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ORIGINAL ARTICLE

Axisymmetric stagnation-point flow over a stretching/shrinking plate with second-order velocity slip

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Temperature jump

Abstract The axisymmetric stagnation point flow over a stretching/shrinking surface with second-order slip and temperature jump is studied numerically. The governing partial differential equations are transformed into ordinary (similarity) differential equations. These equations along with the corresponding boundary conditions are solved numerically using a boundary value problem solver bvp4c in Matlab software. It is observed that dual (first and second) solutions exist for the similarity equations. The effects of different parameters on the velocity and the temperature distributions as well as the skin friction coefficient and the Nusselt number are analyzed and discussed.

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1. Introduction

The study of boundary layer flow and heat transfer toward a stagnation point has attracted the interest of many researchers due to its important applications in engineering and industry. The stagnation region characterizes the highest pressure, the highest heat transfer, and the highest rates of mass deposition. Hiemenz [1] is the pioneer, who

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obtained an exact solution for the steady two-dimensional stagnation point flow, and then Homann [2] has extended this work to axisymmetric case. Later, Howarth [3] investigated the flow near the steady axisymmetric stagnation point. Since then, in parallel with these applications, many authors have actively investigated the nature of solution structure viewed from the fundamental point, such as Chiam [4], Mahapatra and Gupta [5,6], Nazar et al. [7], Reza and Gupta [8], Lok et al. [9,10] and Yacob and Ishak [11], etc. These authors studied the problems of boundary layer flow and heat transfer on a stretching sheet. Chiam [4] combined the stretching sheet problem with two-dimensional stagnation point flow, and found that the flow near the stretching surface is the same as the inviscid flow far from the surface resulting in a no boundary layer flow. It is worth mentioning that the flow and heat transfer over a stretching/shrinking sheet were considered by Zheng et al. [12,13] and Yacob et al. [14].

Quite recently, the flow past a shrinking sheet was taken into consideration because of its essential applications in industries such as product packaging in obtaining proper wrapping, manufacturing of certain polymers and high-performance materials for aerospace coatings [15]. There are two conditions that allow the existence of the solutions for a shrinking sheet which whether an adequate suction on the boundary is imposed [16] or a stagnation flow is added [17] that causes the vorticity of the shrinking sheet is confined in the boundary layer [18]. Wang [17] is the first who introduced the concept of flow past a shrinking sheet and he investigated the stagnation point flow towards a shrinking sheet for both two-dimensional and axisymmetric cases. He obtained dual solutions and unique solution for specific values of the shrinking parameter. Wang's [17] work has been extended by Ishak et al. [19] and then by Bhattacharyya and Layek [20], Bhattacharyya et al. [21] and Lok et al. [18] with different physical conditions.

All of the above-mentioned papers considered the no-slip boundary condition which is known as a central tenet of the Navier–Stokes theory [22]. However, there are some applications where the slip condition cannot be ignored. For instance, rarefied gas flows with slip boundary conditions are applied in the micro-scale devices and low-pressure situations [23–25]. Furthermore, partial velocity slip often occurs for inhomogeneous fluid, especially slurries, gels, emulsions and foams [26]. Wall slip may happen when a thin film of light oil is attached to the plate or when the plate is coated with special coatings such as a thick monolayer of hydrophobic octadecylthichlorosilane [27]. It is worth mentioning that very recently, Rosca et al. [28] studied the mixed convection boundary layer flow near the lower stagnation point of a horizontal circular cylinder with a second-order slip velocity and reported the existence of multiple solutions. A stability analysis was performed to determine which solution is stable and thus physically reliable.

