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General displacement arch-cantilever element method for stress analysis of arch dam

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Abstract: Based on the general displacement method and the basic hypothesis of the trial load method, a new advanced trial load method, the general displacement arch-cantilever element method, was proposed to derive the transformation relation of displacements and loads between the surface nodes and middle plane nodes. This method considers the nodes on upstream and downstream surfaces of the arch dam to be exit nodes (master nodes), and the middle plane nodes to be slave nodes. According to the derived displacement and load transformation matrices, the equilibrium equation treating the displacement of middle plane nodes as a basic unknown variable is transformed into one that treats the displacement of upstream and downstream nodes as a basic unknown variable. Because the surface nodes have only three degrees of freedom (DOF), this method can be directly coupled with the finite element method (FEM), which is used for foundation simulation to analyze the stress of the arch dam with consideration of dam-foundation interaction. Moreover, using the FEM, the nodal load of the arch dam can be easily obtained. Case studies of a typical cylindrical arch dam and the Wudongde arch dam demonstrate the robustness and feasibility of the proposed method.

Key words: *arch dam; arch-cantilever element method; general displacement method; finite element method* (*FEM*); *coupled analysis; stress analysis; Wudongde arch dam*

1 Introduction

The trial load and finite element methods are the main existing stress analysis methods for arch dams. The assigned load is an unknown variable in the traditional trial load method. The paucity of zero-elements in the coefficient matrix of equations necessitates a high amount of computation. In addition, a homogenous, half-infinite, elastic foundation is always assumed, whereas many arch dams are built on complex geology. For these reasons, the traditional trial load method is not adequate for dynamic analysis of arch dams. Therefore, many scholars have improved the traditional trial load method and developed new methods, such as the internal force equilibrium arch-cantilever method (Chen 1988), the strip mode synthesis method of arch dam analysis (Lin et al. 1985), the load distribution displacement method (Lin and Yang 1987), the improved load distribution displacement method (Zhu et al. 1988, 1991), and the arch-cantilever element method (Chen et al. 2003).

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In this paper, the basic hypothesis of the trial load method is used along with the general displacement method (Zhong 1981; Zhong and Li 1981; Zhong and Zhu 1985) to develop a new general displacement arch-cantilever element method, and further to derive the transformation relation of displacements and loads between the surface nodes and middle plane nodes. Due to the fact that there are only three DOF of surface nodes, this method can be directly coupled with the FEM and facilitate the stress analysis of the arch dam with consideration of dam-foundation interaction.

2 Formulation of improved general displacement arch-cantilever element method

2.1 Fundamental formulation of arch-cantilever element method

Based on the trial load method, some researchers (Chen et al. 2003; Xu et al. 2005) have developed an arch-cantilever element method. The arch dam is divided into arch-cantilever elements. Fig. 1 shows an arch-cantilever element. Nodes 1 through 8 are the surface nodes. Nodes *i*, *j*, *m* and *n* are the middle plane nodes. In this system, the arch direction is expressed as v_a, the cantilever direction is expressed as v_b, and the normal direction of the middle plane of the element is defined as v_r.

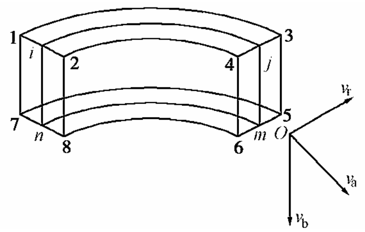


Fig. 1 Arch-cantilever element

The equilibrium equation of an arch-cantilever element is

$$\boldsymbol{F} = \boldsymbol{F}_a + \boldsymbol{F}_b = \boldsymbol{K}\boldsymbol{\delta} \tag{1}$$

where $\boldsymbol{\delta}$ and $\boldsymbol{F}$ are the displacement and load of the middle plane nodes, respectively; $\boldsymbol{F}_a$ and $\boldsymbol{F}_b$ are the nodal loads of the arch and cantilever, respectively; and $\boldsymbol{K}$ is the element stiffness matrix, which is as follows:

