Alexandria Engineering Journal (2013) 52, 259–265

FACULTY OF ENGINEERING
ALEXANDRIA UNIVERSITY

Alexandria University

Alexandria Engineering Journal

www.elsevier.com/locate/aej
www.sciencedirect.com

ORIGINAL ARTICLE

MHD boundary layer flow and heat transfer over an exponentially stretching sheet embedded in a thermally stratified medium

Swati Mukhopadhyay *

Department of Mathematics, The University of Burdwan, Burdwan 713 104, WB, India

Received 13 March 2012; revised 2 February 2013; accepted 10 February 2013
Available online 13 March 2013

KEYWORDS

Boundary layer flow;
Exponentially stretching sheet;
MHD;
Suction;
Thermally stratified medium

Abstract MHD boundary layer flow and heat transfer towards an exponentially stretching sheet embedded in a thermally stratified medium subject to suction are presented in this analysis. Suitable transformations are used to convert the partial differential equations corresponding to the momentum and energy equations into highly nonlinear ordinary differential equations. Numerical solutions of these equations are obtained by shooting method. It is found that the heat transfer rate at the surface increases in presence of thermal stratification. Fluid velocity decreases with increasing magnetic parameter.

1. Introduction

The study of laminar flow and heat transfer over a stretching sheet in a viscous fluid is of considerable interest because of its ever increasing industrial applications and important bearings on several technological processes. The production of sheeting material arises in a number of industrial manufacturing processes and includes both metal and polymer sheets. Crane [1] investigated the flow caused by the stretching of a sheet. Many researchers such as Gupta and Gupta [2], Dutta et al. [3], Chen and Char [4], Andersson [5] extended the work of Crane [1] by including the effect of heat and mass transfer analysis under different physical situations. On the other hand, Gupta and Gupta [2] stressed that realistically, stretching surface is not necessarily continuous. Most of the available literature deals with the study of boundary layer flow over a stretching surface where the velocity of the stretching surface is assumed linearly proportional to the distance from the fixed origin. However, it is often argued that (Gupta and Gupta [2]) realistically, stretching of plastic sheet may not necessarily be linear. This situation was beautifully dealt by Kumaran and Ramanaiah [6] in their work on boundary layer fluid flow where, probably first time, a general quadratic stretching sheet has been assumed. Recently, various aspects of such problem have been investigated by many authors such as Xu and Liao [7], Cortell [8,9], Hayat et al. [10] and Hayat and Sajid [11].

Ali [12] has investigated the thermal boundary layer flow by considering the nonlinear stretching surface. A few years later,

* Tel.: +91 342 2657741; fax: +91 342 2530452.
E-mail address: swati_bumath@yahoo.co.in

Peer review under responsibility of Faculty of Engineering, Alexandria University.

ELSEVIER Production and hosting by Elsevier

1110-0168 © 2013 Faculty of Engineering, Alexandria University. Production and hosting by ElsevierB.V. All rights reserved.
http://dx.doi.org/10.1016/j.aej.2013.02.003

Nomenclature

M	magnetic parameter	*Greek symbols*	
Pr	Prandtl number	η	similarity variable
S	suction/injection parameter	κ	the coefficient of thermal conductivity
St	stratification parameter	μ	dynamic viscosity
T	temperature of the fluid	ν	kinematic viscosity
$T_w(x)$	prescribed surface temperature	ψ	stream function
$T_\infty(x)$	variable free-stream temperature	ρ	density of the fluid
u, v	components of velocity in x and y directions	θ	non-dimensional temperature

Magyari and Keller [13] also focused on heat and mass transfer on boundary layer flow due to an exponentially continuous stretching sheet.

