Identification of Power System Brittle Risk Sources Based on DC Current and Distribution Factor

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Abstract

The occurrence of cascading failure in electric power system due to the brittle sources of the system was excited by the brittleness theory, so how to find the brittle source of the system is the key to avoid the brittle risk occurring. This text forwards the method of the brittle source identification based on the combination of DC flow and distribution factor and proves its effectiveness and practicality through the IEEE-5 node system.

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Keywords: component; brittle risk; brittle source identification; DC flow; distribution factor

1. Introduction

In recent years, some catastrophic blackout of power system all over the world attracted people’s attention [1-5]. From the theoretical point of brittleness, the catastrophic power failure result from the power system’s brittle.

Brittleness theory points out whether one of the subsystems is the brittle source of the systems and the subsystems will arouse the chain collapse of the whole system from itself to others within the system when it was influenced by internal or external factors. It finally makes the whole system crash and.

This text analyzes the brittle risk of power system and identifies the brittle source for the IEEE-5 node system based on combination of DC power flow and distribution factor.

2. The Brittle Risk of Power System

The brittle risk of complex system is composed by two parts: the internal structure and external brittle environment system. The so-called internal structure refers to the structure of the organization between the
subsystems and the external brittle environment is the events and factors which affect the brittle risk of the system. Furthermore, the brittle risk model’s structure of complex system can be divided into four stages as the following shown in Figure 1.

![Figure 1: brittle risk model of complex system](image)

It can be seen that the brittle model of complex system is composed by the brittle input, system structure and the brittle risk of the three fundamental of brittle composition actually, which is shown in Figure 2.

![Figure 2: the upper brittle structure of complex system](image)

Brittle structure of the system contains two aspects: First, the brittle state of each subsystem assume complex system $S$ contains $m$ subsystems, expressed as: $S = \{S_1, S_2, ..., S_m\}$, so $S'_i (i = 1, 2, ..., m)$ represents that the brittle state of the corresponding subsystem $S_i$. $S'_i$ can be written: $S'_i \{a'_1', a'_2', ..., a'_r'\}$ is used to describe the brittle state variables of the subsystem $S_i$. These brittle states of those subsystems are defined as the structure of the brittle risk for the part of the system. Second, the various brittle contacts style of subsystems, including the brittle relationship between subsystems as well as the proliferate manners of brittle within the system, which is the connection of the brittle risk structure to the system.

The brittle relationship between subsystems $R_{ij}(t)$ refers to the time $t$, when a state variable $S''_i(t)$ which characterize the brittle character of a subsystem $S_i(t)$ changes, the other brittle state characteristic quantities $S''_j(t)$ of the corrective subsystems $S_j(t)$ also change, so we definite the subsystem $S_i(t)$ is brittle source and the subsystem $S_j(t)$ is brittle receiver.

3. **The Mathematical Model of DC Power Flow and Distribution Factor Method**

3.1 **The mathematical model of DC power flow**

DC power flow equation [6] is obtained through the simplification of the AC power flow equation under certain condition.

The AC power flow equation for the electric power system expression is:

$$ P_i = V_i \sum_{j \in i} V_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) $$

Branch power flow expression is:

$$ P_{ij} = V_i V_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) - t_{ij} G_{ij} V_j^2 $$

In it, $N$ is the number of nodes, $P_i$ is the active power flow injected into the node $i$. $V_i$ and $V_j$ are voltage amplitude of the node $i$ and $j$, $j \in i$ means all nodes connected to the node $i$ directly, $t_{ij}$ means branch transformer non-standard ratio unit value, $\theta_{ij}$ means the phase angle difference of the branch node voltage, $G_{ij}$ and $B_{ij}$ mean the corresponding real and imaginary parts of the node admittance matrix respectively.
In the equation, $r_{ij}$ and $x_{ij}$ represent the resistance and reactance of the line respectively. The application of DC power flow is based on the following assumptions in actual running of the high-voltage power systems.

1) The resistance $R$ is much smaller than its reactance $X$ in branch: $r_{ij}=0$, the susceptance $b_{ij}=-1/x_{ij}$.