$$\boldsymbol{K} = \int \boldsymbol{T}_a^{\mathrm{T}} \boldsymbol{H}_a^{\mathrm{T}} \boldsymbol{K}_a(\xi) \boldsymbol{H}_a \boldsymbol{T}_a \mathrm{d}\xi + \int \boldsymbol{T}_b^{\mathrm{T}} \boldsymbol{H}_b^{\mathrm{T}} \boldsymbol{K}_b(\eta) \boldsymbol{H}_b \boldsymbol{T}_b \mathrm{d}\eta \tag{2}$$

where $\boldsymbol{T}_a$ and $\boldsymbol{T}_b$ are the transformation matrices of arch and cantilever coordinate systems, respectively; $\boldsymbol{H}_a$ and $\boldsymbol{H}_b$ are the interpolation functions along the arch and cantilever directions, respectively; $\boldsymbol{K}_a$ and $\boldsymbol{K}_b$ are the stiffness matrices of arch and cantilever slices, respectively; and $\mathrm{d}\xi$ and $\mathrm{d}\eta$ are the micro-sections in arch and cantilever planes, respectively.

An equation relating displacements and loads at each node in the arch-cantilever system can be obtained by assemblage of the around nodes. In this way, the method resembles the finite element method. It determines the displacement by transforming loads acting on the dam (e.g., water pressure and temperature) into nodal forces. The elastic beam and arch theories (Panda and Natarajan 1981) are used to calculate stresses.

2.2 Displacement transformation equation between surface nodes and middle plane nodes

We assume that the displacement vector of the middle plane node i is $\boldsymbol{\delta}_i = \left(u_i, v_i, w_i, \theta_{xi}, \theta_{yi}\right)^{\mathrm{T}}$, and the displacement vectors of upstream surface node 1 and downstream surface node 2 are, respectively, $\boldsymbol{\delta}_1 = \left(u_1, v_1, w_1\right)^{\mathrm{T}}$ and $\boldsymbol{\delta}_2 = \left(u_2, v_2, w_2\right)^{\mathrm{T}}$, in the local coordinate system $x'y'z'$ (Fig. 2). x' is the direction from node i to node n, y' is the outer normal direction of the cross section of the arch-cantilever element, and z' is the outer normal direction of the middle plane at node i (Fig. 3).

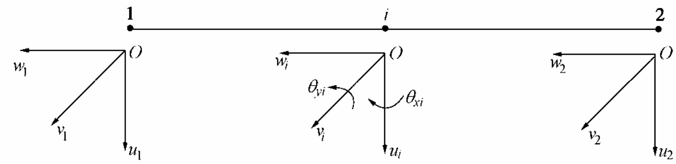


Fig. 2 Displacements of surface nodes and middle plane node

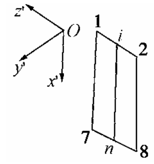


Fig. 3 Local coordinate system

Using the general displacement method, the kinematics formulas are obtained:

$$u_1 = u_i + z'_1\,\theta_{yi}, \quad v_1 = v_i - z'_1\,\theta_{xi}, \quad w_1 = w_i \tag{3}$$

$$u_2 = u_i + z'_2\,\theta_{yi}, \quad v_2 = v_i - z'_2\,\theta_{xi}, \quad w_2 = w_i \tag{4}$$

where z'_1 and z'_2 are the z' coordinates of node 1 and node 2. From Eqs. (3) and (4), it can be shown that

$$\begin{cases} u_i = \dfrac{z'_2 u_1 - z'_1 u_2}{z'_2 - z'_1}, \quad v_i = \dfrac{z'_2 v_1 - z'_1 v_2}{z'_2 - z'_1} \\ w_i = \dfrac{w_1 + w_2}{2}, \ w_1 = w_2 \\ \theta_{xi} = -\dfrac{v_2 - v_1}{z'_2 - z'_1}, \ \theta_{yi} = \dfrac{u_2 - u_1}{z'_2 - z'_1} \end{cases} \tag{5}$$

