A Toolkit for Generating and Displaying Proof Scores in the OTS/CafeOBJ Method*

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Abstract

The OTS/CafeOBJ method can be used to model, specify and verify distributed systems. Specifications are written in equations, which are regarded as rewrite rules and used to verify specifications. The usefulness of the method is demonstrated by applying the method to nontrivial problems such as electronic commerce protocols and railroad signaling systems. In this paper we describe a toolkit called Buffet, which assists verification in the method. Given predicates used to split cases and lemmas, Buffet automatically generates proofs (called proof scores) and checks the proof scores using the CafeOBJ system. Buffet also has facilities to display proof scores generated and verification results on a web browser.

Keywords: Algebraic specification, CafeOBJ, observational transition system, proof scores, verification.

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1 Introduction

Abstract machines as well as abstract data types can be specified in CafeOBJ\cite{4}, an algebraic specification language. Algebraic specifications of abstract machines are called behavioral specifications. Behavioral specifications are written in equations, which are regarded as rewrite rules and used to verify behavioral specifications. Rewriting is an efficient way of implementing equational reasoning, which is the most fundamental way of reasoning and can moderate the difficulties of proofs that might otherwise become too hard to understand.

We use observational transition systems (OTSs; which are transition systems that can be straightforwardly written in equations) as abstract machines and have been developing a method of verifying behavioral specifications. The method is called the OTS/CafeOBJ method\cite{15}. In the OTS/CafeOBJ method, a system is modeled as an OTS, the OTS is written in CafeOBJ and it is verified that the OTS has properties by writing proofs (called proof scores) in CafeOBJ and checking the proof scores by means of rewriting. We have been demonstrating its usefulness by doing case studies, among which are \cite{13,14,17}. In the case studies, however, basically proof scores were entirely written by hand using usual text editors such as Emacs, which is subject to human errors such that some cases to consider may be overlooked.

We have then designed and implemented a toolkit called Buffet, which assists verification in the OTS/CafeOBJ method. Given predicates used to split cases and lemmas, Buffet automatically generates proof scores and checks the proof scores using the CafeOBJ system. Although the success of a proof depends on given predicates for case analysis and lemmas, it is guaranteed that generated proof scores cover all cases, excluding human errors. Buffet also has facilities to display proof scores generated and verification results on a web browser. Since Buffet only displays by default parts of a proof score hierarchically for which further case analysis should be done and/or lemmas should be used, the facilities can help users find how to split cases and what lemmas to use. The facilities can also help users read and understand proof scores. In this paper we describe Buffet and report on a case study that Buffet has been applied to a simple mutual exclusion protocol.

2 Preliminaries

We assume that there exists a universal state space denoted by $\Upsilon$ and data types used, including the equivalence relation denoted by $=$ for each data type, have been defined. An OTS\cite{15} $S$ consists of $\langle O, I, T \rangle$ such that 1) $O$:
a set of observers; each \( o \in O \) is a function \( o : \Upsilon \rightarrow D \), where \( D \) is a data type and may differ from observer to observer; given two states \( v_1, v_2 \in \Upsilon \), the equivalence \( (v_1 =_S v_2) \) between them wrt \( S \) is defined as \( \forall o \in O. o(v_1) = o(v_2) \),

2) \( \mathcal{I} \): the set of initial states such that \( \mathcal{I} \subseteq \Upsilon \), and

3) \( \mathcal{T} \): a set of conditional transitions; each \( \tau \in \mathcal{T} \) is a function \( \tau : \Upsilon \rightarrow \Upsilon \) such that \( \tau(v_1) =_S \tau(v_2) \) for each \( [v] \in \Upsilon /=_S \) and each \( v_1, v_2 \in [v] \); \( \tau(v) \) is called the successor state of \( v \in \Upsilon \) wrt \( \tau \); the condition \( c_\tau \) of \( \tau \) is called the effective condition.

An execution of \( S \) is an infinite sequence \( v_0, v_1, \ldots \) of states satisfying \( \text{Initiation} \ (v_0 \in \mathcal{I}) \) and \( \text{Consecution} \ (\forall i \in \{0, 1, \ldots \}. \exists \tau \in \mathcal{T}. (v_{i+1} =_S \tau(v_i))) \). A state \( v \) is called reachable wrt \( S \) iff there exists an execution of \( S \) in which \( v \) appears.

Properties discussed in this paper are invariants only. A predicate \( p \) is called invariant wrt \( S \) iff \( p(v) \) holds for every reachable state \( v \) wrt \( S \). Observers and transitions may be parameterized, which are generally expressed as \( o_{i_1}, \ldots, i_m : \Upsilon \rightarrow D_k \) and \( \tau_{j_1}, \ldots, j_n : \Upsilon \rightarrow \Upsilon \), provided that \( m, n \geq 0 \) and there exists a data type \( D_k \) such that \( k \in D_k \) for \( k = i_1, \ldots, i_m, j_1, \ldots, j_n \).

