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Interacting fluids generating identical, dual and phantom cosmologies

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Abstract

We find the group of symmetry transformations generated by interacting fluids in spatially flat Friedmann–Robertson–Walker (FRW) spacetime which links cosmologies with the same scale factor (*identity*) or with scale factors a and a^{-1} (*duality*). There exists a duality between contracting and superaccelerated expanding scenarios associated with (*phantom*) cosmologies. We investigate the action of this symmetry group on self-interacting minimally (conformally) coupled quintessence and k-essence cosmologies.

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1. Introduction

Symmetry transformations preserving the form of the spatially flat FRW equations introduce an alternative concept of equivalence between different physical problems [1]. This means that a set of cosmological models are equivalent when their dynamical equations are form invariant under the action of that group. It suggests that any of this equivalent cosmologies can be used to describe the present accelerated expansion of the universe. Hence, it turns out to be interesting to investigate the consequences of this group.

Due to the additivity of the stress-energy tensor a cosmological model with one fluid in flat FRW spacetime can be seen as a model of two interacting fluid components. For instance, a scalar field minimally coupled to gravity has been described as a stiff fluid interacting with vacuum energy [1]. The stress-tensor of the tachyon field could be considered as the sum of two components, one behaving like a pressureless dust and the other having a negative pressure [2]. In this Letter we investigate and extend these results splitting the source in two components in a manner compatible with two discrete symmetries of the Einstein equations in flat FRW spacetimes. These symmetries are the structural invariance of the scale factor as a function of the cosmological time, i.e., $\bar{a}(t) = a(t)$ [3–6] and the duality between expanding and contracting backgrounds $\bar{a} = 1/a$ [3,5,7,8]. In addition, the dual transformation mapping contracting into superaccelerated expanding backgrounds provides the link between a standard and a phantom cosmology [5,9]. A phantom source with sufficiently negative pressure violates the weak energy condition but could describe adequately current observations [10]. Other characteristics of phantom cosmologies have been investigated in [11].

In Section 2 we develop the interacting framework and illustrate it with simple examples, such as, self-interacting minimally (conformally) coupled quintessence field $\phi(\psi)$ and k-essence field φ. In Section 3 we present a linear transformations which preserve the form of the dynamical equations and apply them to those scalar fields. In Section 4 the conclusion are stated.

2. Interacting framework

The Einstein equations in the flat FRW spacetime with scale factor a and a perfect fluid read

$$3H^2 = \rho, \tag{1}$$

$$\dot{\rho} + 3H(\rho + p) = 0, \tag{2}$$

where ρ is the energy density, p the pressure and $H = \dot{a}/a$. There are two independent Einstein equations for the three

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quantities a, p, and ρ. Usually, the system of equations (1), (2) is closed with an equation of state $p = p(\rho)$.

We assume that T_{ik} splits into two perfect fluid parts, $T_{ik} = T^1_{ik} + T^2_{ik}$, with $T^{1,2}_{ik} = (\rho_{1,2} + p_{1,2})u_i u_k + p_{1,2} g_{ik}$, where $\rho_{1,2}$ and $p_{1,2}$ are the energy density and the equilibrium pressure of fluids 1 and 2 respectively, u^i is the four-velocity. Therefore, Eqs. (1), (2) become

$$3H^2 = \rho_1 + \rho_2, \tag{3}$$

$$\dot\rho_1 + \dot\rho_2 + 3H(\rho_1 + \rho_2 + p_1 + p_2) = 0. \tag{4}$$

Eq. (4) shows the interaction between the fluid components allowing the mutual exchange of energy and momentum. Consequently, there will be no local energy–momentum conservation for the fluids separately. To preserve the degree of freedom of the original system of equations (1), (2), we introduce an equation of state for each fluid component $p_{1,2} = (\gamma_{1,2} - 1)\rho_{1,2}$, where $\gamma_{1,2}$ are the barotropic indexes of fluids 1 and 2, respectively. As the energy–momentum tensor of the system as a whole is conserved, we assume an effective perfect fluid description with equation of state $p = (\gamma - 1)\rho$, where $\gamma = (\gamma_1\rho_1 + \gamma_2\rho_2)/\rho$ is the effective barotropic index. For this effective perfect fluid the dynamical equations are identical to (1), (2).