Based on Eqs. (3) through (5), the displacement transformation matrix can be expressed as follows:

$$\boldsymbol{T}_{\mathrm{d}i} = \begin{bmatrix} \dfrac{z'_2}{z'_2 - z'_1} & 0 & 0 & \dfrac{-z'_1}{z'_2 - z'_1} & 0 & 0 \\ 0 & \dfrac{z'_2}{z'_2 - z'_1} & 0 & 0 & \dfrac{-z'_1}{z'_2 - z'_1} & 0 \\ 0 & 0 & \dfrac{1}{2} & 0 & 0 & \dfrac{1}{2} \\ 0 & \dfrac{1}{z'_2 - z'_1} & 0 & 0 & -\dfrac{1}{z'_2 - z'_1} & 0 \\ -\dfrac{1}{z'_2 - z'_1} & 0 & 0 & \dfrac{1}{z'_2 - z'_1} & 0 & 0 \end{bmatrix} \tag{6}$$

The displacement transformation relation between the middle plane node i and the surface nodes 1 and 2 can be expressed as

$$\boldsymbol{\delta}_i = \boldsymbol{T}_{\mathrm{d}i}\begin{bmatrix}\boldsymbol{\delta}_1\\ \boldsymbol{\delta}_2\end{bmatrix}, \text{ and } \ w_1 = w_2 \tag{7}$$

For the upstream surface node and downstream surface node, there are two other conditions: $z'_1 = \frac{t_i}{2}$ and $z'_2 = -\frac{t_i}{2}$, in which t_i is the dam thickness in the normal direction of the middle plane. Therefore, $\boldsymbol{T}_{\mathrm{d}i}$ can be simplified as

$$\boldsymbol{T}_{\mathrm{d}i} = \begin{bmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & -\frac{1}{t_i} & 0 & 0 & \frac{1}{t_i} & 0 \\ \frac{1}{t_i} & 0 & 0 & -\frac{1}{t_i} & 0 & 0 \end{bmatrix} \tag{8}$$

The relationships and conditions described above are applied to the other nodes as well, so the displacement transformation matrix of an arch-cantilever element is

$$\boldsymbol{T}_{\mathrm{d}} = \begin{bmatrix} \boldsymbol{T}_{\mathrm{d}i} & 0 & 0 & 0 \\ 0 & \boldsymbol{T}_{\mathrm{d}j} & 0 & 0 \\ 0 & 0 & \boldsymbol{T}_{\mathrm{d}m} & 0 \\ 0 & 0 & 0 & \boldsymbol{T}_{\mathrm{d}n} \end{bmatrix} \tag{9}$$

and the size of the matrix $\boldsymbol{T}_{\mathrm{d}}$ is 20×24. The displacement transformation relation between master nodes and slave nodes of an arch-cantilever element is expressed as

$$\boldsymbol{\delta}^{\mathrm{e}} = \boldsymbol{T}_{\mathrm{d}}\widetilde{\boldsymbol{\delta}^{\mathrm{e}}} \tag{10}$$

where $\boldsymbol{\delta}^{\mathrm{e}}$ is the displacement column matrix of middle plane nodes, and $\widetilde{\boldsymbol{\delta}^{\mathrm{e}}}$ is the displacement column matrix of upstream and downstream surface nodes. There are additional constraint conditions: $w_1 = w_2$, $w_3 = w_4$, $w_5 = w_6$, and $w_7 = w_8$. These conditions are applied based on the fiber hypothesis of arch-cantilever deformation.

2.3 Load transformation equation between surface nodes and middle plane nodes

It is assumed that the concentrated force at node *i* is $\boldsymbol{F}_i = \left(N_i, Q_{yi}, Q_{zi}, M_{xi}, M_{yi}\right)$, and the equivalent loads at upstream and downstream surface exit nodes are, respectively, $\boldsymbol{F}_1 = \left(f_{x1}, f_{y1}, f_{z1}\right)^{\mathrm{T}}$ and $\boldsymbol{F}_2 = \left(f_{x2}, f_{y2}, f_{z2}\right)^{\mathrm{T}}$. The load transformation equation can be derived with the static equivalent principle and rigid equilibrium equations as follows:

$$\begin{bmatrix}\boldsymbol{F}_1\\ \boldsymbol{F}_2\end{bmatrix} = \boldsymbol{T}_{\mathrm{f}i}\boldsymbol{F}_i^{\mathrm{T}} \tag{11}$$

$$\boldsymbol{T}_{\mathrm{f}i}=\begin{bmatrix} \frac{z'_2}{z'_2-z'_1} & 0 & 0 & 0 & -\frac{1}{z'_2-z'_1} \\ 0 & \frac{z'_2}{z'_2-z'_1} & 0 & \frac{1}{z'_2-z'_1} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 \\ \frac{-z'_1}{z'_2-z'_1} & 0 & 0 & 0 & \frac{1}{z'_2-z'_1} \\ 0 & \frac{-z'_1}{z'_2-z'_1} & 0 & -\frac{1}{z'_2-z'_1} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 \end{bmatrix} \tag{12}$$

