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A method for group decision-making based on determining weights of decision makers using TOPSIS

Zhongliang Yue

College of Science, Guangdong Ocean University, Zhanjiang 524088, China

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ABSTRACT

In general, weights of decision makers (DMs) play a very important role in multiple attribute group decision-making (MAGDM), how to measure the weights of DMs is an interesting research topic. This paper presents a new approach for determining weights of DMs in group decision environment based on an extended TOPSIS (Technique for Order Preference by Similarity to an Ideal Solution) method. We define the positive ideal solution as the average of group decision. The negative ideal solution includes two parts: left and right negative ideal solution, which are the minimum and maximum matrixes of group decision, respectively. We give an example to illustrate the developed approach. Finally, the advantages and disadvantages of this study are also compared.

1. Introduction

Multiple attribute decision-making (MADM) is an important part of modern decision science. It always involves multiple decision attributes and multiple decision alternatives. The purpose of the decision-making is finding the most desirable alternative(s) from a discrete set of feasible alternatives with respect to a finite set of attributes. It has been extensively applied to various areas such as society, economics, military, management, etc. [1–6], and has been receiving more and more attention over the last decades [7,8].

The increasing complexity of the socio-economic environment makes it less and less possible for single decision maker (DM) to consider all relevant aspects of a problem. As a result, many decision-making processes, in the real world, take place in group settings [9]. Moving from single DM's setting to group members' setting introduces a great deal of complexity into the analysis. For example, consider that these DMs usually come from different specialty fields, and thus each DM has his/her unique characteristics with regard to knowledge, skills, experience and personality, which implies that the DM usually has different influence in overall decision result. That is, the weights of DMs are different. Therefore, how to determine the weights of DMs will be an interesting and important research topic. At present, many methods have been proposed to determine the weights of DMs, for example, French [10] proposed a method to determine the relative importance of the groups members by using the influence relations, which may exist between the members. Theil [11] proposed a method based on the correlation concepts when the member's inefficacy is measurable. Keeney and Kirkwood [12], and Keeney [13] suggested the use of interpersonal comparisons to obtain the values of scaling constants in the weighted additive social choice function. Bodily [14] derived the member weight as a result of designation of voting weights from a member to a delegation subcommittee made up of other members of the group. By using the deviation measures between additive linguistic preference

E-mail address: zhongliangyue@gmail.com

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relations, Xu [15] gave a straightforward method to determine the weights of DMs by Bodily' method [14]. Mirkin and Fishburn [16] proposed two approaches which use the eigenvectors method to determine the relative importance of the group's members. Martel and Ben Khélifa [17] proposed a method to determine the relative importance of groups members by using individual outranking indexes. Van den Honert [18] used the REMBRANDT system (multiplicative AHP and associated SMART model) to quantify the decisional power vested in each member of a group, based on subjective assessments by the other group members. Jabeur and Martel [19] proposed a procedure which exploits the idea of Zeleny [3] to determine the relative importance coefficient of each member. Brock [20] used a Nash bargaining based approach to estimating the weights of group members intrinsically. Ramanathan and Ganesh [21] proposed a simple and intuitively appealing eigenvector based method to intrinsically determine the weights of group members using their own subjective opinions. Chen and Fan [22] proposed a factor score method for obtaining a ranking of the assessment levels of experts in group-decision analysis. Yue [23] developed a method for determining weights of DMs with interval numbers. In this article, we shall discuss the weights of DMs based on the Technique for Order Preference by Similarity to an Ideal Solution (TOPSIS).

TOPSIS, the well-known classical MADM methods, was first developed by Hwang and Yoon [24]. It helps DMs organizing the problems to be solved, and carry out analysis, comparing and rankings of the alternatives. Accordingly, the selection of a suitable alternative(s) will be made.

The basic idea of TOPSIS is rather straightforward. It simultaneously considers the distances to both positive ideal solution (PIS) and negative ideal solution (NIS), and a preference order is ranked according to their relative closeness, and a combination of these two distance measures [24–32]. That is, the best alternative has simultaneously the shortest distance from the PIS and the farthest distance from the NIS. The PIS is identified with a "hypothetical alternative" that has the best values for all considered attributes whereas the NIS is identified with a "hypothetical alternative" that has the worst attribute values.

The existing approaches have significant contributions to solving the weights of DMs problems. Most of the literature mentioned above described the individual decision information by a multiplicative preference matrix. Until now there has been little investigation of the weights of DMs based on individual decision information, in which the attribute values are given as observations in nonnegative real numbers, and the DMs have their subjective preferences on alternatives. The aim of this paper is to propose a novel approach to determining the weights of DMs. The extended TOPSIS technique is also called group TOPSIS in this article. For the given individual decision matrixes, the PIS of group opinion is depicted by a matrix, in which elements are expressed in average of all individual decisions. The NIS includes two parts: left and right negative ideal solutions, which are also depicted by a matrix, respectively. Specifically, for the normalized group decision matrixes, the left negative ideal solution (L-NIS) is the minimum matrix of group decision matrixes, the right negative ideal solution (R-NIS) is the maximum matrix of group decision matrixes, and both are expressed in maximum separation from the PIS.