CafeOBJ[4] (see www.ldl.jaist.ac.jp/cafeobj/) is an algebraic specification language/system mainly based on order-sorted algebras[6] and hidden-sorted algebras[5,9]. Abstract machines as well as abstract data types can be specified in CafeOBJ, which has two kinds of sorts: visible and hidden sorts denoting abstract data types and the state spaces of abstract machines, and two kinds of operators wrt hidden sorts: action and observation operators that denote state transitions of abstract machines and let us know the state of abstract machines. Both an action operator and an observation operator take a state of an abstract machine and zero or more data, an action operator returns the successor state and an observation operator returns a value that characterizes the state of an abstract machine. The syntax of operator declarations is

\[
[b]\text{op} \ OpName : \text{Sort}^* \rightarrow \text{Sort}
\]

\( b \)op is used for action and observation operators, while \( \text{op} \) for others. Operators are defined with equations. The syntax of equation declarations is

\[
[c]\text{eq} \ Term = Term \ [\text{if} \ Term] .
\]

c\text{eq} is used for conditional equations, while \( \text{eq} \) for non-conditional ones. The CafeOBJ system uses equations as rewrite rules and rewrites terms. CafeOBJ is also based on rewriting logic. The syntax of rewriting rules is

\[
\text{trans} \ Term \Rightarrow Term .
\]

In Buffet, rewriting rules are used to instruct Buffet to generate proof scores.

Basic units of CafeOBJ specifications are modules. The CafeOBJ system provides built-in modules where basic data types such as truth values are specified. The module of truth values is \( \text{BOOL} \). Since truth values are indispensable for conditional equations, \( \text{BOOL} \) is automatically imported by almost
every module unless otherwise stated. The import of \texttt{BOOL} lets us use visible sort \texttt{Bool} denoting truth values, constants \texttt{true} and \texttt{false} denoting true and false, and operators denoting some basic logical operators. Among the operators are \texttt{not}, \texttt{and}, \texttt{or}, \texttt{xor}, \texttt{implies} and \texttt{iff} denoting negation (\(\neg\)), conjunction (\(\land\)), disjunction (\(\lor\)), exclusive disjunction (\(\oplus\)), implication (\(\Rightarrow\)) and logical equivalence (\(\Leftrightarrow\)), respectively. The conditional choice operator \texttt{if.then_else.fi} is also available. An underscore _ indicates the place where an argument is put. \texttt{BOOL} plays an essential role in verification with the CafeOBJ system. If the equations available in the module are regarded as rewrite rules, they are complete \textit{wrt} propositional logic. Therefore, any term denoting a propositional formula that is always true (or false) surely reduces to \texttt{true} (or \texttt{false}). Generally, a term of \texttt{Bool} reduces to an exclusive disjunction of conjunctions.

\(S\) is written in CafeOBJ. \(\Upsilon\) is denoted by a hidden sort, say \(H\), \(o_{d_1,\ldots,d_m}\) by a CafeOBJ observation operator, say \(o\), and \(\tau_{d_1,\ldots,d_j}\) by a CafeOBJ action operator, say \(\tau\); \(o\) and \(\tau\) are declared as

\[
\text{bop } o : H V_{i_1} \ldots V_{i_m} \rightarrow V_i \quad \text{bop } \tau : H V_{j_1} \ldots V_{j_n} \rightarrow H
\]

\(V_k\) is a visible sort corresponding to \(D_k\) for \(k = i_1,\ldots,i_m, j_1,\ldots,j_n\). Any state in \(I\), i.e. any initial state, is denoted by a constant, say \texttt{init} declared as

\[
\text{op } \text{init} : \rightarrow H
\]

We suppose that the initial value of each \(o_{i_1,\ldots,i_m}\) is \(f(i_1,\ldots,i_m)\). The initial value of each \(o_{i_1,\ldots,i_m}\) is specified with the equation

\[
\text{eq } o(\texttt{init},X_{i_1},\ldots,X_{i_m}) = f(X_{i_1},\ldots,X_{i_m}).
\]

\(X_k\) is a CafeOBJ variable of sort \(V_k\) for \(k = i_1,\ldots,i_m\) and \(f(X_{i_1},\ldots,X_{i_m})\) is a CafeOBJ term denoting \(f(i_1,\ldots,i_m)\). Each \(\tau_{j_1,\ldots,j_n}\) may change the value of each \(o_{i_1,\ldots,i_m}\) if it is applied in a state \(v\) such that \(c_{\tau_{j_1,\ldots,j_n}}\) holds, which can be written as

\[
\text{ceq } o(a(S,X_{j_1},\ldots,X_{j_n}),X_{i_1},\ldots,X_{i_m}) = e-a(S,X_{j_1},\ldots,X_{j_n},X_{i_1},\ldots,X_{i_m})
\]

if \(c-a(S,X_{j_1},\ldots,X_{j_n})\).