Wang [29] studied the stagnation-point flow with first-order velocity slip of rarefied gases and he derived the exact solution of the Navier–Stokes equation. He found that when the slip parameter goes to infinity, the flow behaves as though it were inviscid. The slip flow under different flow configurations has been studied recently in Refs. [30–35]. Further, Fang et al. [36] presented the exact solution of the MHD flow and mass transfer under slip condition over a stretching sheet. It was shown that there exists a unique solution for any combination of the slip, magnetic and the mass transfer parameters. Then, Hafidzuddin et al. [37] investigated a permeable exponentially stretching/shrinking sheet with generalized slip velocity. They noticed that dual solutions exist for a certain range of the suction and stretching/shrinking parameters.

Wu [38] proposed a new second-order slip velocity model which matches better with the Fukui–Kaneko results based on the direct numerical simulation of the linearized Boltzmann equation [39]. Many authors applied Wu's model to study the behavior of fluid flow on a solid surface [40–42]. Moreover, there are many cases where temperature jump along with velocity slip should be considered. Some of the applications are in flow of rarefied gases at low Knudsen numbers [43], flow past superhydrophobic micro-surfaces [44,45] or flow against rough surfaces [46]. The research on the forced convection stagnation flow on a solid surface with slip boundary condition was done by Wang [47]. Later, Wang and Ng [48] considered the three-dimensional axisymmetric stagnation flow on a heated vertical surface by considering first-order velocity slip and

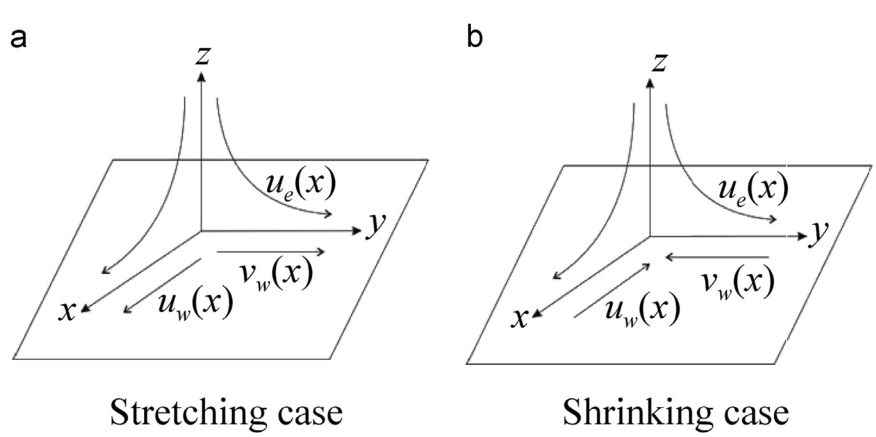

Figure 1 Physical model and coordinate system. (a) Stretching case and (b) shrinking case.

temperature jump. With this awareness, recently many authors [49–51] have considered the effects of velocity slip on the flow and temperature jump on the heat transfer characteristics.

Our aim is to extend the work done by Wang and Ng [48] to consider the second-order velocity slip condition on the stretching/shrinking surface. Using a similarity transformation, the governing partial differential equations are transformed into a system of nonlinear ordinary differential (similarity) equations. The obtained results are compared with those reported by Wang and Ng [48] for a particular case.

2. Mathematical formulation

We consider an axisymmetric stagnation-point flow of a viscous and incompressible fluid near a stretching/shrinking surface with the xy-plane describing the horizontal surface. The stagnation flow is symmetrical about the z-axis far from the surface. It is assumed that the stretching/shrinking velocities of the surface in the x and y directions are $u_w(x)$ and $v_w(y)$, respectively. Far from the surface ($z \to \infty$) the flow is a potential stagnation flow $u_e(x) = ax$, $v_e(y) = ay$ and $w_e(z) = -2az$, where $a > 0$ is the strength of the flow. It is also assumed that the constant plate temperature is T_w and the ambient fluid is T_∞ as shown in Figure 1.

The Navier–Stokes and energy equations are given by (Wang and Ng [48]; Rosca and Pop [52]):

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) \tag{2}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) \tag{3}$$

$$u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) \tag{4}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) \tag{5}$$

We assume that Eqs. (1)–(5) are subjected to the boundary conditions

Table 1 Initial values for axisymmetric stagnation flow on a horizontal plate for $K = 1$, $\lambda = 0$ and $\Lambda = 0$.