The paper is organized as follows. In the next section, briefly introduces the traditional TOPSIS and multiple attribute group decision-making (MAGDM) method. In Section 3, we present an algorithm to determine weights of DMs based on an extended TOPSIS method. In Section 4, we illustrate our proposed algorithmic method with an example. In Section 5, we compare the proposed method with other methods. The final section concludes.

2. TOPSIS method and MAGDM problems

In this section, we review the TOPSIS method and MAGDM problems.

For convenience, we first let $M = \{1,2,\ldots,m\}$, $N = \{1,2,\ldots,n\}$ and $T = \{1,2,\ldots,t\}$; $i \in M$, $j \in N$, $k \in T$. Let $A = \{A_1,A_2,\ldots,A_m\}$ $(m \geqslant 2)$ be a discrete set of m feasible alternatives, $U = \{u_1,u_2,\ldots,u_n\}$ be a finite set of attributes, and $D = \{d_1,d_2,\ldots,d_t\}$ be a group of DMs, and $\lambda = (\lambda_1,\lambda_2,\ldots,\lambda_t)^T$ is the weight vector of DMs, where $\lambda_k \geqslant 0,\ \sum_{k=1}^{n}\lambda_k = 1$.

2.1. Representation of TOPSIS method

For a MADM problem, suppose each alternative is evaluated with respect to the n attributes, whose values constitute a decision matrix denoted by

$$X = (x_{ij})_{m\times n} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} u_1 & u_2 & \cdots & u_n \end{array} \\ \begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{pmatrix} \end{array}. \tag{1}$$

The TOPSIS method (see Fig. 1) consists of the following steps [32,33]:

1. Normalize the decision matrix.
   In general, there are benefit attributes and cost attributes in the MADM problems. In order to measure all attributes in dimensionless units and facilitate inter-attribute comparisons, we introduce the following formulas to normalize each attribute value x_{ij} in decision matrix $X = (x_{ij})_{m\times n}$ into a corresponding element r_{ij} in normalized decision matrix given by Eq. (2).

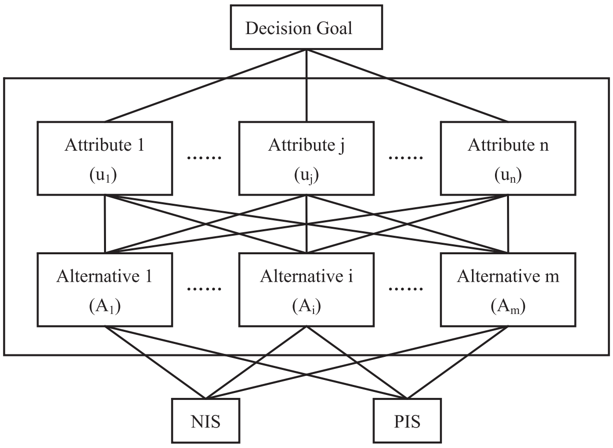

Fig. 1. Hierarchical structure of the traditional TOPSIS.

$$R = (r_{ij})_{m\times n} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} u_1 & u_2 & \cdots & u_n \end{array} \\ \begin{pmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ r_{m1} & r_{m2} & \cdots & r_{mn} \end{pmatrix} \end{array}, \tag{2}$$

where

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m}(x_{ij})^2}}, \quad \text{for benefit attribute } x_{ij},\ i \in M,\ j \in N \tag{3}$$

and

$$r_{ij} = 1 - \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m}(x_{ij})^2}}, \quad \text{for cost attribute } x_{ij},\ i \in M,\ j \in N. \tag{4}$$

2. Calculate the weighted normalized decision matrix. Suppose that $W = (w_1, w_2, \ldots, w_n)^T$ is the weight vector of the attributes, where $w_j \geqslant 0,\ \sum_{j=1}^{n} w_j = 1$, we can construct the weighted normalized decision matrix as

$$Y = (w_j r_{ij})_{m\times n} = (y_{ij})_{m\times n} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} u_1 & u_2 & \cdots & u_n \end{array} \\ \begin{pmatrix} y_{11} & y_{12} & \cdots & y_{1n} \\ y_{21} & y_{22} & \cdots & y_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ y_{m1} & y_{m2} & \cdots & y_{mn} \end{pmatrix} \end{array}. \tag{5}$$

3. Determine the positive and negative ideal solutions.
The PIS A^+ and NIS A^- are determined, respectively, as follows:

$$A^+ = \{y_1^+, y_2^+, \ldots, y_n^+\} \tag{6}$$

and

$$A^- = \{y_1^-, y_2^-, \ldots, y_n^-\}, \tag{7}$$

where $y_j^+ = \max\limits_{1\leqslant i\leqslant m}\{y_{ij}\}(j \in N)$ and $y_j^- = \min\limits_{1\leqslant i\leqslant m}\{y_{ij}\}(j \in N)$.