\(S\) is a CafeOBJ variable of \(H\) and each \(X_k\) is a CafeOBJ variable of \(V_k\). \(a(S,X_{j_1},\ldots,X_{j_n})\) denotes the successor state of \(S\) \textit{wrt} \(\tau_{j_1,\ldots,j_n}\), \(e-a(S,X_{j_1},\ldots,X_{j_n},X_{i_1},\ldots,X_{i_m})\) denotes the value of \(o_{i_1,\ldots,i_m}\) in the successor state. \(c-a(S,X_{j_1},\ldots,X_{j_n})\) denotes \(c_{\tau_{j_1,\ldots,j_n}}\). \(\tau_{j_1,\ldots,j_n}\) changes nothing if it is applied in a state \(v\) such that \(c_{\tau_{j_1,\ldots,j_n}}\) does not hold, which can be written as

\[
\text{ceq } a(S,X_{j_1},\ldots,X_{j_n}) = S \text{ if not } c-a(S,X_{j_1},\ldots,X_{j_n}).
\]

If the value of \(o_{i_1,\ldots,i_m}\) is not affected by applying \(\tau_{j_1,\ldots,j_n}\) in any state (regardless of the truth value of \(c_{\tau_{j_1,\ldots,j_n}}\)), the following equation may be declared:

\[
\text{eq } o(a(S,X_{j_1},\ldots,X_{j_n}),X_{i_1},\ldots,X_{i_m}) = o(S,X_{i_1},\ldots,X_{i_m}).
\]
3 Proof Scores

We describe proof scores showing that a predicate $p_1$ is invariant wrt $S$, which are written in CafeOBJ. We often need other predicates, say $p_2, \ldots, p_n$, for the verification, although such predicates should be found during the verification. Let $x_{i1}, \ldots, x_{imi}$, whose types are $D_{i1}, \ldots, D_{imi}$, be all free variables in $p_i$ except $\nu$, whose type is $\Upsilon$, for $i = 1, \ldots, n$. $p_i$ may be written as $p_i(\nu, x_{i1}, \ldots, x_{imi})$.

Although some invariant properties may be proved by rewriting and case analysis only with other proved invariant properties, we often need induction, especially simultaneous induction[15] on the number of transitions applied.

We first declare the operators denoting $p_1, \ldots, p_n$ and the equations defining the operators. The operators and equations are declared in a module, say INV (which imports the module where $S$ is written), as

\begin{align*}
\text{op inv}_i : & H \times V_{i1} \ldots V_{imi} \rightarrow \text{Bool} \\
\text{eq inv}_i(s, x_{i1}, \ldots, x_{imi}) &= p_i(s, x_{i1}, \ldots, x_{imi}) .
\end{align*}

for $i = 1, \ldots, n$. $V_k$ is a visible sort denoting $D_k$ and $X_k$ is a CafeOBJ variable of $V_k$ for $k = i1, \ldots, im_i$. $p_i(s, x_{i1}, \ldots, x_{imi})$ is a CafeOBJ term denoting $p_i$. In module INV, we also declare a constant $x_k$ denoting an arbitrary value of $V_k$ for $k = 1, \ldots, n$.

We then declare the operators denoting basic formulas to show in the inductive cases and the equations defining the operators. The operators and equations are declared in a module, say ISTEP (which imports INV), as follows:

\begin{align*}
\text{op istep}_i : & V_{i1} \ldots V_{imi} \rightarrow \text{Bool} \\
\text{eq istep}_i(x_{i1}, \ldots, x_{imi}) = & \text{implies \text{inv}_i(s, x_{i1}, \ldots, x_{imi}) implies inv}_i(s', x_{i1}, \ldots, x_{imi}) .
\end{align*}

for $i = 1, \ldots, n$. $s$ and $s'$, which are declared in module ISTEP, are constants of $H$; $s$ denotes an arbitrary state and $s'$ a successor state of the state.

For the base case, we write

\begin{align*}
\text{open INV} \\
\text{red inv}_i(\text{init}, x_{i1}, \ldots, x_{imi}) .
\end{align*}

for $i = 1, \ldots, n$. CafeOBJ command open makes a temporary module that imports a module given as an argument and CafeOBJ command close destroys the temporary module. Parts enclosed with open and close are basic units of proof scores, which are called proof passages in the OTS/CafeOBJ method.