Because of the conditions $z'_1=\frac{t_i}{2}$ and $z'_2=-\frac{t_i}{2}$, $\boldsymbol{T}_{\mathrm{f}i}$ can be simplified as

$$\boldsymbol{T}_{\mathrm{f}i}=\begin{bmatrix} \frac{1}{2} & 0 & 0 & 0 & \frac{1}{t_i} \\ 0 & \frac{1}{2} & 0 & -\frac{1}{t_i} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & -\frac{1}{t_i} \\ 0 & \frac{1}{2} & 0 & \frac{1}{t_i} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 \end{bmatrix} \tag{13}$$

Other nodes can be transformed the same way. Then, the load transformation matrix of an arch-cantilever element with a size of 24×20 is obtained:

$$\boldsymbol{T}_{\mathrm{f}}=\begin{bmatrix} \boldsymbol{T}_{\mathrm{f}i} & 0 & 0 & 0 \\ 0 & \boldsymbol{T}_{\mathrm{f}j} & 0 & 0 \\ 0 & 0 & \boldsymbol{T}_{\mathrm{f}m} & 0 \\ 0 & 0 & 0 & \boldsymbol{T}_{\mathrm{f}n} \end{bmatrix} \tag{14}$$

The load transformation relation between master nodes and slave nodes of an arch-cantilever element is expressed as

$$\widetilde{\boldsymbol{F}^{\mathrm{e}}}=\boldsymbol{T}_{\mathrm{f}}\boldsymbol{F}^{\mathrm{e}} \tag{15}$$

where $\boldsymbol{F}^{\mathrm{e}}$ is the load column matrix of middle plane nodes, and $\widetilde{\boldsymbol{F}^{\mathrm{e}}}$ is the load column matrix of surface nodes.

Obviously, $\boldsymbol{T}_{\mathrm{d}}$ and $\boldsymbol{T}_{\mathrm{f}}$ are a pair of transposed matrices. They guarantee the symmetry and efficiency of the augmented stiffness matrix of the arch-cantilever element.

2.4 Stiffness matrix and balance equation of improved arch-cantilever element method

Inserting Eqs. (10) and (15) into Eq. (1), the following equation is obtained:

$$\widetilde{\boldsymbol{K}}^{\mathrm{e}}\widetilde{\boldsymbol{\delta}}^{\mathrm{e}}=\widetilde{\boldsymbol{F}}^{\mathrm{e}} \tag{16}$$

Therefore, the stiffness matrix of surface nodes is

$$\widetilde{\boldsymbol{K}}^{\mathrm{e}}=\boldsymbol{T}_{\mathrm{f}}\boldsymbol{K}^{\mathrm{e}}\boldsymbol{T}_{\mathrm{d}} \tag{17}$$

Eq. (16) is the balance equation of an arch-cantilever element that treats the displacement of upstream and downstream nodes as a basic unknown variable. Here the element stiffness matrix $\widetilde{\boldsymbol{K}}^{\mathrm{e}}$ is not full-rank; therefore, the additional constraint conditions of $w_1=w_2$, $w_3=w_4$, $w_5=w_6$, and $w_7=w_8$ can eliminate the ill-conditioned problem of this matrix.

2.5 Coupled analysis

In the conventional analysis, the nodes in the arch-cantilever system and the foundation simulated with the FEM have different DOF. Generally, the node in the arch-cantilever system has five DOF, but the node in the foundation has only three. The different DOF makes the coupling of the trial load method with the FEM relatively complex and difficult to realize (Zhao and Lin 1990; Lin and Su 2002). In the improved method, both the surface node and the node of the foundation have three DOF, so they can be coupled directly. Moreover, at the interface between the dam and the foundation, the locally incompatible mesh is permitted, and the displacement coordination method can be used to deal with the locally incompatible meshes to coordinate the displacement field and stress field (Li et al. 2003).