4. Measure the distance from positive and negative ideal solutions.
The separation of each alternative form the PIS, S_i^+, is given as

$$S_i^+ = \left(\sum_{j=1}^{n}(y_{ij} - y_j^+)^2\right)^{\frac{1}{2}}, \quad i \in M. \tag{8}$$

Similarly, the separation form the NIS, S_i^-, is given as

$$S_i^- = \left(\sum_{j=1}^{n}(y_{ij} - y_j^-)^2\right)^{\frac{1}{2}}, \quad i \in M. \tag{9}$$

5. Calculate the closeness coefficient to the ideal solutions.
   The closeness coefficient of the ith alternative A_i with respect to the ideal solutions is defined as

$$C_i = \frac{S_i^-}{S_i^+ + S_i^-}, \quad i \in M. \tag{10}$$

   Since $S_i^+ \geqslant 0\ (i \in M)$ and $S_i^- \geqslant 0\ (i \in M)$, then, clearly, $C_i \in [0,1]\ (i \in M)$.
6. Rank the preference order.
   A set of alternatives then can be ranked by preference according to the descending order of C_i; in other words, larger C_i means better alternative.

2.2. Representation of MAGDM problem

A MAGDM problem can be described as follows:
Let

$$X_k = (x_{ij}^{(k)})_{m\times n} = \begin{array}{c c} & \begin{array}{cccc} u_1 & u_2 & \cdots & u_n \end{array} \\ \begin{array}{c} A_1 \\ A_2 \\ \vdots \\ A_m \end{array} & \begin{pmatrix} x_{11}^{(k)} & x_{12}^{(k)} & \cdots & x_{1n}^{(k)} \\ x_{21}^{(k)} & x_{22}^{(k)} & \cdots & x_{2n}^{(k)} \\ \vdots & \vdots & \vdots & \vdots \\ x_{m1}^{(k)} & x_{m2}^{(k)} & \cdots & x_{mn}^{(k)} \end{pmatrix} \end{array}, \quad k \in T, \tag{11}$$

be decision matrix of kth DM. First of all, we normalize X_k into R_k in Eq. (12) by Eqs. (3) and/or (4).

$$R_k = (r_{ij}^{(k)})_{m\times n} = \begin{array}{c c} & \begin{array}{cccc} u_1 & u_2 & \cdots & u_n \end{array} \\ \begin{array}{c} A_1 \\ A_2 \\ \vdots \\ A_m \end{array} & \begin{pmatrix} r_{11}^{(k)} & r_{12}^{(k)} & \cdots & r_{1n}^{(k)} \\ r_{21}^{(k)} & r_{22}^{(k)} & \cdots & r_{2n}^{(k)} \\ \vdots & \vdots & \vdots & \vdots \\ r_{m1}^{(k)} & r_{m2}^{(k)} & \cdots & r_{mn}^{(k)} \end{pmatrix} \end{array}, \quad k \in T. \tag{12}$$

For a given weight vector $W = (w_1, w_2, \ldots, w_n)^T$ of the attributes, we can construct the weighted normalized decision matrix as

$$Y_k = (y_{ij}^{(k)})_{m\times n} = \left(w_j r_{ij}^{(k)}\right)_{m\times n} = \begin{array}{c c} & \begin{array}{cccc} u_1 & u_2 & \cdots & u_n \end{array} \\ \begin{array}{c} A_1 \\ A_2 \\ \vdots \\ A_m \end{array} & \begin{pmatrix} y_{11}^{(k)} & y_{12}^{(k)} & \cdots & y_{1n}^{(k)} \\ y_{21}^{(k)} & y_{22}^{(k)} & \cdots & y_{2n}^{(k)} \\ \vdots & \vdots & \vdots & \vdots \\ y_{m1}^{(k)} & y_{m2}^{(k)} & \cdots & y_{mn}^{(k)} \end{pmatrix} \end{array}, \ k \in T. \tag{13}$$

Then, we can obtain a group decision matrix $Y = (y_{ij})_{m\times n}$ by using following formula

$$Y = \sum_{k=1}^{t} \lambda_k Y_k = (y_{ij})_{m\times n}, \tag{14}$$

where $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_t)^T$ is the weight vector of DMs, where $\lambda_k \geqslant 0,\ \sum_{k=1}^{n}\lambda_k = 1$, and $y_{ij} = \sum_{k=1}^{t}\lambda_k y_{ij}^{(k)}$.
Further, by using the aggregation operator

$$y_i = \sum_{j=1}^{n} y_{ij}, \quad i \in M, \tag{15}$$

to aggregate all the elements in the ith row of $Y = (y_{ij})_{m\times n}$, and then get the overall attribute values $y_i (i \in M)$ of the alternatives $A_i (i \in M)$.

In view of this, the weight vector $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_t)^T$ of DMs plays a very important role in MAGDM. The question is how to determine the weight vector?

3. Determining weights of DMs by an extended TOPSIS method

Based on the analysis above, this section will present an approach to determining the weighs of DMs. Firstly, we will explain why such a method is proposed. Then, an extended TOPSIS method is established. Finally, an algorithm for determining the weighs of DMs is given.

3.1. Idea of the developed approach

The traditional TOPSIS method introduces two "reference" points: PIS and NIS in order to ranking of alternatives (see Fig. 1). Moving from single DM's setting to multiple DMs' setting, a key issue is determination of "reference" points (or a benchmark) of all individual decision matrixes in order to comparing the decision levels among DMs. We propose the average matrix of group decision as the PIS of group TOPSIS. The reasons are that: (1) the PIS is the maximum compromise (in mean sense) among all individual decisions of group; (2) the PIS is adopted as the final decision (outcome) of group in most of the situations where a group decision must be taken. For example, for a teaching competition participated in by young teachers in a university, if there are t DMs, the final score of each competitor is the average of t scores given by the DMs; and (3) the NIS is the maximum individual regret (the farthest distance from PIS) for some DMs.