The scalar field ϕ with energy density and pressure

$$\rho_\phi = \frac{1}{2}\dot\phi^2 + V(\phi), \qquad p_\phi = \frac{1}{2}\dot\phi^2 - V(\phi), \tag{5}$$

can be represented in terms of two interacting fluids namely $\rho_1 = \dot\phi^2/2$ and $\rho_2 = V(\phi)$, with equations of state $p_1 = \rho_1$ and $p_2 = -\rho_2$, meaning that $\gamma_1 = 2$ (*stiff matter*) and $\gamma_2 = 0$ (*vacuum energy*). Due to the interactions between the two fluid components the energy–momentum tensor conservation of the system as a whole is equivalent to the Klein–Gordon equation

$$\ddot\phi + 3H\dot\phi + \frac{dV}{d\phi} = 0, \tag{6}$$

while $\gamma = 2\rho_1/\rho_\phi$ is the effective barotropic index.

The conformal scalar field ψ driven by the potential

$$\mathcal{V}(\psi) = \lambda\psi^4 + \mathcal{V}_0, \quad \mathcal{V}_0 > 0, \tag{7}$$

is an interesting model, because this potential has received much attention in the literature in connection with the early inflationary epoch [12]. This simplified model leads to a final accelerated expansion phase retaining the essentials of minimally coupled approaches. The energy density and the pressure of the conformal scalar field

$$\rho_\psi = \frac{1}{2}(\dot\psi + H\psi)^2 + \lambda\psi^4 + \mathcal{V}_0, \tag{8}$$

$$p_\psi = \frac{1}{6}(\dot\psi + H\psi)^2 + \frac{\lambda}{3}\psi^4 - \mathcal{V}_0, \tag{9}$$

can be represented as the sum of two interacting fluids namely $\rho_1 = (\dot\psi + H\psi)^2/2 + \lambda\psi^4$ and $\rho_2 = \mathcal{V}_0$, with equations of state $p_1 = \rho_1/3$ and $p_2 = -\rho_2$ representing (*radiation*) $\gamma_1 = 4/3$ and (*vacuum energy*) $\gamma_2 = 0$, while $\gamma = 4\rho_1/3\rho_\psi$ is the effective barotropic index. The energy conservation of the effective fluid is given by the Klein–Gordon equation

$$\ddot\psi + 3H\dot\psi + \frac{1}{6}R\psi + 4\lambda\psi^3 = 0, \tag{10}$$

whose first integral is

$$\frac{1}{2}(\dot\psi + H\psi)^2 + \lambda\psi^4 = \frac{b}{a^4}, \tag{11}$$

where b is an integration constant. After combining Eqs. (1), (8), (11) and integrating, we get the scale factors

$$a_c^\pm = \left[\pm\sqrt{\frac{b}{\mathcal{V}_0}}\sinh\sqrt{\frac{4\mathcal{V}_0}{3}}t\right]^{1/2} \qquad b > 0, \tag{12}$$

$$a_c = \left[\sqrt{-\frac{b}{\mathcal{V}_0}}\cosh\sqrt{\frac{4\mathcal{V}_0}{3}}t\right]^{1/2} \qquad b < 0. \tag{13}$$

Defining the conformal time $\eta = \int dt/a$ and the new field $\chi = \psi a$, Eq. (10) becomes $\chi'' + 4\lambda\chi^3 = 0$, where $' \equiv d/d\eta$. Its general solution can be expressed in terms of the Jacobi functions. For $\lambda > 0$, the qualitative aspects of ψ are obtained from the first integral (11), which reads $\chi'^2/2 + \lambda\chi^4 = b$. Hence, χ oscillates between $-b^{1/4} \leqslant \chi \leqslant b^{1/4}$ and $\psi = \chi/a$ becomes a decreasing oscillating function with a vanishing final limit at the minimum of the potential. On the other hand, the bouncing solution (13) avoids the initial singularity.

