum_{\{n_i(x)\in\mathbb{Z}\}} \exp\left[2\pi i \sum_x a^d \frac{n_i(x)}{a^p} f_i(x)\right] = \sum_{\{m_i(x)\in\mathbb{Z}\}} \prod_{(x,i)} \delta\left(f_i(x) - \frac{m_i(x)}{a^{d-p}}\right), \tag{A.3}$$

where we have used the fact that the exponential argument must be nondimensional, hence $a^{d-p+[f]} \equiv a^0 = 1 \Rightarrow [f] = a^{p-d}$.

The continuum limit corresponds to make the number N of lattice sites go to infinity while keeping the lattice hypervolume V_d fixed which gives the condition $a \to 0$. In this limit we formally define the Poisson-dual current by

$$\theta^V_i\big(x; \xi^p\big) := \lim_{\substack{a\to 0\\ N\to\infty\\ V_d\,\text{cte}}} \vartheta^V_i(x) = 2\pi \lim_{\substack{a\to 0\\ N\to\infty\\ V_d\,\text{cte}}} \frac{n_i(x)}{a^p}. \tag{A.4}$$

The object $\theta^V_i(x; \xi^p)$ has dimension a^{-p} and is singular over a region ξ^p of dimension p on the lattice where $\{n_i(x \in \xi^p) :\neq 0\}$. In the rest of the lattice, where $\{n_i(x \notin \xi^p) := 0\}$, we have from (A.1) that $\vartheta^V_i(x) = 0$ such that $\theta^V_i(x; \xi^p)$ vanishes outside the region ξ^p. Thus we identify the object $\theta^V_i(x; \xi^p)$ with a delta configuration that localizes the p-brane ξ^p:

$$\theta^V_i\big(x; \xi^p\big) = 2\pi \delta_i\big(x; \xi^p\big). \tag{A.5}$$

Hence in the continuum limit the identity (A.3) is given by

$$\sum_{\{\xi^p\}} e^{2\pi i \int d^d x\, \delta_i(x;\xi^p) f_i(x)} = \sum_{\{\chi^{d-p}\}} \delta\big[f_i(x) - \delta_i\big(x; \chi^{d-p}\big)\big], \tag{A.6}$$

which is the GPI.

The brane proliferation–dilution interpretation of the GPI (A.6) follows from the following reasoning: if $\{\chi^{d-p}\} \to \emptyset$ then $\delta_i(x; \chi^{d-p}) \to 0$ (there are no $\{\chi^{d-p}\}$ branes in the space to be localized) and

$$\sum_{\{\chi^{d-p}\}} \delta\big[f_i(x) - \delta_i\big(x; \chi^{d-p}\big)\big] \to \delta[f_i] \equiv \int \mathcal{D}\tau_i e^{i\int d^d x\, \tau_i f_i}. \tag{A.7}$$

Comparing (A.6) and (A.7) we see that in the limit of dilution of the $\{\chi^{d-p}\}$ branes we have $\theta^V_i(x; \xi^p) = 2\pi\delta_i(x; \xi^p) \to \tau_i$ and $\sum_{\{\xi^p\}} \to \int \mathcal{D}\tau_i$ which corresponds to the proliferation of the $\{\xi^p\}$ branes.

Conversely, in the limit of proliferation of the $\{\chi^{d-p}\}$ branes, $\theta^V_i(x; \chi^{d-p}) = 2\pi\delta_i(x; \chi^{d-p}) \to \gamma_i$ and $\sum_{\{\chi^{d-p}\}} \to \int \mathcal{D}\gamma_i$ we have

$$\sum_{\{\chi^{d-p}\}} \delta\big[f_i(x) - \delta_i\big(x; \chi^{d-p}\big)\big] \to \int \mathcal{D}\gamma_i \delta[f_i - \gamma_i] = \mathbb{1}. \tag{A.8}$$

Comparing (A.6) and (A.8) we see that in the limit of proliferation of the $\{\chi^{d-p}\}$ branes we have $\theta^V_i(x; \xi^p) = 2\pi\delta_i(x; \xi^p) \to 0$ which corresponds to the dilution of the $\{\xi^p\}$ branes.

It is important to notice that the information about which brane configurations are accessible by the system in the brane sums in the GPI (A.6) is not present in this formulation, being an external input controlled by hands as when we considered previously, for example, the extreme cases of prolific or diluted accessible brane configurations.

References

[1] H.B. Nielsen, P. Olesen, Nucl. Phys. B 61 (1973) 45;
Y. Nambu, Phys. Rev. D 10 (1974) 4262;
M. Creutz, Phys. Rev. D 10 (1974) 2696;
G. T'Hooft, High Energy Physics, Editorice Compositori, Bologna, 1975;
G. Parisi, Phys. Rev. D 11 (1975) 970;
A. Jevicki, P. Senjanovic, Phys. Rev. D 11 (1975) 860;
S. Mandelstam, Phys. Rep. C 23 (1976) 245.
[2] C. Montonen, D.I. Olive, Phys. Lett. B 72 (1977) 117.
[3] T. Banks, R. Myerson, J.B. Kogut, Nucl. Phys. B 129 (1977) 493.
[4] B. Julia, G. Toulouse, J. Phys. Lett. 40 (1979) 395.
[5] H. Kleinert, Lett. Nuovo Cimento 35 (1982) 405.
[6] H. Kleinert, Multivalued Fields in Condensed Matter, Electromagnetism and Gravitation, World Scientific Publishing, 2007;
H. Kleinert, Gauge Fields in Condensed Matter, World Scientific Publishing, 1989.

[7] F. Quevedo, C.A. Trugenberger, Nucl. Phys. B 501 (1997) 143, arXiv:hep-th/9604196.
[8] P.A.M. Dirac, Proc. Roy. Soc. A 133 (1931) 60;
P.A.M. Dirac, Phys. Rev. 74 (1948) 817.
[9] A good review about these issues is given in G. Ripka, Dual Superconductor Models of Color Confinement, Springer-Verlag, 2005, arXiv:hep-ph/0310102.
[10] J. Gamboa, L.S. Grigorio, M.S. Guimaraes, F. Mendez, C. Wotzasek, Phys. Lett. B 668 (2008) 447, arXiv:0805.0626 [hep-th].
[11] L.S. Grigorio, M.S. Guimaraes, C. Wotzasek, Phys. Lett. B 674 (2009) 213, arXiv:0808.3698 [hep-th].
[12] M.S. Guimaraes, Confinement, duality and violation of Lorentz symmetry, PhD thesis, 2009 (in Portuguese).
[13] B. Felsager, Geometry, Particles and Fields, Springer-Verlag, 1998.