TOPSIS method is suitable for cautious (risk avoider) DM(s), because the DM(s) might like to have a decision which not only makes as much profit as possible, but also avoids as much risk as possible [29]. The developed approach in this paper assigns high weights to those DMs if the DMs want to have maximum group utility (majority/group), and minimum individual risk (minority/individual) in mean sense.

In order to realize the idea above, in the following, we will establish an extended TOPSIS model in a group decision environment.

3.2. An extended TOPSIS method

Let $Y_k = (y_{ij}^{(k)})_{m\times n}$ be weighted normalized decision matrix of kth $(k \in T)$ DM. As described in above section, in mean sense, the best result of group decision making should be the average matrix of group decision:

$$Y^* = (y_{ij}^*)_{m\times n} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} u_1 & u_2 & \cdots & u_n \end{array} \\ \begin{pmatrix} y_{11}^* & y_{12}^* & \cdots & y_{1n}^* \\ y_{21}^* & y_{22}^* & \cdots & y_{2n}^* \\ \vdots & \vdots & \vdots & \vdots \\ y_{m1}^* & y_{m2}^* & \cdots & y_{mn}^* \end{pmatrix} \end{array}, \tag{16}$$

where $Y^* = \frac{1}{t}\sum_{k=1}^{t} Y_k$, and $y_{ij}^* = \frac{1}{t}\sum_{k=1}^{t} y_{ij}^{(k)}$ $(i \in M, j \in N)$.

In other words, a DM is higher decision level because his/her opinion is closer to average. So we define $Y^* = (y_{ij}^*)_{m\times n}$ as the PIS of MAGDM. For the decision matrix Y_k of kth DM, the closer the average matrix Y^*, the more the weight of kth DM.

In order to measure decision level of each DM, we can calculate the distance between each individual decision matrix $Y_k (k \in T)$ and average matrix Y^*. Consider that the Euclidean distance is the most widely used tool to measure the separation of two objects in practical applications, we utilize it to measure the separation between Y_k and Y^*.

The separation of each individual decision Y_k form the Y^*, S_k^+, is given as:

$$S_k^+ = \|Y_k - Y^*\| = \left(\sum_{i=1}^{m}\sum_{j=1}^{n}(y_{ij}^{(k)} - y_{ij}^*)^2\right)^{\frac{1}{2}}, \quad k \in T. \tag{17}$$

In this sense, the smaller distance S_k^+, the better decision Y_k of kth DM.

Considering that, in mean sense, for a DM, the maximum risk is the maximum separation from the average matrix of group decision. And the average matrix of group decision is the distributing center of all matrixes of group decision, for this reason, we define following left and right maximum separation from the average matrix of group decision. That is, we divide the NISs into two parts: L-NIS Y_l^- and R-NIS Y_r^-, respectively, as follows:

$$Y_l^- = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} u_1 & u_2 & \cdots & u_n \end{array} \\ \begin{pmatrix} y_{11}^l & y_{12}^l & \cdots & y_{1n}^l \\ y_{21}^l & y_{22}^l & \cdots & y_{2n}^l \\ \vdots & \vdots & \vdots & \vdots \\ y_{m1}^l & y_{m2}^l & \cdots & y_{mn}^l \end{pmatrix} \end{array} \tag{18}$$

and

$$Y_r^- = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} u_1 & u_2 & \cdots & u_n \end{array} \\ \begin{pmatrix} y_{11}^r & y_{12}^r & \cdots & y_{1n}^r \\ y_{21}^r & y_{22}^r & \cdots & y_{2n}^r \\ \vdots & \vdots & \vdots & \vdots \\ y_{m1}^r & y_{m2}^r & \cdots & y_{mn}^r \end{pmatrix} \end{array}, \tag{19}$$

where $y_{ij}^l = \min\limits_{1 \leqslant k \leqslant t} \left\{ y_{ij}^{(k)} | y_{ij}^{(k)} \leqslant y_{ij}^* \right\}$ and $y_{ij}^r = \max\limits_{1 \leqslant k \leqslant t} \left\{ y_{ij}^{(k)} | y_{ij}^{(k)} \geqslant y_{ij}^* \right\}$.

In fact, Y_l^- and Y_r^- are the minimum and maximum matrixes of group decision, respectively (see Fig. 2).