For the k-essence field φ with Lagrangian $\mathcal{L} = -U(\varphi)F(x)$, where $U(\varphi)$ is the potential, $F(x)$ is a function of the kinetic term $x = g^{ik}\varphi_i\varphi_k$ and $\varphi_i = \partial\varphi/\partial x^i$, we associate the energy–momentum tensor of a perfect fluid. The energy density and pressure of the k field are

$$\rho_\varphi = U(\varphi)[F - 2xF_x], \qquad p_\varphi = -U(\varphi)F, \tag{14}$$

with $F_x = dF/dx$. They can be split as two interacting perfect fluids such that $\rho_1 = U(\varphi)F(x)$ and $\rho_2 = -2U(\varphi)xF_x$ with equations of state $p_1 = -\rho_1$ and $p_2 = 0$. They play the role of (*vacuum energy*) $\gamma_1 = 0$ and (*dust*) $\gamma_2 = 1$. The energy–momentum tensor conservation of the effective fluid is the k-field equation

$$[F_x + 2xF_{xx}]\ddot\varphi + 3HF_x\dot\varphi + \frac{U'}{2U}[F - 2xF_x] = 0, \tag{15}$$

while $\gamma = \rho_2/\rho_\varphi$ is the effective barotropic index.

3. Form invariance symmetry

Here, we will find a symmetry transformation that preserves the form of the system of equations (3) and (4). To begin with, we observe that the total energy density is form invariant under the linear transformations

$$\begin{pmatrix}\bar\rho_1\\ \bar\rho_2\end{pmatrix} = \begin{pmatrix}\alpha & 1-\beta\\ 1-\alpha & \beta\end{pmatrix}\begin{pmatrix}\rho_1\\ \rho_2\end{pmatrix}, \tag{16}$$

that is, $\bar\rho = \bar\rho_1 + \bar\rho_2 = \rho_1 + \rho_2 = \rho$ for any α and β. These form invariant transformations constitute a group and they induce the transformations $\bar H = H$ or $\bar H = -H$ in Eq. (3). The former leads to the *identity* $\bar a = a$ and the latter to the *duality* $\bar a = 1/a$. The duality between contracting and superaccelerated expanding ($H>0$ and $\dot H>0$) phases gives rise to a *phantom* transformation [5]. Assuming equations of state for the interacting fluids, $p_{1,2}=(\gamma_{1,2}-1)\rho_{1,2}$ in the initial configuration and $\bar p_{1,2}=(\bar\gamma_{1,2}-1)\bar\rho_{1,2}$ in the final configuration, then the conservation equation (4) remains form invariant when

$$\alpha_i=\frac{\bar\gamma_2-\gamma_1}{\bar\gamma_2-\bar\gamma_1},\qquad \beta_i=\frac{\gamma_2-\bar\gamma_1}{\bar\gamma_2-\bar\gamma_1},\qquad \bar H=H, \tag{17}$$

$$\alpha_d=\frac{\bar\gamma_2+\gamma_1}{\bar\gamma_2-\bar\gamma_1},\qquad \beta_d=-\frac{\gamma_2+\bar\gamma_1}{\bar\gamma_2-\bar\gamma_1},\qquad \bar H=-H. \tag{18}$$

The duality $H\to -H$, acting on Eq. (2) induces the transformation $\rho+p\to-(\rho+p)$ and the associated matter violates the weak energy condition. Such model could explain current observations and it is referred to as phantom cosmology [13]. When both ρ and p diverge the dual transformation trades a final big crunch by a final big rip. However, when ρ is finite but p diverges [14], the dual transformation interchanges this finite time singularity by other of the same kind. These sudden future singularities can occur even when the matter obeys $\rho>0$ and $\rho+3p>0$.

Below, we show the action of the form invariance symmetry group on the three fields ϕ, ψ and φ. The associated effective perfect fluid description will be used splitting each fluid as two interacting ones with indexes (γ_1,γ_2), i.e., $Q(\phi)\equiv(2,0)$, $C(\psi)\equiv(4/3,0)$ and $k(\varphi)\equiv(0,1)$. Also, we allow real or imaginary fields.

$k\to Q$: Let us suppose that we are interested in seeking the set of k-essence models having the same scale factor that the quintessence ones. To this end we use Eqs. (5), (14), (16) and (17). Then, we get

$$\frac12\dot\phi^2=\dot\varphi^2UF_x,\qquad V=U[F-F_x x]. \tag{19}$$

We are demanding that the last be an identity, i.e., it does not impose any relation between the fields ϕ, φ and their derivatives. Under this requirement the second equation (19) translates into conditions

$$V(\phi)=U(\varphi),\qquad F-F_x x=1. \tag{20}$$

The first equation indicates that both potentials are the same when written as functions of cosmological time, but different when written as functions of the individual fields. Integrating the second equation (20) we get $F=1+mx$ with m an arbitrary integration constant. Inserting F into the first equation (19) we find the following relationship between the fields ϕ and φ [15]

$$\phi=\sqrt{2m}\int\sqrt{U}\,d\varphi. \tag{21}$$

It shows the *identity* between quintessence and k-essence models in flat FRW. For instance, the inverse square potential $U\propto\varphi^{-2}$ correspond to an exponential potential $V\propto e^{-\sqrt{2}\phi/\sqrt{mU_0}}$ [16] and $U\propto\varphi^{2n}$ to $V\propto\phi^{2n/(n+1)}$.