2.6 Calculation method of nodal load

In the traditional trial load method, the formulas for calculating the nodal loads of the dam middle plane are complex and the calculation accuracy is low. Using Eq. (15), the calculation of the nodal loads of the dam middle plane can be converted to the calculation of those of upstream and downstream surfaces. Therefore, in the actual calculation, it is unnecessary to calculate the nodal loads of the dam middle plane, and the arch-cantilever element can be considered an eight-node isoparametric element. The finite element method is used to obtain the surface nodal loads of the arch-cantilever element, in which the water load is a surface force, the dead weight of the dam is a body force, and the temperature load can be calculated with corresponding methods. In fact, the water load acts on the dam surface, so the numerical results are relatively more accurate.

2.7 Calculation program

In this study, the calculation program FEMHmadrid, developed by Pastor et al. (1997), was used for 3D finite element simulation analysis of the arch dam. The program of the general displacement arch-cantilever element method, presented in this paper, was compiled by Fortran 95 and added to the program FEMHmadrid as a module.

3 Case study

3.1 Example 1

As a test of this method, a typical simple cylindrical arch dam of consistent thickness, one that has been studied by many researchers (Zinkiewicz and Cheung 1964), was investigated. The shape is described in Fig. 4. The elastic modulus and Poisson's ratio of the dam material are 20 GPa and 0.15, respectively. The density of the cylindrical arch dam is 2 400 kg/m^3. The elastic modulus of the dam foundation is 20 GPa. For the convenience of comparing the calculated result with others, only the water-pressure and dead weight of the dam and foundation were considered.

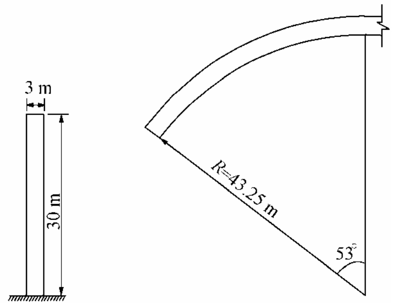


Fig. 4 Arch and cantilever scheme

The dam was discretized into 120 arch-cantilever elements (10 arch layers and 19 cantilever columns), while the foundation of the dam was discretized into 1 848 isoparametric elements. Then, these two types of elements were combined with the software GID. The configuration of the calculation model is shown in Fig. 5. The computation results are given in Figs. 6 through 8. In Figs. 7 and 8, the tensile stress is positive and the compressive stress is negative.

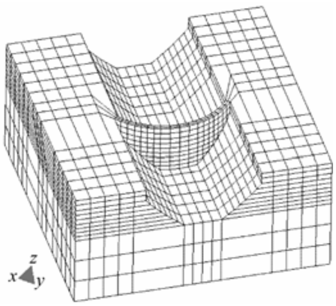


Fig. 5 Calculation model

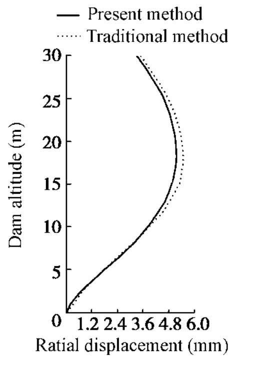


Fig. 6 Radial displacement of central cantilever

As shown in Figs. 6 through 8, the radial displacement of the central cantilever and hoop and vertical equivalent stresses of the central cantilever are compared with the results obtained by the U.S. Bureau of Reclamation with the traditional trial load method (Zinkiewicz and Cheung 1964).

The radial displacement of the central cantilever calculated with the present method is lower than that calculated with the traditional trial load. The reason is that the stiffness of the foundation simulated by the FEM is greater than that of the tradition method, and the stiffness of the dam body is increased by fully considering the dimensional effect of the dam. The stresses calculated in the paper agree well with those of the traditional method. Overall, the variation patterns of the displacement and stresses calculated with the two methods are basically identical.