Similarly, the separations of each individual decision form the NISs, S_k^{l-} and S_k^{r-}, are given respectively as

$$S_k^{l-} = \|Y_k - Y_l^-\| = \left(\sum_{i=1}^m \sum_{j=1}^n \left(y_{ij}^{(k)} - y_{ij}^l \right)^2 \right)^{\frac{1}{2}}, \quad k \in T \tag{20}$$

and

$$S_k^{r-} = \|Y_k - Y_r^-\| = \left(\sum_{i=1}^m \sum_{j=1}^n \left(y_{ij}^{(k)} - y_{ij}^r \right)^2 \right)^{\frac{1}{2}}, \quad k \in T. \tag{21}$$

Clearly, the larger the separations S_k^{l-} and S_k^{r-}, the better the decision Y_k of kth DM. Therefore, similar to (10), a closeness coefficient is defined to determine the ranking order of all DMs once the S_k^+, S_k^{l-} and S_k^{r-} have been calculated. The closeness coefficient of the kth DM (weighted normalized decision matrix Y_k) with respect to Y^* is defined as:

$$C_k = \frac{S_k^{l-} + S_k^{r-}}{S_k^+ + S_k^{l-} + S_k^{r-}}, \quad k \in T. \tag{22}$$

Since $S_k^{l-} \geqslant 0,\ S_k^{r-} \geqslant 0$ and $S_k^+ \geqslant 0\ (k \in T)$, then, clearly, $C_k \in [0, 1]\ (k \in T)$.

Obviously, an individual decision matrix Y_k is closer to the Y^* and farther from Y_k^{l-} as well as Y_k^{r-}, and as C_k approaches to 1. Therefore, according to the closeness coefficient, we can determine the order of all DMs.

In order to get the weight vector of DMs from the closeness coefficient, we can make the following transformation

$$\lambda_k = \frac{C_k}{\sum_{k=1}^t C_k}, \quad k \in T, \tag{23}$$

such that $\lambda_k \geqslant 0,\ \sum_{k=1}^t \lambda_k = 1$.

3.3. The presented algorithm

In sum, an algorithm to determine the weight vector of DMs, based TOPSIS approach, can be shown as the following six steps.

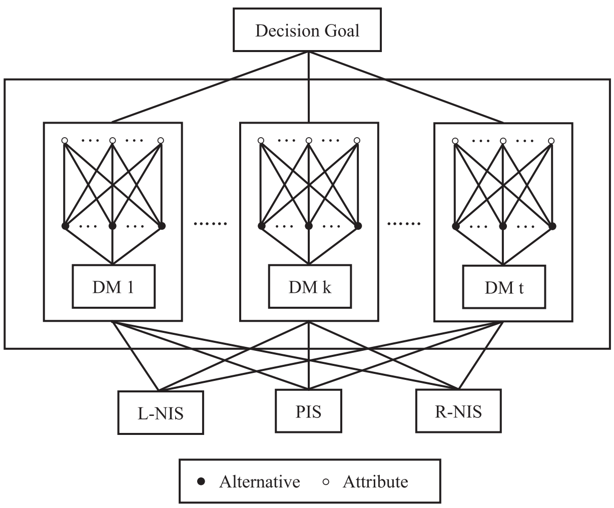

Fig. 2. Hierarchical structure of the extended TOPSIS.

Step 1: Normalize the decision matrices X_k $(k \in T)$ in Eq. (11) into R_k $(k \in T)$ in Eq. (12) by Eqs. (3) and/or (4), where $X_k(k \in T)$ is decision matrix of the kth $(k \in T)$ DM.
Step 2: Calculate the weighted normalized decision matrices $Y_k(k \in T)$ by Eq. (13).
Step 3: Determine the PIS Y^*, L-NIS Y_l^- and R-NIS Y_r^- for all individual decisions by Eqs. (16), (18) and (19), respectively.
Step 4: Calculate the separation measures from the PIS, L-NIS and R-NIS, S_k^+, S_k^{l-} and S_k^{r-}, by Eq. (17), Eqs. (20) and (21), respectively.
Step 5: Calculate the closeness coefficient C_k to the ideal solutions by Eq. (22).
Step 6: Determine weight vector $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_t)^T$ of DMs by Eq. (23).

4. Illustrative example

In the following, an instance (adapted from Shih et al. [34]) is provided to illustrate the proposed approach.

Example. A human resources selection example.

A local chemical company tries to recruit an on-line manager. The company's human resources department provides some relevant selection tests as the benefit attributes to be evaluated. These objective test include knowledge tests (language test, professional test and safety rule test), skill tests (professional skills and computer skills). After these objective tests, there are 17 qualified candidates (as alternatives marked by $A_1, A_2, \ldots, A_{17}$, or briefly marked by $1, 2, \ldots, 17$) on the list for the selection. Then four DMs (marked by d_1, d_2, d_3, d_4) are responsible for the selection from among them based on subjective tests. The basic data of subjective attributes, including panel interview and 1-on-1 interview tests (only quantitative information here) for the decision, are listed in Table 1.

Following the suggested steps, each DM will construct a normalized decision matrix. Since all listed attributes are benefit attributes, by Eq. (3), we first normalize Table 1 into Table 2 according to Step 1. Table 2 includes 4 normalized decision matrixes.

In addition, the weights of attributes, elicited by DMs, are shown in Table 3.

By Step 2, the each column/attribute vector in Table 2 is respectively multiplied by the associated weight given by DM in Table 3. Then, the weighted normalized decision results are obtained which are shown in Table 4.

The ideal solutions Y^*, Y_l^- and Y_r^-, by Step 3, are summarized in Table 5.

By Step 4, We can respectively calculate the separations of each weighted normalized decision matrix from these ideal solutions, which are summarized in Table 6.