Δ	Wang and Ng [48]		Present study	
	$f''(0)$	$-\theta'(0)$	$f''(0)$	$-\theta'(0)$
0	1.31194	0.3995	1.311938	0.399536
0.1	1.21009	0.4135	1.210087	0.413499
0.5	0.86688	0.4435	0.866879	0.443918
1.0	0.61730	0.4590	0.617300	0.458984
5.0	0.17928	0.4789	0.179287	0.478939
10.0	0.09460	0.4822	0.094597	0.482177

$$\begin{aligned} &u = u_w(x) + u_{slip}(x), \quad v = v_w(y) + v_{slip}(y), \quad T = T_w + T_{jump}(z) \\ &\text{at} \quad z = 0 \\ &u \to u_e(x), \quad v \to v_e(y), \quad w \to w_e(z), \quad T \to T_\infty \\ &\text{as} \quad z \to \infty \end{aligned} \tag{6}$$

where $u_w(x) = cx$, $v_w(y) = cy$ and $T_{jump}(z) = C\partial T/\partial z$ with C being the temperature jump coefficient.

Here u, v and w are the velocity components along the x, y and z-axes, T is the fluid temperature, p is the pressure, α is the thermal diffusivity of the fluid, ν is the kinematic viscosity of the fluid and ρ is the density of the fluid. Further, $u_{slip}(x)$ and $v_{slip}(y)$ are the slip velocities at the stretching/shrinking surface, which due to Wu [38] are given as

$$\begin{aligned} &u_{slip}(x) = \\ &\frac{2}{3}\left(\frac{3-\varepsilon l^2}{\varepsilon} - \frac{3}{2}\frac{1-l^2}{K_n}\right)\delta\frac{\partial u}{\partial z} - \frac{1}{4}\left[l^4 + \frac{2}{K_n^2}(1-l^2)\right]\delta^2\frac{\partial^2 u}{\partial z^2} = \\ &A\frac{\partial u}{\partial z} + B\frac{\partial^2 u}{\partial z^2} \\ &v_{slip}(y) = \\ &\frac{2}{3}\left(\frac{3-\varepsilon l^2}{\varepsilon} - \frac{3}{2}\frac{1-l^2}{K_n}\right)\delta\frac{\partial v}{\partial z} - \frac{1}{4}\left[l^4 + \frac{2}{K_n^2}(1-l^2)\right]\delta^2\frac{\partial^2 v}{\partial z^2} = \\ &A\frac{\partial v}{\partial z} + B\frac{\partial^2 v}{\partial z^2} \end{aligned} \tag{7}$$

where A and B are constants, K_n is Knudsen number, $l = \min(1/K_n, 1)$, ε is the momentum accommodation coefficient with $0 < \varepsilon \le 1$, and δ is the molecular mean free path. Based on the definition of l, it is seen that for any given value of K_n, we have $0 \le l \le 1$. Since the molecular mean free path δ is always positive, it results in such that B is a negative number.

We look for a similarity solution of Eqs. (1)–(5) of the following form:

$$\begin{aligned} &u = axf'(\eta), \quad v = ayf'(\eta), \quad w = -2\sqrt{a\nu}f(\eta), \\ &\theta(\eta) = (T - T_\infty)/(T_w - T_\infty), \quad \eta = z\sqrt{a/\nu} \end{aligned} \tag{8}$$

where prime denotes differentiation with respect to η. The pressure p can be derived from Eq. (4) and it is given by

$$\frac{p}{\rho} = \nu\frac{\partial w}{\partial z} - \frac{w^2}{2} + \text{constant} \tag{9}$$