Extension to that, Elbashbeshy [14] added new dimension to the study of Ali [12] on exponentially continuous stretching surface. Vajravelu [15] and Vajravelu and Cannon [16] also considered the flow over a nonlinear stretching sheet. Khan [17] and Sanjayanand and Khan [18] studied the viscous-elastic boundary layer flow and heat transfer due to an exponentially stretching sheet. Later, Sajid and Hayat [19] considered the influence of thermal radiation on the boundary layer flow due to an exponentially stretching sheet by solving the problem analytically via homotopy analysis method (HAM). Recently, Bidin and Nazar [20] analysed the effect of thermal radiation on the steady laminar two-dimensional boundary layer flow and heat transfer over an exponentially stretching sheet, which has been solved analytically by Sajid and Hayat [19]. Akyildiz et al. [21] consider the velocity $u = cx^n$ at $y = 0$, which was employed for positive odd integer values of n. It is clear that such a profile would fail for even integer values of n, as the flow at $y = 0$ would be in the wrong direction in the case of $-\infty < x < 0$ (see Van Gorder and Vajravelu [22]). With the help of the modification provided in Van Gorder and Vajravelu [22], one can account for any values of $n \geqslant 1$, even non-integers. Pal [23] reported mixed convection flow past an exponentially stretching surface in the presence of a magnetic field. Nadeem et al. [24] addressed the flow of Jeffrey fluid and heat transfer past an exponentially stretching sheet. Ishak [25] discussed the combined effects of magnetic field and thermal radiation on flow and heat transfer over an exponentially stretching sheet. Sahoo and Poncet [26] addressed the flow of third grade fluid past an exponentially stretching sheet with slip condition. Recently, Mukhopadhyay and Gorla [27] analysed the effects of partial slip on flow past an exponentially stretching sheet. Bhattacharyya [28] discussed the mass transfer in case of boundary layer flow over an exponentially stretching sheet with an exponentially moving free stream in a reactive species.

Suction/injection (blowing) of a fluid through the bounding surface can significantly change the flow field. In general, suction tends to increase the skin friction, whereas injection acts in the opposite manner. The process of suction/blowing has also its importance in many engineering activities such as in the design of thrust bearing and radial diffusers, and thermal oil recovery [29]. Suction is applied to chemical processes to remove reactants [30].

The flow due to a heated surface immersed in a stable stratified viscous fluid has been investigated experimentally and analytically in several studies such as Yang et al. [31], Jaluria and Gebhart [32] and Chen and Eichhorn [33]. Thermal stratification may arise when there is a continuous discharge of the thermal boundary layer into the medium, for example, a heated vertical surface embedded in a porous bed which is of limited extent in the direction of the plate. In such case, the thermal boundary layer eventually hits the ceiling, and at that point, it falls horizontally into the medium since it contains hotter fluid than the rest of the medium (hotter fluid is lighter than the colder fluid) [34]. However, convective flow in a stratified media has not received much attention.

The study of magneto-hydrodynamic (MHD) flow of an electrically conducting fluid is of considerable interest in modern metallurgical and metal-working processes. The process of fusing of metals in an electrical furnace by applying a magnetic field and the process of cooling of the first wall inside a nuclear reactor containment vessel where the hot plasma is isolated from the wall by applying a magnetic field are some examples of such fields [35]. In controlling momentum and heat transfers in the boundary layer flow of different fluids over a stretching sheet, applied magnetic field may play an important role [36]. Kumaran et al. [37] reported that magnetic field makes the streamlines steeper which results the boundary layer thinner.

The purpose of this present work is to extend the flow and heat transfer analysis in boundary layer over an exponentially stretching sheet embedded in a stratified medium. Using suitable transformations, a third order ordinary differential equation corresponding to the momentum equation and a second order differential equation corresponding to the heat equation are derived. Using shooting method, numerical calculations up to desired level of accuracy were carried out for different values of dimensionless parameters of the problem under consideration for the purpose of illustrating the results graphically. The analysis of the results obtained shows that the flow field is influenced appreciably by the stratification parameter in presence of suction at the wall. Estimation of heat transfer coefficient which is very important from the industrial application point of view is also presented in this analysis. It is hoped that the results obtained will not only provide useful information for applications, but also serve as a complement to the previous studies.

2. Mathematical model

Consider the flow of an incompressible viscous electrically conducting fluid past a flat heated sheet coinciding with the plane $y = 0$. The flow is confined to $y > 0$. Two equal and opposite forces are applied along the x-axis, so that the wall is stretched

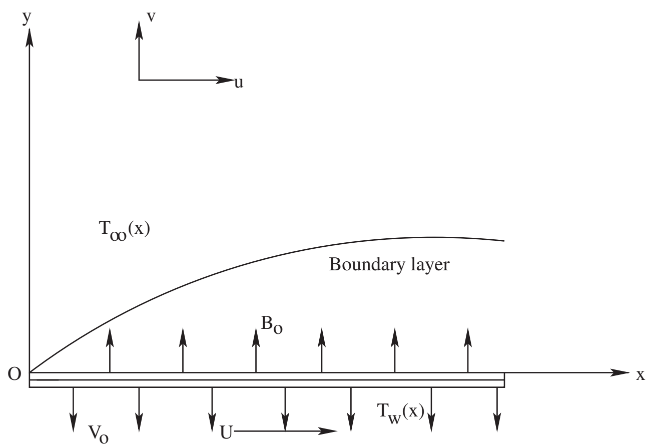

Figure 1 Sketch of the physical problem.

keeping the origin fixed (see Fig. 1). A variable magnetic field $B(x) = B_0 e^{\frac{x}{2L}}$ is applied normal to the sheet, B_0 being a constant [25]. The sheet is of temperature $T_w(x)$ and is embedded in a thermally stratified medium of variable ambient temperature $T_\infty(x)$ where $T_w(x) > T_\infty(x)$. It is assumed that $T_w(x) = T_0 + be^{\frac{x}{2L}}$, $T_\infty(x) = T_0 + ce^{\frac{x}{2L}}$ where T_0 is the reference temperature, $b > 0$, $c \geqslant 0$ are constants.