Further, we can respectively calculate the closeness coefficients by Step 5, the weight vector of DMs by Step 6, and DMs' ranking, which are organized in Table 6. The final DMs' priority ranking produced by the extended TOPSIS in this paper is as

$$d_2 \succ d_4 \succ d_3 \succ d_1.$$

The 6th column of Table 6 has illustrated that the vector $(0.2350, 0.2601, 0.2485, 0.2564)^T$ is weight vector of DMs. We utilize the Eq. (14) to aggregate all the individual decisions in Table 4 into the collective decisions in the columns 2 and 3 of Table 7. Then, summing all elements in each line of columns 2 and 3 of Table 7, the integrated assessment of 17 candidates are obtained as illustrated in column 4 of Table 7. The ranking of 17 candidates are also shown in last column of Table 7. We can see that the 16th candidate is ranked first, and the 12th candidate is ranked last.

Table 1
Decision matrixes of example–subjective attributes.

No. of candidates	X_1		X_2		X_3		X_4	
	Panel interview	1-on-1 interview	Panel interview	1-on-1 interview	Panel interview	1-on-1 interview	Panel interview	1-on-1 interview
1	80	75	85	80	75	70	90	85
2	65	75	60	70	70	77	60	70
3	90	85	80	85	80	90	90	95
4	65	70	55	60	68	72	62	72
5	75	80	75	80	50	55	70	75
6	80	80	75	85	77	82	75	75
7	65	70	70	60	65	72	67	75
8	70	60	75	65	75	67	82	85
9	80	85	95	85	90	85	90	92
10	70	75	75	80	68	78	65	70
11	50	60	62	65	60	65	65	70
12	60	65	65	75	50	60	45	50
13	75	75	80	80	65	75	70	75
14	80	70	75	72	80	70	75	75
15	70	65	75	70	65	70	60	65
16	90	95	92	90	85	80	88	90
17	80	85	70	75	75	80	70	75

Note: (1) There are four DMs selected for the evaluation. (2) There are a total of 17 candidates for evaluation. (3) All listed attributes are benefit attributes.

Table 2
Normalized decision matrixes.

No.	R_1		R_2		R_3		R_4	
	Panel interview	1-on-1 interview	Panel interview	1-on-1 interview	Panel interview	1-on-1 interview	Panel interview	1-on-1 interview
1	0.2624	0.2416	0.2747	0.2565	0.2552	0.2297	0.2988	0.2683
2	0.2132	0.2416	0.1939	0.2245	0.2382	0.2526	0.1992	0.2209
3	0.2952	0.2738	0.2585	0.2726	0.2722	0.2953	0.2988	0.2998
4	0.2132	0.2255	0.1777	0.1924	0.2314	0.2362	0.2058	0.2272
5	0.2460	0.2577	0.2424	0.2565	0.1702	0.1805	0.2324	0.2367
6	0.2624	0.2577	0.2424	0.2726	0.2620	0.2690	0.2490	0.2367
7	0.2132	0.2255	0.2262	0.1924	0.2212	0.2362	0.2224	0.2367
8	0.2296	0.1933	0.2424	0.2084	0.2552	0.2198	0.2722	0.2683
9	0.2624	0.2738	0.3070	0.2726	0.3063	0.2789	0.2988	0.2904
10	0.2296	0.2416	0.2424	0.2565	0.2314	0.2559	0.2158	0.2209
11	0.2296	0.2416	0.2004	0.2084	0.2042	0.2133	0.2158	0.2209
12	0.1968	0.2094	0.2101	0.2405	0.1702	0.1969	0.1494	0.1578
13	0.2460	0.2416	0.2585	0.2565	0.2212	0.2461	0.2324	0.2367
14	0.2624	0.2255	0.2424	0.2309	0.2722	0.2297	0.2490	0.2367
15	0.2296	0.2094	0.2424	0.2245	0.2212	0.2297	0.1992	0.2051
16	0.2952	0.3061	0.2973	0.2886	0.2893	0.2625	0.2922	0.2840
17	0.2624	0.2738	0.2262	0.2405	0.2552	0.2625	0.2324	0.2367

Note: (1) There are four DMs selected for the evaluation. (2) There are a total of 17 candidates for evaluation.

Table 3
Weights on attributes of example.

No.	Attributes	The weights of the group			
		d_1	d_2	d_3	d_4
1	Panel interview	0.5243	0.4574	0.4160	0.4503
2	1-on-1 interview	0.4757	0.5426	0.5840	0.5497

Note: There are four DMs selected for the evaluation.

Table 4
Weighted normalized decision matrixes.