2) The voltage phase angle difference between two nodes of branch is very small: $\sin \Theta_{ij} = \Theta_i - \Theta_j = \Theta_{ij}$, $\cos \Theta_{ij} = 1.0$.

3) The susceptance of node to the ground is negligible: $b_{i0}=b_{j0}=0$.

4) The voltage of each node is close to the rated voltage: $V_i=V_j=1.0$.

5) The relative transformers’ ratio $\tau_{ij}=1$.

Then the equation (1) can be simplified:

$$ P_i = \sum_{j \neq i} B_{ij} \theta_{ji} $$

The matrix form: $P = B \theta$ (5)

The equation (2) can be simplified:

$$ P_{ij} = -B_{ij} \theta_{ij} = \frac{\Theta_i - \Theta_j}{x_{ij}} $$

3.2 The mathematical model of the distribution factor method

Sometimes, we need to know active power flow change of branch, which may be caused by one or more branches breaking in the power system analysis and can be described by the distribution factor method [7].

It is illustrated that the active power flow the branch $l$ is $P_l$ and the branch $l$ breaking will cause the power flow of branch $k$ changing. The intermediate variable is $\eta_i=XM_i$, in which $M_i$ is the corresponding (n-1)-order vector matrix associated with the branch $l$:

$$ M_i^T = [...1...1...-1...] $$

The self-impedance and mutual impedance of branch $k$ after branch $l$ breaking:

$$ X_{i-l} = M_i^T \eta_l $$

$$ X_{k-l} = M_k^T \eta_k $$

The power flow distribution factor of branch $k$ ($k \neq l$) to the branch $l$ which broken off:

$$ D_{k-l} = \frac{X_{k-l} / x_k}{1 - X_{l-l} / x_l} $$

The power flow of $k$:

$$ P_{k}^l = P_k + D_{k-l} P_l $$

4. Identification for Brittle Source of Power System

The identification method of power brittle source is carried out by fault simulation of breaking all of the components. If one component’s breaking causes other components breaking and makes the failure chain of transmission in the entire system, then leads to all lines broken off eventually, which indicates that the element is the brittle source of system[8].

Identification of brittle source is based on power flow calculation. First, calculate the power flow distribution of the initial system using DC power flow method quickly in the range of permitted error, then
calculate the power flow redistribution of lines caused by one or more branches broken off by using distribution factor method. Broken off the branch if the re-allocation result over the transmission border and re-distribute power flow again. So back and forth, the steps complete until all of the lines broken off or all the flow are less than the limited value.

The steps of the brittle source identification based on the combination of DC flow and distribution factor as following:

1) Select one node as the reference node and its voltage phase angle: $\theta = 0$;
2) Form the nodal admittance matrix $B_\theta$ except for reference node:

$$B_\theta(i, i) = \sum_{j\neq i} 1/x_{ij}$$
$$B_\theta(i, j) = -1/x_{ij}$$

(11)

3) Calculate matrix $X$ of $B_\theta$: $X = B_\theta^{-1}$;
4) The node voltage phase angle matrix:

$$\theta = XP_{\theta}^S$$

(12)

5) The active power flow $P_{ij}$;
6) Break off the branch $l$ ($l = 1, 2, \ldots, m$) one by one calculate self-impedance and mutual impedance of other branches to the branch $l$;
7) Calculate the flow distribution factor $D_{k-l}$ of each branch $k$ ($k \neq l$) to the broken one $l$;
8) Calculate the flow $P_{k-l}$ of other branches after branch $l$ broken;
9) If the branch’s flow of the re-distribution over the transmission border, this branch must be broken and come back to step (6) or end.

5. Example

The IEEE-5 node system[6] is shown below in this paper. The numbers of node and branch are also marked in Figure 3.