The continuity, momentum, and energy equations governing such type of flow are written as

$$\frac{\partial u}{\partial x} + \frac{\partial \upsilon}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + \upsilon\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2}{\rho}u \tag{2}$$

$$u\frac{\partial T}{\partial x} + \upsilon\frac{\partial T}{\partial y} = \frac{\kappa}{\rho c_p}\frac{\partial^2 T}{\partial y^2} \tag{3}$$

where u and υ are the components of velocity respectively in the x and y directions, $\nu = \frac{\mu}{\rho}$ is kinematic viscosity, ρ is fluid density, μ is coefficient of fluid viscosity, c_p is specific heat at constant pressure and κ is the thermal conductivity of the fluid. Detail about the horizontally stratified medium can be found in the work of Nielsen and Balling [38].

2.1. Boundary conditions

The appropriate boundary conditions for the problem are given by

$$u = U, \quad \upsilon = -V(x), \quad T = T_w(x) \quad \text{at } y = 0 \tag{4a}$$

$$u \to 0, \quad T = T_\infty(x) \quad \text{as } y \to \infty \tag{4b}$$

here $U = U_0 e^{\frac{x}{L}}$ is the stretching velocity, U_0 is reference velocity, $V(x) > 0$ is velocity of suction and $V(x) < 0$ is velocity of blowing, $V(x) = V_0 e^{\frac{x}{2L}}$, a special type of velocity at the wall is considered. V_0 is the initial strength of suction.

2.2. Method of solution

Introducing the suitable transformations as

$$\begin{aligned} &\eta = \sqrt{\frac{U_0}{2\nu L}}e^{\frac{x}{2L}}y, \quad u = U_0 e^{\frac{x}{L}} f'(\eta) \\ &\upsilon = -\sqrt{\frac{\nu U_0}{2L}}e^{\frac{x}{2L}}\{f(\eta) + \eta f'(\eta)\}, \quad \frac{T - T_\infty}{T_w - T_0} = \theta(\eta) \end{aligned} \tag{5}$$

and upon substitution of (5) in Eqs. (2) and (3), the governing equations transform to

$$f''' + ff'' - 2f'^2 - \mathrm{M}f' = 0 \tag{6}$$

$$\theta'' + \mathrm{Pr}(f\theta' - f'\theta) - \mathrm{PrSt}f' = 0 \tag{7}$$

and the boundary conditions take the following form:

$$f' = 1, \quad f = \mathrm{S}, \quad \theta = 1 - \mathrm{St} \quad \text{at } \eta = 0 \tag{8}$$

and

$$f' \to 0, \quad \theta \to 0 \quad \text{as} \quad \eta \to \infty \tag{9}$$

where the prime denotes differentiation with respect to η, $\mathrm{M} = \frac{2\sigma B_0^2 L}{\rho U_0}$ is the magnetic parameter, $\mathrm{S} = \frac{V_0}{\sqrt{\frac{U_0 \nu}{2L}}} > 0$ (or <0) is the suction (or blowing) parameter, $\mathrm{St} = c/b$ is the stratification parameter and $\mathrm{Pr} = \frac{\mu c_p}{\kappa}$ is the Prandtl number. $\mathrm{St} > 0$ implies a stably stratified environment, while $\mathrm{St} = 0$ corresponds to an unstratified environment.