No.	Y_1		Y_2		Y_3		Y_4	
	Panel interview	1-on-1 interview	Panel interview	1-on-1 interview	Panel interview	1-on-1 interview	Panel interview	1-on-1 interview
1	0.1376	0.1149	0.1256	0.1392	0.1062	0.1341	0.1345	0.1475
2	0.1118	0.1149	0.0887	0.1218	0.0991	0.1475	0.0897	0.1214
3	0.1548	0.1303	0.1182	0.1479	0.1133	0.1724	0.1345	0.1648
4	0.1118	0.1073	0.0813	0.1044	0.0963	0.1380	0.0927	0.1249
5	0.1290	0.1226	0.1109	0.1392	0.0708	0.1054	0.1046	0.1301
6	0.1376	0.1226	0.1109	0.1479	0.1090	0.1571	0.1121	0.1301
7	0.1118	0.1073	0.1035	0.1044	0.0920	0.1380	0.1002	0.1301
8	0.1204	0.0920	0.1109	0.1131	0.1062	0.1284	0.1226	0.1475
9	0.1376	0.1303	0.1404	0.1479	0.1274	0.1629	0.1345	0.1596
10	0.1204	0.1149	0.1109	0.1392	0.0963	0.1495	0.0972	0.1214
11	0.0860	0.0920	0.0916	0.1131	0.0849	0.1245	0.0972	0.1214
12	0.1032	0.0996	0.0961	0.1305	0.0708	0.1150	0.0673	0.0867
13	0.1290	0.1149	0.1182	0.1392	0.0920	0.1437	0.1046	0.1301
14	0.1376	0.1073	0.1109	0.1253	0.1133	0.1341	0.1121	0.1301
15	0.1204	0.0996	0.1109	0.1218	0.0920	0.1341	0.0897	0.1128
16	0.1548	0.1456	0.1360	0.1566	0.1203	0.1533	0.1316	0.1561
17	0.1376	0.1303	0.1035	0.1305	0.1062	0.1533	0.1046	0.1301

Note: (1) There are four DMs selected for the evaluation. (2) There are a total of 17 candidates for evaluation.

5. Comparisons the extended TOPSIS with other methods

In this section we compare the extended TOPSIS method with other methods. The methods to be compared here are the traditional TOPSIS method [24] and the another extended TOPSIS method proposed by Shih et al. [34].

Table 5
Ideal solutions.

No.	PIS Y^*		L-NIS Y_l^-		R-NIS Y_r^-	
	Panel interview	1-on-1 interview	Panel interview	1-on-1 interview	Panel interview	1-on-1 interview
1	0.1260	0.1339	0.1062	0.1149	0.1376	0.1475
2	0.0973	0.1264	0.0887	0.1149	0.1118	0.1475
3	0.1302	0.1539	0.1133	0.1303	0.1548	0.1724
4	0.0955	0.1186	0.0813	0.1044	0.1118	0.1380
5	0.1038	0.1243	0.0708	0.1054	0.1290	0.1392
6	0.1174	0.1394	0.1090	0.1226	0.1376	0.1571
7	0.1019	0.1199	0.0920	0.1044	0.1118	0.1380
8	0.1150	0.1202	0.1062	0.0920	0.1226	0.1475
9	0.1350	0.1502	0.1274	0.1303	0.1404	0.1629
10	0.1062	0.1313	0.0963	0.1149	0.1204	0.1495
11	0.0899	0.1128	0.0849	0.0920	0.0972	0.1245
12	0.0843	0.1080	0.0673	0.0867	0.1032	0.1305
13	0.1110	0.1320	0.0920	0.1149	0.1290	0.1437
14	0.1185	0.1242	0.1109	0.1073	0.1376	0.1341
15	0.1032	0.1171	0.0897	0.0996	0.1204	0.1341
16	0.1357	0.1529	0.1203	0.1456	0.1548	0.1566
17	0.1130	0.1360	0.1035	0.1301	0.1376	0.1533

Note: There are a total of 17 candidates for evaluation.

Table 6
Separations, closeness coefficients, weights and ranking.

DMs	S_k^+	S_k^{l-}	S_k^{r-}	C_k	λ_k	Ranking
d_1	0.0902	0.1284	0.1342	0.7444	0.2350	4
d_2	0.0460	0.1092	0.1065	0.8242	0.2601	1
d_3	0.0675	0.1246	0.1252	0.7874	0.2485	3
d_4	0.0523	0.1116	0.1150	0.8125	0.2564	2

Table 7
Integrated assessment of 17 candidates.

No. of candidates	Panel interview	1-on-1 interview	Sum	Ranking
1	0.1259	0.1344	0.2603	4
2	0.0970	0.1265	0.2235	12
3	0.1298	0.1542	0.2840	3
4	0.0951	0.1187	0.2138	15
5	0.1036	0.1246	0.2281	11
6	0.1170	0.1397	0.2567	5
7	0.1017	0.1200	0.2217	13
8	0.1149	0.1207	0.2357	10
9	0.1350	0.1505	0.2855	2
10	0.1060	0.1315	0.2375	9
11	0.0901	0.1131	0.2032	14
12(#)	0.0841	0.1082	<u>0.1923</u>	17
13	0.1108	0.1323	0.2431	7
14	0.1181	0.1245	0.2426	8
15	0.1030	0.1173	0.2203	16
16(*)	0.1354	0.1531	<u>0.2885</u>	1
17	0.1125	0.1360	0.2485	6

Note: (1) There are a total of 17 candidates for evaluation. (2) "*" and "#" mark the first and the last candidate, respectively.

Each of methods has its advantages and disadvantages and none of them can always perform better than the others in any situations. It all depends on how we look at things, and not on how they are themselves.