Substituting Eq. (8) into Eqs. (2), (3) and (5), the following set of ordinary differential equations reduce to

$$f''' + 2ff'' + 1 - f'^2 = 0 \tag{10}$$

$$\theta'' + 2Prf\theta' = 0 \tag{11}$$

and the boundary conditions (6) become

$$\begin{aligned} &f(0) = 0, \quad f'(0) = \lambda + \Delta f''(0) + \Lambda f'''(0), \quad \theta(0) = 1 + K\theta'(0), \\ &f'(\eta) \to 1, \quad \theta(\eta) \to 0 \quad \text{as} \quad \eta \to \infty \end{aligned} \tag{12}$$

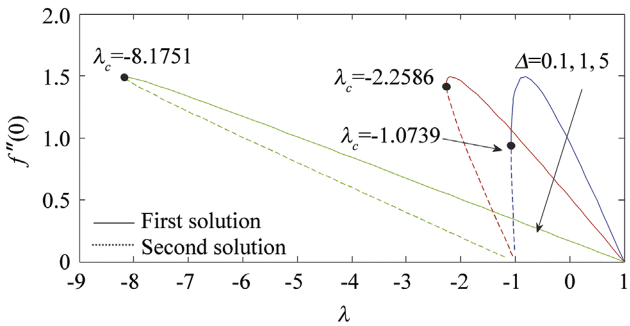

Figure 2 Variation of the skin friction coefficient $f''(0)$ with λ for various values of first-order velocity slip parameter Δ when $\Lambda = -0.3$.

where $Pr = \nu/\alpha$ is the Prandtl number, $\lambda = c/a$ is a dimensionless constant parameter with $\lambda > 0$ for a stretching surface and $\lambda < 0$ for a shrinking surface, $\Delta = A\sqrt{a/\nu} > 0$ is the first-order velocity slip parameter, $\Lambda = Ba/\nu < 0$ is the second-order velocity slip parameter and $K = C\sqrt{a/\nu} > 0$ is the temperature jump parameter.

The physical quantities of interest are the skin friction coefficient C_f and the local Nusselt number Nu_x, which are defined as

$$C_f = \frac{\tau_w}{\rho u_e^2(x)}, \quad Nu_x = \frac{x q_w}{k(T_w - T_\infty)} \tag{13}$$

where τ_w is the skin friction or shear stress along the stretching/shrinking surface and q_w is the heat flux from the stretching/shrinking surface, which are given by

$$\tau_w = \mu\left(\frac{\partial u}{\partial z}\right)_{z=0}, \quad q_w = -k\left(\frac{\partial T}{\partial z}\right)_{z=0} \tag{14}$$

Using the similarity variables (8), we obtain

$$Re_x^{1/2} C_f = f''(0), \quad Re_x^{-1/2} Nu_x = -\theta'(0) \tag{15}$$

where $Re_x = u_e(x)\,x/\nu$ is the local Reynolds number.

3. Results and discussion

The nonlinear ordinary differential Eqs. (10) and (11) along with the boundary conditions (12) were solved numerically using the bvp4c package in Matlab. In this method, since the present problem may have more than one solution, the bvp4c needs more than one initial guesses that satisfy the boundary conditions (12). Determining the initial guesses for the first solution is easier than that of the second solution, which is quite challenging due to the converging issue. To overcome this difficulty, we start with a set of parameter values for which the solution is easier to appear. Then, we use the obtained results and consider it as an initial guess for the next values of parameters. This technique is called a continuation [53]. In our numerical computations, we have chosen the condition $\eta \to \infty$ as $\eta = 10$ for the first solution, while for the second solution, $\eta = 40$ was found sufficient for the profiles to reach the far field boundary conditions asymptotically.