3. Numerical method for solution

The above Eqs. (6) and (7) along with the boundary conditions are solved by converting them to an initial value problem. We set

$$f' = z, \quad z' = p, \quad p' = [2z^2 + \mathrm{M}z - fp] \tag{10}$$

$$\theta' = q, \quad q' = -\mathrm{Pr}(fq - z\theta) + \mathrm{PrSt}\,z \tag{11}$$

with the boundary conditions

$$f(0) = \mathrm{S}, \quad f'(0) = 1, \quad \theta(0) = 1 - \mathrm{St}. \tag{12}$$

In order to integrate (10) and (11) as an initial value problem, one requires a value for $p(0)$, that is, $f''(0)$ and $q(0)$, that is, $\theta'(0)$ but no such values are given at the boundary. The suitable guess values for $f''(0)$ and $\theta'(0)$ are chosen and then integration is carried out. Comparing the calculated values for f' and θ at $\eta = 10$ (say) with the given boundary conditions $f'(10) = 0$ and $\theta(10) = 0$ and adjusting the estimated values, $f''(0)$ and $\theta'(0)$, we apply the fourth order classical Runge–Kutta method with step-size $h = 0.01$. The above procedure is repeated until we get the converged results within a tolerance limit of 10^{-5}.

4. Results and discussion

In order to analyse the results, numerical computation has been carried out using the method described in the previous section for various values of suction parameter (S), stratification parameter (St), magnetic parameter (M) and Prandtl number (Pr). For illustrations of the results, numerical values are plotted in Figs. 2a–5.

For the verification of accuracy of the applied numerical scheme, a comparison of the present results corresponding to the values of heat transfer coefficient $[-\theta'(0)]$ for $\mathrm{St} = 0$ and $\mathrm{S} = 0$ (i.e. in absence of thermal stratification and suction) with the available published results of Bidin and Nazar [20]

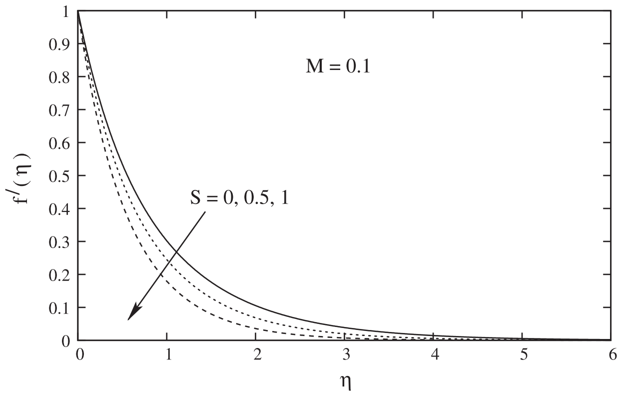

Figure 2a Variation of horizontal velocity $f'(\eta)$ with η for several values of suction parameter S.

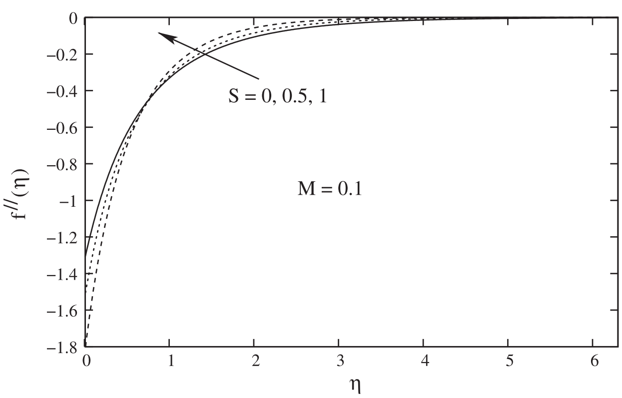

Figure 2b Variation of shear stress $f''(\eta)$ with η for several values of suction parameter S.

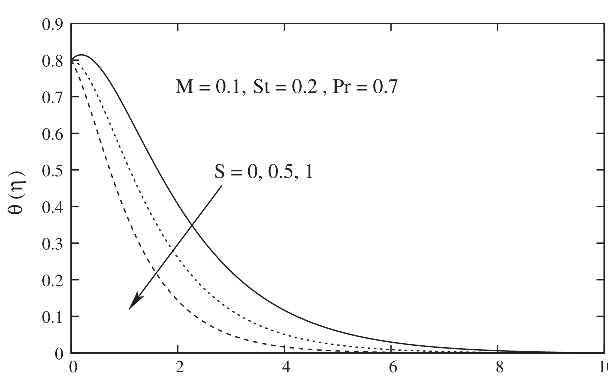

Figure 2c Variation of temperature $\theta(\eta)$ with η for several values of suction parameter S.

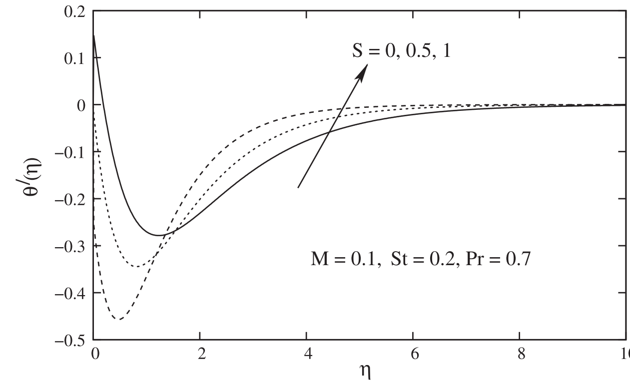

Figure 2d Variation of temperature gradient $\theta'(\eta)$ with η for several values of suction parameter S.