For the induction case showing that each $\tau_{j1, \ldots, jm_j}$ (denoted by action operator $a$) preserves each $p_i$, we often need case analysis. We suppose that the state space is split into $l$ sub-spaces for the induction case, although such case analysis should be done during the verification. Each of the $l$ subcases is supposed to be characterized by a predicate $\text{case}_{ik}$ for $k = 1, \ldots, l$; the predicates
should satisfy \((case_{i_1} \lor \ldots \lor case_{i_l}) \iff \text{true}\). For the induction case, we then write

\begin{verbatim}
open ISTEP
    -- arbitrary objects
    op y_{1m_1} : -> V_{1m_1} . . . op y_{jm_j} : -> V_{jm_j} .
    -- assumptions
    Declaration of equations denoting case_{i_k}.
    -- successor state
    eq s' = a(s, y_{j_1}, . . . , y_{j_m_j}) .
    -- check
    red SIH_i implies istep_i(x_{i_1}, . . . , x_{i_m_i}) .
close
\end{verbatim}

for \(i = 1, \ldots, n\) and \(k = 1, \ldots, l\). A comment starts with `--` and terminates at the end of the line. \(SIH_i\) is used to strengthen the induction hypothesis \(inv_i(s, x_{i_1}, \ldots, x_{i_{m_i}})\) and is the form \(inv_{i_1}(s, t_{i_1}, \ldots)\) and \(\ldots\) and \(inv_{i_k}(s, t_{i_k}, \ldots)\), where \(1 \leq i_1, \ldots, i_k \leq n\) and each \(t_k\) is a term of sort \(V_k\).

## 4 Buffet: A Toolkit for the OTS/CafeOBJ Method

Buffet is a toolkit for generating and displaying proof scores. An overview of Buffet is shown in Fig. 1. Buffet consists of the Buffet server, Gateau (a Buffet client), PSP (Proof Score Presenter) and the CafeOBJ system. Gateau takes three kinds of files: `spec.mod` in which an OTS \(S\) is specified in CafeOBJ, `inv.mod` in which modules INV and ISTEP are declared, and `script.mod` in which a script to instruct Gateau to generate a proof score is written. Gateau communicates with the CafeOBJ system via the Buffet server using the HTTP protocol and an inter-process communication (IPC) method. Given `spec.mod`, `inv.mod` and `script.mod`, then Gateau passes them into the CafeOBJ system, generates a proof score based on some information extracted from the three files and passes the proof score into the CafeOBJ system. Gateau then receives the results of rewriting the proof score from the CafeOBJ system, generates a file `proof.xml` in XML from the proof score and the results, and passes the
file to PSP. PSP then generates a file proof.html in HTML from proof.xml to display the proof score and the results on a web browser. In the rest of the section, we describe the Buffet server, Gateau and PSP.

4.1 The Buffet Server

The Buffet server provides the five services: 1) to create a new session, 2) to have the CafeOBJ system load files, 3) to obtain a module information from the CafeOBJ system, reconstruct the module as an XML document and pass it to a client, 4) to have the CafeOBJ system reduce a term under a module and pass the result reconstructed as an XML document to a client, and 5) to finish the current session.

We describe the five services in turn. When requested by a client, the Buffet server creates a new session for the client, starting the CafeOBJ system as its child process and establishing an IPC connection between the server process and the child process. After that, the client can communicate with the CafeOBJ system via the Buffet server. CafeOBJ command in is used to load files into the CafeOBJ system. The Buffet server has the CafeOBJ system load a file by sending command in and the file name to the CafeOBJ system. CafeOBJ command show is used to parse modules and use the results of parsing them, provided that switch tree print is set to on. The Buffet server has the CafeOBJ system parse a module by sending command show and the module name to the CafeOBJ system, reconstructs the module as an XML document based on the result of parsing the module and passes it to a client. The Buffet server has the CafeOBJ system reduce a term under a module by sending command red, the term and the module name, reconstructs the result term as an XML document and passes it to a client. At the end of a session, the Buffet server finishes the session by stopping the CafeOBJ system.

The current implementation of the Buffet server is written in Perl and consists of about 1,200 lines.

4.2 Gateau

Gateau has the five commands new, input, parse, verify and quit. Commands new and quit correspond to the first and fifth services provided by the Buffet server. Command input takes a file name as its argument and uploads the file to the Buffet server, which does the second service. Command parse takes the name of a module in which an OTS $S$ is supposed to be specified and passes it to the Buffet server, which does the third service; Gateau extracts the action operators denoting the transitions of $S$ and their (effective) conditions from the module (which is an XML document) returned by the Buffet
server. The action operators and their conditions are used to generate proof scores showing that predicates are invariant wrt $S$. Command verify takes the name of a module in which a proof script is supposed to be written for the proof that a predicate is invariant wrt $S$ and passes it to the Buffet server, which does the third service; Gateau generates a proof score based on the module (which is an XML document) returned by the Buffet server and the information obtained by command parse. For each proof passage of the proof score, Gateau makes a module corresponding to the proof passage (excluding the statement containing CafeOBJ command red), uploads the module to the Buffet server so as to have the CafeOBJ system load the module, asks the Buffet server to have the CafeOBJ system reduce the term (appearing in the proof passage and denoting the formula to be proved) under the module, and receives the result (which is an XML document) from the CafeOBJ system via the Buffet server. Based on the results and the proof score, Gateau makes an XML document of them, saves it as a file and passes the file name to PSP.

The current implementation of Gateau is written in Perl and consists of about 1,400 lines. In the rest of this subsection, we describe proof scripts and how to generate proof scores based on proof scripts.