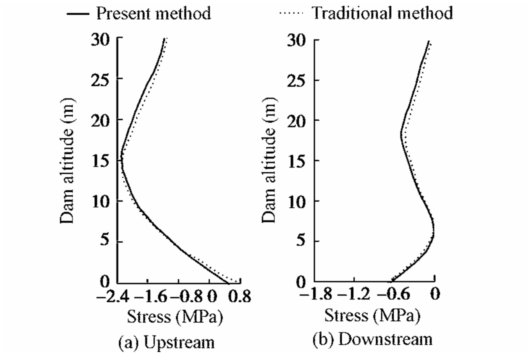


Fig. 7 Hoop equivalent stress of central cantilever

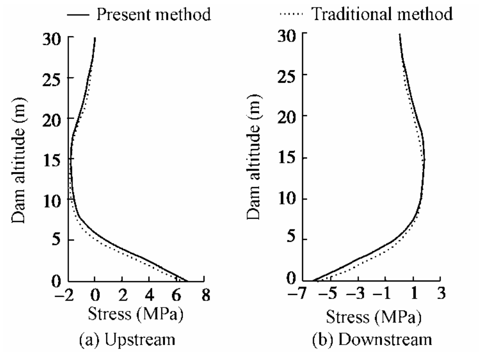


Fig. 8 Vertical equivalent stress of central cantilever

3.2 Example 2

The Wudongde arch dam is a 265-m-high double-curvature arch dam. The crest elevation is 988 m. It has a crest length of 304.8 m, and the thickness of the center cantilever varies from 48 m at the base to 10 m at the crest. The main shape parameters and schematic diagrams are shown in Table 1 and Figs. 9 and 10.

Table 1 Shape parameters of Wudongde arch dam

Elevation (m)	Z_0 (m)	T_c (m)	T_l (m)	T_r (m)	R_{cl} (m)	R_{cr} (m)	φ_l (°)	φ_r (°)
958.00	30.036	10.000	11.420	10.554	170.749	255.144	36.511	40.046
935.00	20.268	16.262	18.950	16.976	148.391	284.869	39.382	42.654
905.00	10.396	22.800	27.314	25.165	127.340	216.539	42.359	45.040
875.00	3.443	27.906	34.218	33.284	113.585	205.452	18.916	46.020
845.00	-0.953	32.027	39.869	40.719	105.081	99.495	44.264	45.370
815.00	-3.252	35.614	44.569	46.680	99.781	96.559	43.251	43.649
785.00	-3.516	36.114	48.404	50.203	95.640	94.531	40.200	40.605
755.00	-2.405	42.976	51.176	50.969	90.612	91.302	33.995	36.201
723.00	0.000	48.000	51.760	50.143	81.967	84.155	20.015	28.743

Note: R_{cl} and R_{cr} are radii of left and right arch abutments.

The normal upstream high water elevation of is 975 m and the corresponding downstream water elevation is 810 m. The sedimentation elevation in front of the dam is 863 m, the buoyant unit density is 750 kg/m^3, and the internal friction angle is 12.5°. The perennial mean air temperature is 20.9 ℃. The mean air temperature is 12.3 ℃ in January and 26.9 ℃ in July, respectively. The annual mean water temperature is 20.9 ℃ and the depth of the temperature-changing water layer is 80.0 m. The joint closure temperature for the arch dam ranges from 12 ℃ to 16 ℃ (Table 2). The material properties of the dam are shown in Table 3.

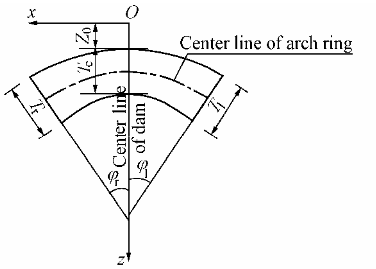


Fig. 9 Plane of arch ring

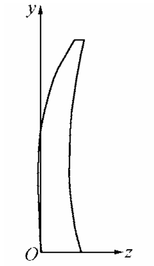


Fig. 10 Section of crown cantilever

Table 2 Joint closure temperature for arch dam

Elevation (m)	Temperature (℃)	Elevation (m)	Temperature (℃)
723.00	12.0	875.00	14.0
755.00	12.0	905.00	14.0
785.00	13.0	935.00	15.0
815.00	13.0	965.00	16.0
845.00	13.0	988.00	16.0

Table 3 Material properties of dam

Item	Modulus of elasticity (GPa)	Poisson's ratio	Density (kg/m^3)	Thermal diffusivity (m^2/month)	Thermal expansion coefficient (10^{-5}/℃)
Foundation	25	0.220	2 700	—	1.0
Dam concrete	20	0.167	2 400	3	1.0

The dam was discretized into 126 arch-cantilever elements (10 arch layers and 23 cantilever columns), while the foundation of the dam was discretized into 3 304 isoparametric elements. These two type elements were combined with the software GID. The calculation model of the Wudongde arch dam is shown in Fig. 11.