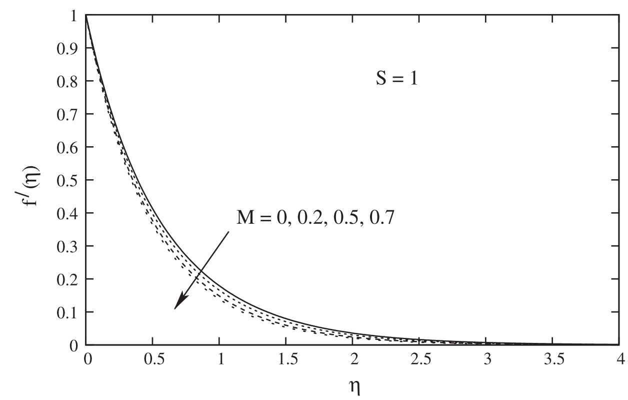

Figure 3 Variation of horizontal velocity $f''(\eta)$ with η for several values of magnetic parameter M.

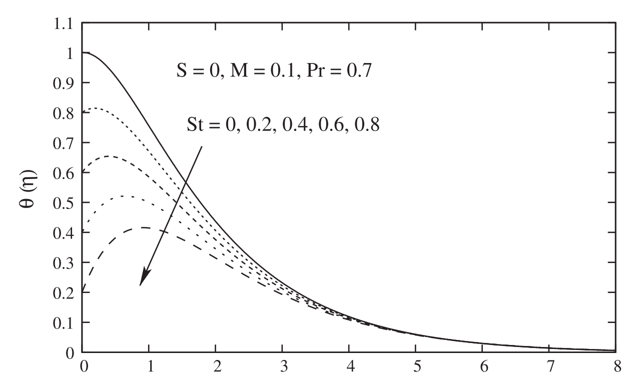

Figure 4a Variation of temperature $\theta(\eta)$ with η for several values of thermal stratification parameter St in the absence of suction.

in the absence of Eckert number (E) and thermal radiation (K) is made and presented in Table 1. The results are found in excellent agreement.

Figs. 2a and 2b depict the effects of suction parameter S on velocity and shear stress profiles, respectively, for exponentially stretching sheet. It is observed that velocity decreases significantly with increasing suction parameter (Fig. 2a). From Fig. 2b, it is very clear that the shear stress decreases initially with the suction parameter S, but shear stress increases significantly after a certain distance η from the sheet. It is observed that, when the wall suction (S > 0) is considered, this causes a decrease in the boundary layer thickness and the

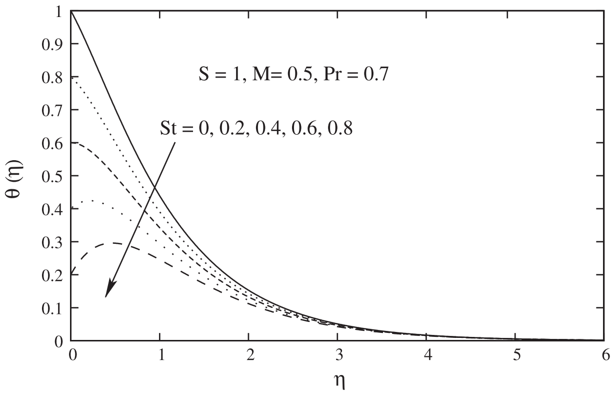

Figure 4b Variation of temperature $\theta(\eta)$ with η for several values of thermal stratification parameter St in presence of suction.

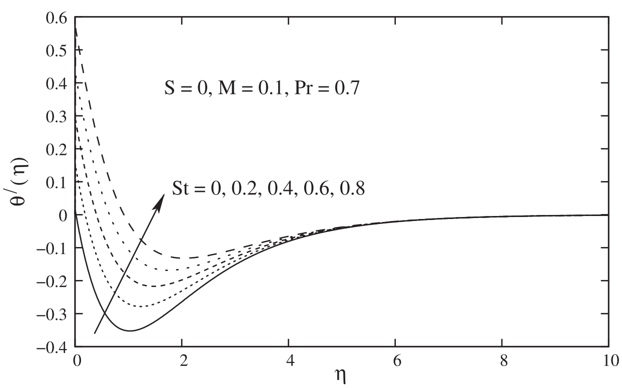

Figure 4c Variation of temperature gradient $\theta'(\eta)$ with η for several values of thermal stratification parameter St in the absence of suction.