4.2.1 Proof Scripts

For each predicate $p_i$ (denoted by operator $\text{inv}_i$) to be verified, we write a proof script from which a proof score of $p_i$ is generated. In a proof script of $p_i$, for each action operator a denoting transition $\tau_{j_1,\ldots,j_m}$, we give predicates such as $c_{i_1},\ldots,c_{i_{n_i}}$ that are used to split cases and formulas such as $\text{inv}_{i_1}(s,t_{i_1},\ldots),\ldots,\text{inv}_{i_{n_i}}(s,t_{i_{n_i}},\ldots)$ that are used to strengthen the induction hypothesis $\text{inv}_i(s,x_{i_1},\ldots,x_{i_{n_i}})$ for the induction case that $\tau_{j_1,\ldots,j_m}$ preserves $p_i$. Such predicates and formulas are given in the form of rewriting rules. The rewriting rules for predicates $c_{i_1},\ldots,c_{i_{n_i}}$ and formulas $\text{inv}_{i_1}(s,t_{i_1},\ldots),\ldots,\text{inv}_{i_{n_i}}(s,t_{i_{n_i}},\ldots)$ look like

\begin{align*}
\text{trans predicates}(a(S,Y_{j_1},\ldots,Y_{j_m})) & \Rightarrow c_{i_1} . \\
\text{...} \\
\text{trans predicates}(a(S,Y_{j_1},\ldots,Y_{j_m})) & \Rightarrow c_{i_{n_i}} . \\
\text{trans lemmas}(a(S,Y_{j_1},\ldots,Y_{j_m})) & \Rightarrow \text{inv}_{i_1}(s,t_{i_1},\ldots) . \\
\text{...} \\
\text{trans lemmas}(a(S,Y_{j_1},\ldots,Y_{j_m})) & \Rightarrow \text{inv}_{i_{n_i}}(s,t_{i_{n_i}},\ldots) .
\end{align*}

Operators predicates and lemmas are used as keywords to write such predicates and formulas. The reason why such predicates and formulas are given in the form of rewriting rules is that we can use the CafeOBJ systems to parse rewriting rules and do not have to implement another parser for such predicates and formulas. Proof scores are generated based on such predicates and formulas, which is next described.
4.2.2 How to Generate Proof Scores

Given predicate \( p_i \) (denoted by operator inv\(_i\)) to be verified, predicates \( c_{i_1}, \ldots, c_{i_n} \) used to split cases and formulas inv\(_i\)(\( s, t_{i_1}, \ldots \)), \( \ldots \) inv\(_{i_k}\)(\( s, t_{i_n}, \ldots \)) used to strengthen the induction hypothesis inv\(_i\)(\( s, x_{i_1}, \ldots, x_{i_m} \)), then a proof score of \( p_i \) is generated and checked as follows:

(i) Base case: Gateau has the CafeOBJ system reduce term inv\(_i\)(init, \( x_{i_1}, \ldots, x_{i_m} \)) under module INV and generates an XML document of the proof passage and the result.

(ii) Induction cases: For each action operator a denoting transition \( \tau_{j_1}, \ldots, j_{m_j} \), for each proof passage to be checked a temporary module PROOF\_TMP is generated and a term denoting a formula to be proved is reduced under PROOF\_TMP in the following way (let \( c_{i_0} \) be \( c_{\tau_{j_1}, \ldots, j_{m_j}} \)):

\[
k := 1; \text{stack} := \text{empty}; \text{push}(\text{stack}, \{c_{i_0}\}); \text{push}(\text{stack}, \neg c_{i_0});
\]

\[
\text{while} \text{stack} \neq \text{empty} \do
\]

\[
\text{Cs} := \text{pop}(\text{stack}); (* \text{let Cs be } \{c'_1, \ldots, c'_{n'}\}. *)
\]

Make the module

\[
\text{mod PROOF\_TMP}{
\text{pr(ISTEP)}
\}
\]

Let \( T \) be istep\(_i\)(\( x_{i_1}, \ldots, x_{i_m} \));

Have the CafeOBJ system reduce \( T \) under PROOF\_TMP;

if the result is true then

(* The proof succeeds in the case Cs. *)

Generate an XML document of the proof passage and the result;

else if the result is false then

(* If the result is true, the proof succeeds in the case Cs. *)

else if \( k \leq i_n \) then

\[
\text{push}(\text{stack}, \text{Cs} \cup \{c_k\}); \text{push}(\text{stack}, \text{Cs} \cup \neg c_k); k := k + 1;
\]

else

(* If not, further case analysis and/or other lemmas may be needed. *)

\[
\text{fi fi fi fi;}
\]

\( k \) is an integer variable and stack is a stack of predicate sets. push and pop are usual operators of stacks. mod is the keyword for declaring modules and pr is the keyword for importing modules.