The numerical computations were performed for several values of the dimensionless parameters which are stretching/shrinking parameter λ, Prandtl number Pr, the first-order velocity slip parameter $\Delta = A\sqrt{a/\nu} > 0$, the second-order velocity slip parameter $\Lambda = B\,a/\nu < 0$ and the temperature jump parameter $K = C\sqrt{a/\nu} > 0$. The value of parameters Pr and K are fixed to 0.7 and 0.2, respectively. Comparative study for the skin friction coefficient $f''(0)$ and the rate of heat transfer $-\theta'(0)$ with the numerical results of Wang and Ng [48] was carried out to validate the numerical results obtained when $\lambda = 0$ (non-stretching/shrinking case) and $\Lambda = 0$ (non-second-order velocity slip case). The comparison shows a favorable agreement, as can be seen in Table 1. Therefore, we can expect that the numerical results for other cases are correct and reliable.

The variations of the skin friction coefficient $f''(0)$ and the local Nusselt number $-\theta'(0)$ against λ for first-order velocity slip $\Delta = 0.1$, 1, 5 are shown in Figures 2 and 3, respectively. The solid lines denote the first solution, while the dash lines indicate the second solution. From both figures, we can observe that there exist dual solutions for Eqs. (10) and (11) subject to the boundary Eq. (12) when $\lambda < 0$. The value of $f''(0)$ being zero when $\lambda = 1$. This is due to the fluid and the solid surface which move in the same velocity, and thus there is no friction at the fluid-solid interface. However, in Figure 3 there is heat transfer at the surface, even though no friction occurred. This happens because of the temperature difference between the fluid and the solid surface.

For the shrinking case, the solutions exist up to a critical values of λ denoted by λ_c. The values of λ_c for the corresponding values of Δ are depicted in Figures 2 and 3. It is remarked that the range of the first and second solutions significantly increase as parameter Δ increases and no similarity solutions exist for $\lambda < \lambda_c$. The wide range of solutions is due to physical phenomena of the slip effect. The generation of vorticity for shrinking velocity is slightly reduced for increasing slip on the solid surface. Hence, that vorticity remains bounded in the boundary layer region for larger shrinking velocity and thus the steady solution is possible for large values of λ [28]. As can be seen in Figure 3, the rate of heat transfer increases with increasing

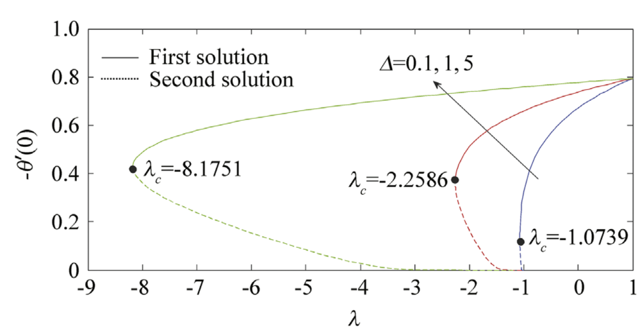

Figure 3 Variation of the local Nusselt number $-\theta'(0)$ with λ for various values of first-order velocity slip parameter Δ when $\Lambda = -0.3$, $K = 0.2$, $Pr = 0.7$.

values of λ and Δ for the first solution and it decreases with increasing λ and Δ for the second solution.

Figures 4 and 5 illustrate the variations of the skin friction coefficient $f''(0)$ and the rate of heat transfer $-\theta'(0)$ with λ for second-order velocity slip $\Lambda = -0.1, \ -1, \ -5$. It is indicated that when the magnitude of second-order parameter increases the solution range for both figures increases. An interesting behavior is found in Figure 4 which the pattern of each curve is significantly different for each value of Λ. For $\Lambda = -0.1$, the curve is similar to parabolic as expected and for $\Lambda = -5$ the curve tends to shape an ellipse but when $\Lambda = -1$, the curve seems to form a straight line. For the value of magnitude of $\Lambda = -0.1$ and -1, the value of the first solution is larger than the second solution as predicted, but differs from those of $\Lambda = -5$. It is noticeable, the range of solution increases whenever the magnitude of Λ is increased. As reported by Rosca and Pop [41] for a similar problem where dual solutions exist, the first solution is physically stable and reliable while the second solution is not. Apparently, Figures 2–5 show that for the first solution, in the presence of slip effects, the skin friction decreases while the heat transfer rate increases.