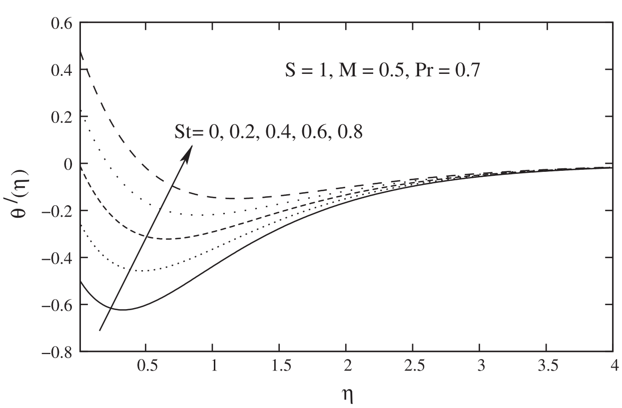

Figure 4d Variation of temperature gradient $\theta'(\eta)$ with η for several values of thermal stratification parameter St in presence of suction.

velocity field is reduced. S = 0 represents the case of non-porous stretching sheet.

Figs. 2c and 2d represent the temperature and temperature gradient profiles for variable suction parameter S. It is seen

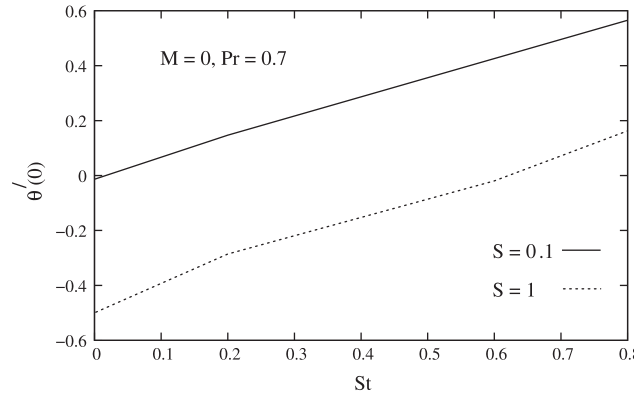

Figure 5 Heat transfer coefficient $\theta'(0)$ against stratification parameter St for two values of suction parameter S.

that temperature decreases with increasing suction parameter (Fig. 2c). The temperature gradient decreases initially with the suction parameter S, but it increases after a certain distance η from the sheet. Far away from the wall, such feature is smeared out Fig. 2d. Thus, suction at the surface has a tendency to reduce both the hydrodynamic and thermal boundary layer thicknesses.

Fig. 3 represents the velocity profiles for the variation of magnetic parameter M. With increasing values of M, fluid velocity is found to decrease. Actually, rate of transport decreases with the increase in M because the Lorentz force which opposes the motion of fluid increases with the increase in M.

Next, we present the effects of thermal stratification parameter (St) on temperature and temperature gradient profiles in the absence and presence of suction at the boundary. Temperature profiles $\theta(\eta)$ for different values of the stratification parameter (St) in the absence of suction (Fig. 4a) and presence of suction (Fig. 4b) are presented. Figs. 4c and 4d are respectively the graphical representations of temperature gradient profiles $\theta'(\eta)$ for several values of stratification parameter for non-porous (for S = 0) and porous (for S > 0) sheet. It is found that the temperature decreases as the stratification parameter St increases (Figs. 4a and 4b). This is quite obvious. Since increase in St means increase in free-stream temperature or decrease in surface temperature. Thermal boundary layer thickness is therefore also decreased with an increase in St values. Profiles all decay from the maximum value at the wall to zero in the free stream, that is, converge at the outer edge of the boundary layer.

The temperature gradient increases considerably with an increase in stratification, St for both cases (Figs. 4c and 4d).

Fig. 5 exhibits the nature of heat transfer coefficient $[\theta'(0)]$ with stratification parameter St for two values of suction parameter S. It is very clear that heat transfer increases with stratification parameter St but decreases with suction. Initially, $\theta'(0)$ is negative but finally it becomes positive for S = 1 but $\theta'(0)$ is almost positive for S = 0.1. Negative value of $\theta'(0)$ indicates that heat is transferred from the fluid to the stretching surface in spite of the excess of surface temperature over that of the free-stream fluid. This phenomenon can be explained as: a fluid particle heated to nearly the wall temperature moves downstream to a location where the wall temperature is lower.