Each predicate \( c'_{k'} \) is the form \( l_1 \land \ldots l_{n'_k} \) or \( \neg(l_1 \land \ldots l_{n'_k}) \), where each \( l_i \) is a literal, namely the form \( \alpha_n \) or \( \neg \alpha_n \), and \( \alpha_n \) is an atomic formula. In the case that \( c'_{k'} \) is the form \( \neg(l_1 \land \ldots l_{n'_k}) \), we declare equation \( c'_{k'} = \text{false} \). In the case that \( c'_{k'} \) is the form \( l_1 \land \ldots l_{n'_k} \), we declare an
equation for each $l_\kappa$. In the case that $l_\kappa$ is the form $-\alpha_\kappa$, we declare equation $l_\kappa = \text{false}$. In the case that $l_\kappa$ is the form $\alpha_\kappa$, if $\alpha_\kappa$ is the form $\text{left} = \text{right}$, we declare equation $\text{left} = \text{right}$, and otherwise we declare equation $l_\kappa = \text{true}$.

4.3 PSP

Given an XML document of a proof score and the results of reducing the proof passages in the proof score, PSP generates an HTML document. When an HTML document generated by PSP is first displayed on a web browser, proof passages for which results are not true and their results are shown, and other proof passages (for which the proof has succeeded) are hidden. Proof passages are hierarchically shown according to the predicates used to split cases and each proof passage is clickable, allowing the proof passage to appear and disappear. The current implementation of PSP is written in XSLT (XSL Transformations; see www.w3.org/TR/xslt) and consists of about 600 lines.

5 A Case Study: A Mutual Exclusion Protocol

We describe a case study that Buffet has been applied to the verification that a simple mutual exclusion protocol has the mutual exclusion property. The protocol repeatedly executed by multiple processes can be written as

11: Remainder Section
12: repeat until $-\text{fetch&store(lock, true)}$
   Critical Section
cs: lock := false

lock is a boolean variable and is initially set to false. $\text{fetch&store}(x, v)$ atomically exchanges the value of variable $x$ with value $v$ and returns the original value of $x$. Each process is initially at location l1.

5.1 Modeling and Specification of the Mutual Exclusion Protocol

Let $B$, $P$ and $L$ be types of boolean values, process IDs and locations (l1, l2 and cs). The mutual exclusion protocol is modeled as the OTS $S_{MX}$ such that 1) $O_{MX}$ consists of $\text{lock} : \Upsilon \rightarrow B$ and $\text{loc}_i : \Upsilon \rightarrow L$ for $i \in P$, 2) $I_{MX}$ is $\{v \in \Upsilon \mid -\text{lock}(v) \land \forall i \in P. (\text{loc}_i(v) = l1)\}$, and 3) $T_{MX}$ consists of $\text{try}_i : \Upsilon \rightarrow \Upsilon$, $\text{enter}_i : \Upsilon \rightarrow \Upsilon$ and $\text{leave}_i : \Upsilon \rightarrow \Upsilon$, for $i \in P$, whose effective conditions are $c_{\text{try}_i}(v) \equiv (\text{loc}_i(v) = l1)$, $c_{\text{enter}_i}(v) \equiv (\text{loc}_i = l2 \land -\text{lock}(v))$ and $c_{\text{leave}_i}(v) \equiv (\text{loc}_i = \text{cs})$ and whose definitions are:

(i) Let $v'$ be $\text{try}_i(v)$. If $c_{\text{try}_i}(v)$ holds, then $\text{lock}(v') = \text{lock}(v)$ and $\text{loc}_j(v') = (\text{if } i = j \text{ then } l2 \text{ else } \text{loc}_j(v))$. Otherwise, nothing changes.
(ii) Let \(v'\) be \(\text{enter}_i(v)\). If \(c_{\text{enter}_i}(v)\) holds, then \(\text{lock}(v') = \text{true}\) and \(\text{loc}_j(v') = (\text{if } i = j \text{ then } \text{cs} \text{ else } \text{loc}_j(v))\). Otherwise, nothing changes.

(iii) Let \(v'\) be \(\text{leave}_i(v)\). If \(c_{\text{leave}_i}(v)\) holds, then \(\text{lock}(v') = \text{false}\) and \(\text{loc}_j(v') = (\text{if } i = j \text{ then } \text{li} \text{ else } \text{loc}_j(v))\). Otherwise, nothing changes.