The velocity $f'(\eta)$ and temperature $\theta(\eta)$ profiles for various values of Λ and λ are portrayed in Figures 6–9. Through Figures 6 and 7, we can see the effects of Λ on both figures. The thickness of the boundary layer decreases as the magnitude of Λ is increased for the first solution and vice versa for the second solution. However, the

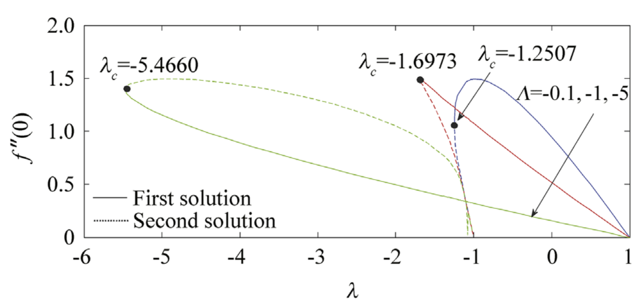

Figure 4 Variation of the skin friction coefficient $f''(0)$ with λ for various values of second-order velocity slip parameter Λ when $\Delta = 0.3$.

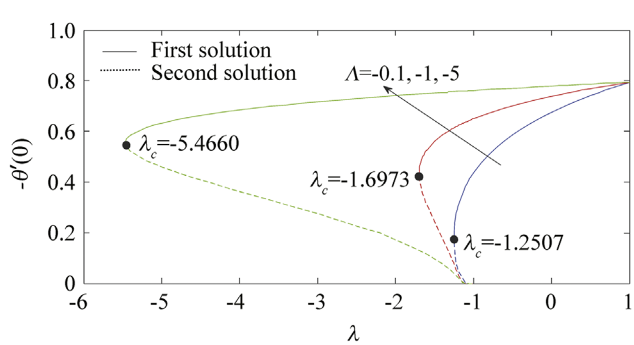

Figure 5 Variation of the local Nusselt number $-\theta'(0)$ with λ for various values of second-order velocity slip parameter Λ when $\Delta = 0.3$, $K = 0.2$, $Pr = 0.7$.

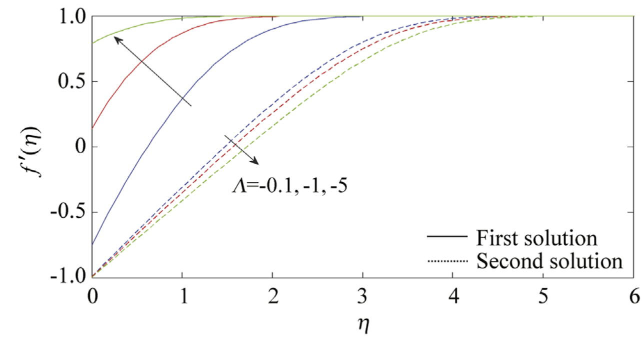

Figure 6 Velocity profiles $f'(\eta)$ for various values of Λ when $\Delta = 0.3$, $K = 0.2$, $\lambda = -1.2$ and $Pr = 0.7$.

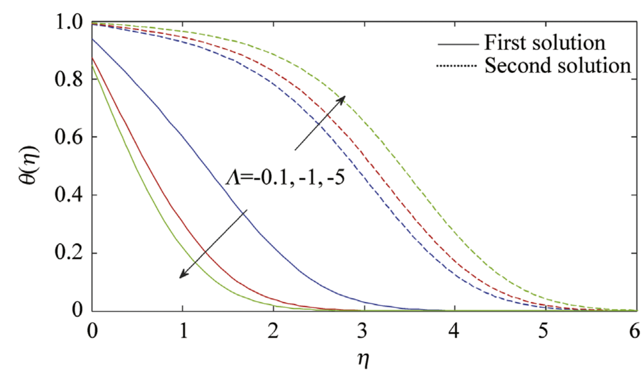

Figure 7 Temperature profiles $\theta(\eta)$ for various values of Λ when $\Delta = 0.3$, $K = 0.2$, $\lambda = -1.2$ and $Pr = 0.7$.