Table 1 Values of $Nu_x Re_x^{-1/2} = -\theta'(0)$ for several values of Prandtl number Pr.

Pr	Bidin and Nazar [20] with $E = 0 = K$	Present study with St = 0 = S = M
1	0.9547	0.9547
2	1.4714	1.4714
3	1.8961	1.8961

Rate of heat transfer increases with Prandtl number (Table 1). An increase in Prandtl number reduces the thermal boundary layer thickness. Prandtl number signifies the ratio of momentum diffusivity to thermal diffusivity. Fluids with lower Prandtl number will possess higher thermal conductivities (and thicker thermal boundary layer structures), so that heat can diffuse from the sheet faster than for higher Pr fluids (thinner boundary layers).

But stratification parameter and Prandtl number have no effects on skin-friction coefficient as the momentum boundary layer equation is independent of θ.

5. Conclusions

The present study gives the numerical solutions for steady MHD boundary layer flow and heat transfer over an exponentially stretching surface embedded in a thermally stratified medium in presence of suction. The effect of suction as well as magnetic parameter on a viscous incompressible fluid is to suppress the velocity field which in turn causes the enhancement of the skin-friction coefficient. Rate of transport is reduced with the increasing magnetic field. The temperature decreases with increasing values of the stratification parameter.

Acknowledgments

The author acknowledges the financial support by UGC, New Delhi, India under the Special Assistance Programme DSA Phase-1.
Thanks are indeed due to the reviewers for their constructive suggestions which helped a lot in improving the quality of the manuscript.

References

[1] L.J. Crane, Flow past a stretching plate, Z. Angew. Math. Phys. 21 (1970) 645–647.
[2] P.S. Gupta, A.S. Gupta, Heat and mass transfer on a stretching sheet with suction or blowing, Can. J. Chem. Eng. 55 (1977) 744–746.
[3] B.K. Dutta, P. Roy, A.S. Gupta, Temperature field in the flow over a stretching sheet with uniform heat flux, Int. Commun. Heat Mass Transfer 12 (1985) 89–94.
[4] C.K. Chen, M.I. Char, Heat transfer of a continuous stretching surface with suction or blowing, J. Math. Anal. Appl. 135 (1988) 568–580.
[5] H.I. Andersson, Slip flow past a stretching surface, Acta Mech. 158 (2002) 121–125.
[6] V. Kumaran, G. Ramanaiah, A note on the flow over a stretching sheet, Acta Mech. 116 (1996) 229–233.
[7] H. Xu, S.J. Liao, Series solutions of unsteady magnetohydrodynamics flows of non-Newtonian fluids caused by an impulsively stretching plate, J. Non-Newtonian Fluid Mech. 159 (2005) 46–55.
[8] R. Cortell, Flow and heat transfer of a fluid through a porous medium over a stretching surface with internal heat generation/absorption and suction/blowing, Fluid Dyn. Res. 37 (2005) 231–245.
[9] R. Cortell, Effects of viscous dissipation and work done by deformation on the MHD flow and heat transfer of a viscoelastic fluid over a stretching sheet, Phys. Lett. A 357 (2006) 298–305.
[10] T. Hayat, Z. Abbas, M. Sajid, Series solution for the upper-convected Maxwell fluid over a porous stretching plate, Phys. Lett. A 358 (2006) 396–403.
[11] T. Hayat, M. Sajid, Analytic solution for axi-symmetric flow and heat transfer of a second grade fluid past a stretching sheet, Int. J. Heat Mass Transfer 50 (2007) 75–84.
[12] M.E. Ali, On thermal boundary layer on a power law stretched surface with suction or injection, Int. J. Heat Fluid Flow 16 (1995) 280–290.
[13] E. Magyari, B. Keller, Heat and mass transfer in the boundary layers on an exponentially stretching continuous surface, J. Phys. D: Appl. Phys. 32 (2000) 577–585.
[14] E.M.A. Elbashbeshy, Heat transfer over an exponentially stretching continuous surface with suction, Arch. Mech. 53 (2001) 643–651.
[15] K. Vajravelu, Viscous flow over a nonlinearly stretching sheet, Appl. Math. Comput. 124 (2001) 281–288.
[16] K. Vajravelu, J.R. Cannon, Fluid flow over a nonlinear stretching sheet, Appl. Math. Comput. 181 (2006) 609–618.
[17] S.K. Khan, Boundary layer viscoelastic fluid flow over an exponentially stretching sheet, Int. J. Appl. Mech. Eng. 11 (2006) 321–335.
[18] E. Sanjayanand, S.K. Khan, On heat and mass transfer in a visco-elastic boundary layer flow over an exponentially stretching sheet, Int. J. Therm. Sci. 45 (2006) 819–828.
[19] M. Sajid, T. Hayat, Influence of thermal radiation on the boundary layer flow due to an exponentially stretching sheet, Int. Commun. Heat Mass Transfer 35 (2008) 347–356.
[20] B. Bidin, R. Nazar, Numerical solution of the boundary layer flow over an exponentially stretching sheet with thermal radiation, Eur. J. Sci. Res. 33 (4) (2009) 710–717.
[21] T. Akyildiz, D.A. Siginer, K. Vajravelu, J.R. Cannon, R.A. Van Gorder, Similarity solutions of the boundary layer equations for a nonlinearly stretching sheet, Math. Methods Appl. Sci. (2009), http://dx.doi.org/10.1002/mma.1181 (www.interscience.wiley.com).
[22] R.A. Van Gorder, K. Vajravelu, A note on flow geometries and the similarity solutions of the boundary layer equations for a nonlinearly stretching sheet, Arch. Appl. Mech. (2009), http://dx.doi.org/10.1007/s00419-009-0370-6.
[23] D. Pal, Mixed convection heat transfer in the boundary layer on an exponentially stretching surface with magnetic field, Appl. Math. Comput. 217 (2010) 2356–2369.
[24] S. Nadeem, S. Zaheer, T. Fang, Effects of thermal radiation on the boundary layer flow of a Jeffrey fluid over an exponentially stretching surface, Numer. Algor. (2011), http://dx.doi.org/10.1007/s11075-010-9423-8.
[25] A. Ishak, MHD boundary layer flow due to an exponentially stretching sheet with radiation effect, Sains Malaysiana 40 (2011) 391–395.
[26] B. Sahoo, S. Poncet, Flow and heat transfer of a third grade fluid past an exponentially stretching sheet with partial slip boundary condition, Int. J. Heat Mass Transfer 54 (2011) 5010–5019.
[27] S. Mukhopadhyay, R.S.R. Gorla, Effects of partial slip on boundary layer flow past a permeable exponential stretching sheet in presence of thermal radiation, Heat Mass Transfer (2012), http://dx.doi.org/10.1007/s00231-012-1024-8.