\(S_{\text{MX}}\) is written in CafeOBJ. The signature of the CafeOBJ specification of \(S_{\text{MX}}\) is

\[
\begin{align*}
\text{-- any initial state} & \quad \text{op init} : \to \text{Sys} \\
\text{-- observation operators} & \quad \text{bop lock} : \text{Sys} \to \text{Bool} \quad \text{bop loc} : \text{Sys} \text{ Pid} \to \text{Loc} \\
\text{-- action operators} & \quad \text{bop try} : \text{Sys} \text{ Pid} \to \text{Sys} \quad \text{bop enter} : \text{Sys} \text{ Pid} \to \text{Sys} \\
& \quad \text{bop leave} : \text{Sys} \text{ Pid} \to \text{Sys}
\end{align*}
\]

\(\text{Sys}\) is the hidden sort denoting \(\Upsilon\), \(\text{Pid}\) is the visible sort denoting \(P\) and \(\text{Loc}\) is the visible sort denoting \(L\). Constant \(\text{init}\) denotes any initial state. Observation operators \(\text{lock}\) and \(\text{loc}\) denote observers \(\text{lock}\) and \(\text{loc}_i\), and action operators \(\text{try}, \text{enter}\) and \(\text{leave}\) denote transitions \(\text{try}_i, \text{enter}_i\) and \(\text{leave}_i\). The three action operators are defined in equations as

\[
\begin{align*}
\text{-- try} & \quad \text{eq } \text{lock}(\text{try}(S,I)) = \text{lock}(S) \\
& \quad \text{ceq } \text{try}(\text{try}(S,I),J) = (\text{if } I = J \text{ then } \text{li} \text{ else } \text{loc}(S,J) \text{ fi}) \\
& \quad \text{ceq } \text{try}(S,I) = S \text{ if not } (\text{c-try}(S,I)) \\
\text{-- enter} & \quad \text{ceq } \text{lock}(\text{enter}(S,I)) = \text{true} \text{ if } \text{c-enter}(S,I) \\
& \quad \text{ceq } \text{loc}(\text{enter}(S,I),J) = (\text{if } I = J \text{ then } \text{cs} \text{ else } \text{loc}(S,J) \text{ fi}) \\
& \quad \text{ceq } \text{enter}(S,I) = S \text{ if not } (\text{c-enter}(S,I)) \\
\text{-- leave} & \quad \text{ceq } \text{lock}(\text{leave}(S,I)) = \text{false} \text{ if } \text{c-leave}(S,I) \\
& \quad \text{ceq } \text{loc}(\text{leave}(S,I),J) = (\text{if } I = J \text{ then } \text{li} \text{ else } \text{loc}(S,J) \text{ fi}) \\
& \quad \text{ceq } \text{leave}(S,I) = S \text{ if not } (\text{c-leave}(S,I))
\end{align*}
\]

Operators \(\text{c-try}, \text{c-enter}\) and \(\text{c-leave}\) denote \(c_{\text{try}_i}, c_{\text{enter}_i}\) and \(c_{\text{leave}_i}\), which are defined as

\[
\begin{align*}
\text{eq } c_{\text{try}}(S,I) = (\text{loc}(S,I) = \text{li}) \\
\text{eq } c_{\text{enter}}(S,I) = (\text{loc}(S,I) = \text{li} \text{ and not } \text{lock}(S)) \\
\text{eq } c_{\text{leave}}(S,I) = (\text{loc}(S,I) = \text{cs})
\end{align*}
\]

5.2 Verification of the Mutual Exclusion Protocol

We describe the verification that \(S_{\text{MX}}\) has the mutual exclusion property. For the verification, all we have to do is to prove predicate \((\text{loc}_i(v) = \text{cs} \land \text{loc}_j(v) = \text{cs}) \Rightarrow (i = j)\) invariant wrt \(S_{\text{MX}}\). The predicate is denoted by operator \(\text{inv1}\) defined as

\[
\text{eq } \text{inv1}(S,I,J) = (\text{loc}(S,I) = \text{cs} \text{ and } \text{loc}(S,J) = \text{cs} \text{ implies } I = J)
\]

The operator is declared and defined in module \(\text{INV}\). In the module, constants \(i\) and \(j\) denoting arbitrary values of sort \(\text{Pid}\) are also declared. The operator
denoting the basic formula to be shown in each induction case is denoted by operator istep1 defined as

\[ \text{eq istep1}(I,J) = \text{inv1}(s,I,J) \implies \text{inv1}(s',I,J) . \]

The operator is declared and defined in module ISTEP. In the module, constants \( s \) and \( s' \) are also declared.

First of all we do not use any predicates to split cases and any formulas to strengthen the induction hypothesis and have Buffet generate and check a proof score of \( \text{inv1}(s,i,j) \) and display the proof score and the results. Buffet reports that seven cases have been checked and the proof has succeeded in three out of the seven cases. We show in Fig. 2 part of the proof score and the results displayed by Buffet. Small triangles are clickable buttons. An upside down triangle means that its contents are shown, and a triangle rotated clockwise by 90 degrees means that its contents are hidden. The first button from top in Fig. 2 corresponds to the induction case of \( \text{enter} \), denoted by \( \text{enter} \). There are two proof passages in the induction case. One corresponding to the second button is shown and the other corresponding to the last button is hidden. The proof succeeds in the second proof passage but for the first proof passage we need case analysis.