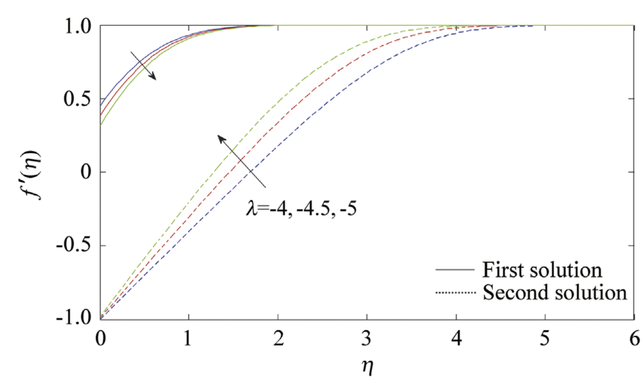

Figure 8 Velocity profiles $f'(\eta)$ for various values of λ when $\Delta = 5$, $\Lambda = -0.3$, $K = 0.2$ and $Pr = 0.7$.

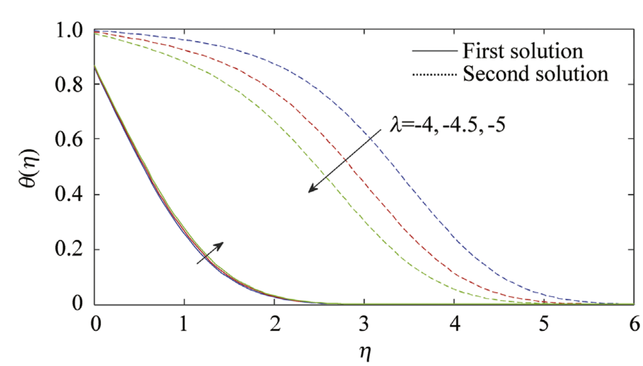

Figure 9 Temperature profiles $\theta(\eta)$ for various values of λ when $\Delta = 5$, $\Lambda = -0.3$, $K = 0.2$ and $Pr = 0.7$.

temperature and the thermal boundary layer decrease with the increment of the magnitude of Λ and conversely for the second solution. In consequence, the velocity at the surface $f'(0)$ increases, which resulted in the decrease of the skin friction coefficient at the surface $f''(0)$, while decreases the surface temperature $\theta(0)$ leads to the increase of the heat transfer rate at the surface $-\theta'(0)$ for the first solution. This happens due to the diminishing and rising values of the skin friction coefficient and the local Nusselt number, respectively as shown in Table 2.

Table 2 Variations of the skin friction coefficient $f''(0)$ and the local Nusselt number $-\theta'(0)$ for $\lambda = -1.2$, $K = 0.2$, $Pr = 0.7$ and $\Delta = 0.3$.

Λ	First solution		Second solution	
	$f''(0)$	$-\theta'(0)$	$f''(0)$	$-\theta'(0)$
-0.1	1.35203151	0.30552092	0.69033355	0.05025554
-1.0	1.19354866	0.62222864	0.64772804	0.03879292
-5.0	0.35497908	0.75748119	0.58721674	0.02444822