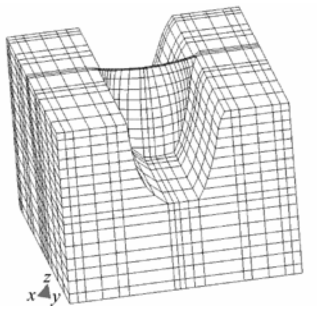
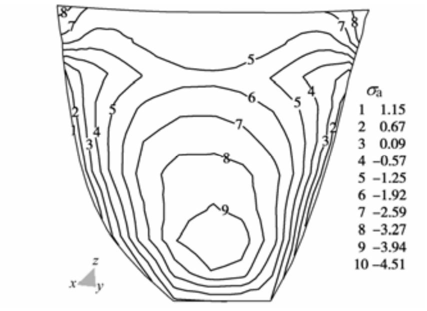


Fig. 11 FEM model of Wudongde arch dam

The computation results are given in Figs. 12 through 15. Comparison of the results calculated using the present method with those calculated using the FEM internal force method (Li et al. 2006) shows that the variation patterns within those results are basically identical, which proves the correctness and applicability of this method.



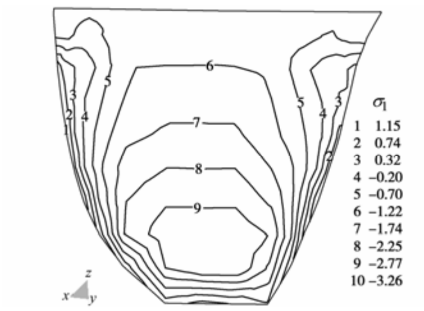
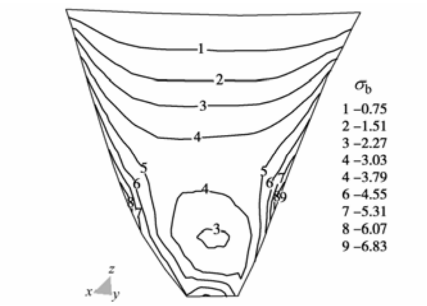


Fig. 12 Hoop stress upstream σ_a (MPa) **Fig. 13** First principal stress upstream σ_1 (MPa)



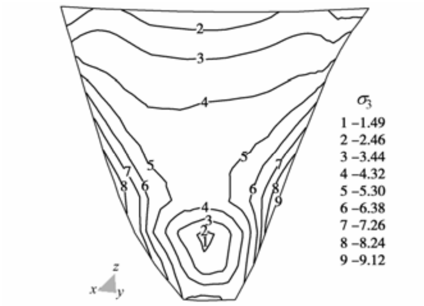


Fig. 14 Vertical stress downstream σ_b (MPa) **Fig. 15** Third principal stress downstream σ_3 (MPa)

4 Conclusions

(1) The general displacement proposed in this paper fully considers the dimensional effect of the dam, and it can be directly coupled with the FEM, which is used for foundation simulation. This coupling method takes into consideration the dam-foundation interaction and overcomes the drawbacks of the traditional method. Case studies in this paper demonstrate that this method is practical. In addition, because the stresses of the arch dam are calculated based on the internal forces which are assumed to be linearly distributed along the radial direction, the results correspond to current design specifications for arch dams.

(2) Through the integration of arch slices and cantilever slices, the equivalent arch-cantilever element is obtained. Using the nodal loads of the upstream and downstream surfaces to replace those of the middle plane and the FEM to obtain the surface nodal loads, the complex load condition can be easily considered.

(3) This method potentially provides a new way to conduct further studies on the seismic analysis of arch dams with consideration of dam-water-foundation interaction.

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