Next we use predicates to split cases and the predicates are given as

\[
\begin{align*}
\text{-- for try} \\
\text{trans predicates(try(S,P))} & \implies (i = \text{pid1}) . \\
\text{-- for enter} \\
\text{trans predicates(enter(S,P))} & \implies (i = \text{pid1}) . \\
\text{trans predicates(enter(S,P))} & \implies (j = \text{pid1}) . \\
\text{trans predicates(enter(S,P))} & \implies (\text{loc}(s,i) = \text{cs}) . \\
\text{trans predicates(enter(S,P))} & \implies (\text{loc}(s,j) = \text{cs}) . \\
\text{-- for leave}
\end{align*}
\]
But we do not use any formulas to strengthen the induction hypothesis. In this case Buffet reports that 18 cases have been checked and the proof has succeeded in 15 out of the 18 cases. We show in Fig. 3 part of the proof score and the results displayed by Buffet.

Looking at the proof passage in Fig. 3, we notice that process j is at location cs and lock(s) is false in state s, which seems contradiction. Therefore we conjecture that predicate \( (loc_i(v) = cs) \Rightarrow lock(v) \) is also invariant. The predicate is denoted by operator inv2 defined (in module INV) as
\[
eq inv2(S,I) = (loc(S,I) = cs \implies lock(S))
\]
We also declare and define operator istep2 denoting the basic formula to be shown in each induction case as istep1 in module ISTEP.

In addition to the predicates to split cases, we also use formulas to strengthen the induction hypothesis inv1(s,i,j) and the formulas are given as
\[
\text{trans lemmas(enter(S,P)) } \Rightarrow \text{inv2(s,i)}.
\]
\[
\text{trans lemmas(enter(S,P)) } \Rightarrow \text{inv2(s,j)}.
\]
In this case Buffet reports that the proof has succeeded in all 18 cases.
For the verification of $\text{inv2}(s, i)$, we use the predicates to split cases and the formulas to strengthen the induction hypothesis:

```
-- for try
trans predicates(try(S,P)) => (pid1 = i) .
trans predicates(try(S,P)) => (loc(s,i) = cs) .
trans predicates(try(S,P)) => lock(s) .
trans predicates(try(S,P)) => (loc(s,i) = l1) .
-- for leave
trans predicates(leave(S,P)) => (pid1 = i) .
trans predicates(leave(S,P)) => (loc(s,i) = cs) .
trans predicates(leave(S,P)) => lock(s) .
trans lemmas(leave(S,P)) => inv1(s,i,pid1) .
```

Buffet reports that 15 cases have been checked and the proof has succeeded in all the cases.

Note that the predicate denoted by $\text{inv2}$ is needed to strengthen the induction hypothesis for the invariant proof of the predicate denoted by $\text{inv1}$ and vice versa, which means that if each of the proof scores is written individually, then simultaneous induction[15] is needed.

6 Related Work

BOBJ[8] is an algebraic specification language based on order-sorted and hidden-sorted algebras, which is a sibling language of CafeOBJ. BOBJ implements conditional circular coinductive rewriting with case analysis (c4rw). Given equations (which are used to split cases) and lemmas, c4rw automatically generates proof scores and checks the proof scores. BOBJ allows us to specify how to split cases in more detail than Buffet, but users who give equations used to split cases are responsible for whether the whole cases are covered by the equations. As shown in [8], some problems can be verified well with c4rw, but it seems that further research should be done to make it clear that BOBJ can be appropriately applied to what types of problems. BOBJ is part of the Tatami system[7]. The Tatami system provides facilities for displaying proofs so as to make them preferably attractive to software engineers based on algebraic semiotics (which combines algebraic specification with social semiotics). The basic idea behind the facilities may be used to improve our way of displaying proof scores.

Several proof assistants have been proposed. Among them are Coq[1] and Isabelle/HOL[12]. They provide some automatic proof mechanisms to some extent, but basically help users construct their proofs. Users feed commands called tactics into a proof assistant to make progress on their proofs. Tactics usually reduce a proof goal into zero or more proof sub-goals, which are hopefully simpler. But users should select appropriate tactics in order to succeed in their proofs. This means that users are basically required to have knowl-
edge and experience to complete their proofs on their own without any proof assistants, although proof assistants prevent users from making mistakes.

Among the existing tools supporting verification of (distributed) systems with algebraic specification languages are Larch Prover (LP)[10] and Maude Inductive Theorem Prover (Maude ITP)[3]. The design policy of LP is to make proof assistants easier-to-use especially for engineers, but users of LP are basically required to have similar skills as those needed to use other proof assistants. Maude ITP assists verification of abstract data types written in Maude[2], an algebraic specification and programming language based on membership equational logic and rewriting logic, but does not assist verification of abstract machines.

7 Conclusion

We have described Buffet for generating and displaying proof scores in the OTS/CafeOBJ method and reported on the case study on the verification of a simple mutual exclusion protocol. In addition to the mutual exclusion protocol, Buffet has been successfully applied to the verification that the NSLPK authentication protocol[11] and the Otway-Rees authentication protocol[16] have the secrecy property.

References