![Figure 3: IEEE-5 node system](image)

**TABLE I. POWER TRANSMISSION BORDER OF LINES**

<table>
<thead>
<tr>
<th>Branch number</th>
<th>Node of Beginning to end</th>
<th>Power transmission border</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>1-2</td>
<td>2</td>
</tr>
<tr>
<td>(2)</td>
<td>1-3</td>
<td>0.65</td>
</tr>
<tr>
<td>(3)</td>
<td>2-3</td>
<td>2</td>
</tr>
<tr>
<td>(4)</td>
<td>2-4</td>
<td>6</td>
</tr>
</tbody>
</table>
Select node 5 as the reference node. The node admittance matrix $B_0$:

$$B_0 = \begin{bmatrix} 6.8571 & -4.0000 & -2.8571 & 0.0000 \\ -4.0000 & 74.0000 & -3.3333 & -66.6667 \\ -2.8571 & -3.3333 & 39.5238 & 0.0000 \\ 0.0000 & -66.6667 & 0.0000 & 66.6667 \end{bmatrix}$$

Calculate the impedance matrix $X$:

$$X = B_0^{-1} = \begin{bmatrix} 0.2439 & 0.1467 & 0.3000 & 0.1467 \\ 0.1467 & 0.2300 & 0.3000 & 0.2300 \\ 0.0300 & 0.0300 & 0.0300 & 0.0300 \\ 0.1467 & 0.2300 & 0.0300 & 0.2450 \end{bmatrix}$$

Calculate the phase angle vector matrix $\theta$:

$$\theta = \begin{bmatrix} -0.0918 \\ 0.3129 \\ 0.0774 \\ 0.3879 \\ 0.0000 \end{bmatrix}$$

Calculate the flow of branches by substituting the matrix $\theta$ into flow equation:

$$P_1 = P_{12} = -1.6189, \quad P_2 = P_{13} = -0.0411$$

$$P_3 = P_{34} = 1.3011, \quad P_4 = P_{34} = -5.0000$$

$$P_5 = P_{55} = -2.5800$$

Discuss each branch whether the brittle source of power system is by breaking each branch respectively.

The intermediate variable when branch (1) breaks off:

$$\eta_1 = XM_1 = \begin{bmatrix} -0.0972 \\ 0.0833 \\ 0.0000 \\ 0.0983 \end{bmatrix}$$

Obtain the self-impedance and mutual impedance to the branch (1) broken:

$$X_{1,1} = 0.1956, \quad X_{2,1} = 0.0972$$

$$X_{3,1} = -0.0833, \quad X_{4,1} = 0.0833$$

$$X_{5,1} = 0.0000$$

Obtain distribution factor of each branch:

$$D_{b,1} = 1.2755, \quad D_{b,1} = -1.2755$$

$$D_{b,1} = -25.5102, \quad D_{b,1} = 0.0000$$

Calculate the rest of the branches’ flow:

$$P_{11} = -2.1060, \quad P_{11} = 3.3660$$

$$P_{11} = 36.2980, \quad P_{11} = -2.5800$$

As can be seen from the results, the breaking of branch (1) resulted in other branches exceeding the transmission borders and breaking, so branch (1) is the brittle source.

The flowing table is the result of chain breaking calculated in the same way.
TABLE II. **CHAIN BREAKING RESULTS**

<table>
<thead>
<tr>
<th>Breaking branch NO.</th>
<th>Node of Beginning to end</th>
<th>Caused Broken branch NO.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1-2</td>
<td>□, □</td>
</tr>
<tr>
<td>□</td>
<td>1-3</td>
<td>□, □</td>
</tr>
<tr>
<td>□</td>
<td>2-3</td>
<td>□, □</td>
</tr>
<tr>
<td>□</td>
<td>2-4</td>
<td>□, □</td>
</tr>
<tr>
<td>□</td>
<td>3-5</td>
<td>□, □</td>
</tr>
</tbody>
</table>

We can conclude that the branches (1), (3) and (5) is the brittle source of this system through the analysis. Any of their failure can cause the brittle risk and make system collapse.

6. **Conclusion**

Any system with high degree of reliability also has the brittle risk by the brittleness theory, but it is limited by the excitation of the factors and conditions which can be excited. Therefore, as long as it can be monitored and controlled the brittle source of the system for real-time, we can eliminate the stimulate trend in the bud.

The combination of DC flow and distribution factor could identify the brittle source of system quickly and accurately in this paper and the method is simple and easy to implement. Finally, it is confirmed the usefulness and effectiveness of this method through the analysis of the IEEE-5 node system example.

**References**