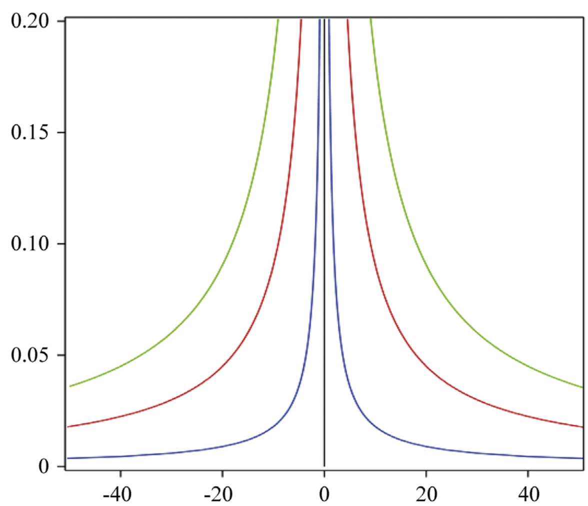

Figure 10 Streamlines in the plane $y = 0$ for $\Delta = 0.3$, $\Lambda = -0.1$ and $\lambda = 1.2$ (stretching surface).

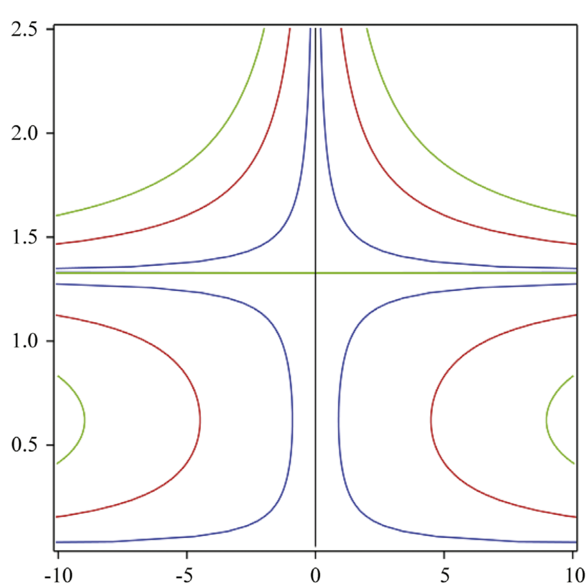

Figure 11 Streamlines in the plane $y = 0$ for $\Delta = 0.3$, $\Lambda = -0.1$ and $\lambda = -1.2$ (shrinking surface).

Moreover, the dual velocity profiles of Figure 8 depicts that the velocity decreases with increasing of the magnitude of λ which corresponds to a faster shrinking velocity for the first solution. Opposite behavior is observed for the second solution. As plotted in Figure 9 for the first solution, the temperature increases slightly while for the second solutions, the temperature decreases and both profiles reduce gradually to zero. For the first solution in Figure 9, the function $\theta(\eta)$ has a negative gradient at the surface and it shows that the heat slowly disperses from the solid surface to the fluid. Apparently, the boundary layer thickness for the first solution is lower than that of the second solution for all profiles. The velocity and temperature profiles satisfy the far field boundary conditions (12) asymptotically, thus support the validity of the numerical results obtained and the existence of the dual solutions displayed in Figures 2–5.

Figures 10 and 11 demonstrate the streamlines in the presence of the first and second order slip parameters for the stretching at $\lambda = 1.2$ and shrinking at $\lambda = -1.2$, respectively. Figure 10 shows a symmetry stagnation flow towards the stretching surface, while different pattern is seen in Figure 11, which shows a double layer. There exists in this figure a horizontal line that separates the flow into two parts where the flow on the upper part seems to have a similar pattern with the stretching case. While obviously seen, a reverse rotating flow is formed in the lower part.

4. Concluding remarks

The problem of a three-dimensional axisymmetric stagnation point flow over a stretching/shrinking surface with first and second-order velocity slip and temperature jump was studied in this paper. The governing boundary layer equations were solved numerically for both stretching/shrinking surface using problem solver bvp4c in Matlab software. The existence and multiplicity (duality) of solutions were displayed and explained by looking at the combined effects of the stretching/shrinking parameter, slip parameters Δ and Λ with temperature jump parameter K and Prandtl number Pr remain unchanged. The dual solutions exist for the shrinking case, while the solution is unique for the stretching case. The results showed that the presence of first and second order slip decreased the skin friction while increased the rate of heat transfer.

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