[28] K. Bhattacharyya, Steady boundary layer flow and reactive mass transfer past an exponentially stretching surface in an exponentially moving free stream, J. Egypt. Math. Soc. 20 (2012) 223–228.

[29] K. Bhattacharyya, Effects of radiation and heat source/sink on unsteady MHD boundary layer flow and heat transfer over a shrinking sheet with suction/injection, Front. Chem. Sci. Eng. 5 (2011) 376–384.

[30] S. Mukhopadhyay, Slip effects on MHD boundary layer flow over an exponentially stretching sheet with suction/blowing and thermal radiation, Ain Shams Eng. J. (2012), <http://dx.doi.org/10.1016/j.asej.2012.10.007>.

[31] K.T. Yang, J.L. Novotny, Y.S. Cheng, Laminar free convection from a non-isothermal plate immersed in a temperature stratified medium, Int. J. Heat Mass Transfer 15 (1972) 1097–1109.

[32] Y. Jaluria, B. Gebhart, Stability and transition of buoyancy-induced flows in a stratified medium, J. Fluid Mech. 66 (1974) 593–612.

[33] C.C. Chen, R. Eichhorn, Natural convection from simple bodies immersed in thermally stratified fluids, ASME J. Heat Transfer 98 (1976) 446–451.

[34] A. Ishak, R. Nazar, I. Pop, Mixed convection boundary layer flow adjacent to a vertical surface embedded in a stable stratified medium, Int. J. Heat Mass Transfer 51 (2008) 3693–3695.

[35] W. Ibrahim, B. Shankar, M.M. Nandeppanavar, MHD stagnation point flow and heat transfer due to nanofluid towards a stretching sheet, Int. J. Heat Mass Transfer 56 (2013) 1–9.

[36] M. Turkyilmazoglu, Exact analytical solutions for heat and mass transfer of MHD slip flow in nanofluids, Chem. Eng. Sci. 84 (2012) 182–187.

[37] V. Kumaran, A.K. Banerjee, A. Vanav Kumar, K. Vajravelu, MHD flow past a stretching permeable sheet, Appl. Math. Comput. 210 (2009) 26–32.

[38] S.B. Nielsen, N. Balling, Transient heat flow in a stratified medium, Tecronophysics 121 (1985) 1–10.