The traditional TOPSIS has solved a MADM problem with just one DM; whereas the extended TOPSIS technique in this paper has solved a MADM problem with multiple DMs. The PIS/NIS in the traditional TOPSIS, as a benchmark of all alternatives (vectors), is expressed by a vector; whereas the PIS and NISs of the extended TOPSIS technique in this paper, as a benchmark of all decision matrixes of DMs, are expressed by three matrixes which are PIS, L-NIS and R-NIS.

As described in the above section, the traditional TOPSIS method is suitable for cautious (risk avoider) DM(s), because the DM(s) might like to have a decision which not only makes as much profit as possible, but also avoids as much risk as possible

Table 8
Comparison with the traditional TOPSIS.

Characteristics	Traditional TOPSIS	Extended TOPSIS
Evaluation objective	Selection and ranking of a number of alternatives	Selection and ranking of a number of DMs
No. of DMs	One	More than one
Weights on attributes	Given	Given
Cardinal information	Alternatives with respect to attributes	Alternatives with respect to attributes of multiple DMs
PIS	The best alternative expressed by a vector	The best decision expressed by a matrix
NIS	The worst alternative expressed by a vector	The worst decisions expressed by L-NIS and R-NIS matrixes
Core process	The distances from each alternative to PIS and NIS (between two vectors)	The distances from each decision to PIS, L-NIS and R-NIS (between two matrixes)

Table 9
Comparison with the method proposed by Shih et al.

Characteristics	Proposed by Shih et al.	Proposed by this paper
Decision information	Decision matrixes $X_1, X_2, \ldots, X_t$ of alternatives with respect to attributes	Decision matrixes $X_1, X_2, \ldots, X_t$ of alternatives with respect to attributes
No. of DMs	$t > 1$	$t > 1$
Weights on attributes	Subjective assessments given by DMs	Given
PIS	The best alternative expressed by a vector	The best decision expressed by a matrix
NIS	The worst alternative expressed by a vector	The worst decision expressed by L-NIS and R-NIS matrixes
Core process	The separations (s_i^- and s_i^+) from each alternative to PIS and NIS (between two vectors)	The separations (S_k^+, S_k^{l-} and S_k^{r-}) from each decision to PIS, L-NIS and R-NIS (between two matrixes)
Relative closeness	$\frac{s_i^-}{s_i^+ + s_i^-}$, for alternative A_i	$\frac{S_k^{l-} + S_k^{r-}}{S_k^+ + S_k^{l-} + S_k^{r-}}$, for DM d_k
Weights on DMs	Same	Different
Key decision	Ranking of a number of alternatives	Ranking of a number of DMs
Final decision	Ranking of a number of alternatives	Ranking of a number of alternatives

[29]. In this sense, the distance measure between an alternative (vector) and PIS reflects the alternative's preference in the traditional TOPSIS, i.e., the smaller the distance measure from PIS, the more the alternative's preference; whereas the distance measure between an individual decision matrix and PIS reflects the DM's profit in this extended TOPSIS technique, i.e., the smaller the distance measure from PIS, the more the DM's profit. On the contrary, the distance measure between an alternative (vector) and NIS reflects the alternative's preference to avoid risk in the traditional TOPSIS, i.e., the larger the distance measure, the more the alternative's preference to avoid risk; whereas the distance measures between an individual decision matrix and NISs, including L-NIS and R-NIS, reflects the DM's decision level to avoid risk in this extended TOPSIS technique, i.e., the larger the distance measure from NIS(s), the higher the DM's decision level to avoid risk, or the smaller the DM's individual regret (the smaller distance from PIS). The above mentioned merits and demerits of this extended TOPSIS method are briefly shown in Table 8.

Another remarkable TOPSIS method is proposed by Shih et al. [34], which is a good MADM technique in a group decision environment.

In the method proposed by Shih et al., the ideal solutions are generated by individual decision matrix, so that the information of the ideal solutions is dispersive. In this paper, the ideal solutions are generated by group decision (all decision matrixes of group), so that the information of the ideal solutions is a whole (divides into three matrixes: PIS, L-NIS and R-NIS) and reflects a group effect.

The DMs' weights are same in the method proposed by Shih et al., whereas the DMs' weights of the proposed method in this paper are different, which are generated from the individual decision matrix. In general, the information of weight from the data (measured values) is more suitable than a priori or same. Additionally, each individual decision matrix provides not only the information of alternatives in group but also the information of individual weight (decision level).

As far as the extended TOPSIS in this paper is concerned, its biggest advantage is that ideal solutions are macroscopic and high-dimension, so it is a clear, convenient and practical approach for dealing with group decision problems.

Others of relative comparison with the method proposed by Shih et al. are shown in Table 9.

6. Conclusions

Many practical problems are often characterized by MAGDM. Evaluating decisional level of DM is important research topic in group decision making. In this paper, we have developed an approach for determining weights of DMs in a group decision environment based on the extended TOPSIS method. Compared to the existing MADM approaches, the method proposed in this paper has certain distinguishing characteristics. The developed algorithm is applicable not only ranking DMs, but also aggregating individual decision into a group decision, then ranking alternatives according to the group decision.

However, it should be made clear that the use of the proposed method is limited by the requirement that the attribute data is in the form of crisp numbers. The proposed method should be extended to support situations in which the information is in other forms, e.g., linguistic variables or fuzzy numbers.

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