For $k\to Q$ the dual transformation (16), (18) gives

$$\frac12\dot\phi^2=-\dot\varphi^2UF_x,\qquad V=U[F-3F_x x]. \tag{22}$$

Requiring that these equations be identities, we have $V(\phi)=U(\varphi)$ and $F=1-mx^{1/3}$. Then, the first equation (22) gives the transformation rule for the kinetic terms

$$\dot\phi^2=\frac{2mU}{3}\dot\varphi^{2/3}. \tag{23}$$

For constant potentials $V=U=V_0>0$, the first integrals of Eqs. (6) and (15) can be written as $\dot\phi^2=2\bar b/\bar a^6$ and $\dot\varphi^{2/3}=2ba^6$ where the integration constants $\bar b$ and b transform as $\bar b=2mV_0b/3$ (see Eq. (23)). After solving Eq. (1), we express the duality as

$$a=\left[\pm\sqrt{\frac{2mb}{3}}\sinh\sqrt{3V_0}t\right]^{-1/3}\to\bar a=\frac1a,\qquad \bar b>0, \tag{24}$$

$$a=\left[\sqrt{-\frac{2mb}{3}}\cosh\sqrt{3V_0}t\right]^{-1/3}\to\bar a=\frac1a,\qquad \bar b<0. \tag{25}$$

The phantom sector of the duality comprises the $(-)$ branch of Eq. (24) for $Q\to k$ and the branch (25) for $k\to Q$. The former represents a cosmology with a future big rip singularity at $t=0$ and a real (imaginary) φ according to $m>0$ ($m<0$). The latter describes a non-singular cosmology with $\dot H>0$ and an imaginary ϕ.

$C\to Q$: We are going to find the relationships between a conformal scalar field model driven by the potential (7) and a scalar field one under an identical transformation. Using Eqs. (5), (8), (9), (16) and (17), we obtain

$$\frac12\dot\phi^2=\frac13(\dot\psi+H\psi)^2+\frac23\lambda\psi^4, \tag{26}$$

$$V(\phi)=\frac16(\dot\psi+H\psi)^2+\frac13\lambda\psi^4+\mathcal{V}_0, \tag{27}$$

the potential $V(\phi)=\mathcal{V}_0+\dot\phi^2/4$ and the scale factors (12), (13). Besides, the potential V can be reconstructed as a function of ϕ and

$$\phi=\ln\tanh\sqrt{\frac{\mathcal{V}_0}{3}}t,\qquad V=\mathcal{V}_0\left[\cosh^2\phi-\frac23\sinh^2\phi\right], \tag{28}$$

correspond to the solutions (12) and

$$\phi=-2i\arctan e^{\sqrt{4\mathcal{V}_0/3}t},\qquad V=\mathcal{V}_0\left[\cos^2 i\phi+\frac23\sin^2 i\phi\right], \tag{29}$$

to the solutions (13).

For $C\to Q$ the dual transformation (16), (18) gives

$$\frac12\dot\phi^2=-\frac13(\dot\psi+H\psi)^2-\frac23\lambda\psi^4, \tag{30}$$

$$V(\phi)=\frac56(\dot\psi+H\psi)^2+\frac53\lambda\psi^4+\mathcal{V}_0. \tag{31}$$

These equations lead to the potential $V(\phi)=\mathcal{V}_0-5\dot\phi^2/4$, so comparing the first integral of Eq. (6) with Eqs. (11) and (30), we have $\dot\phi^2=2\bar b\bar a^4$ with $\bar b=-2b/3$. After solving Eq. (1), we express the duality as

$$a_c^\pm\to\bar a_c^\pm=\left(a_c^\pm\right)^{-1},\qquad \bar b<0, \tag{32}$$

$$a_c\to\bar a_c=(a_c)^{-1},\qquad \bar b>0. \tag{33}$$

The phantom sector of the duality comprises the $(-)$ branch of Eq. (32) for $C \to Q$ and the branch (33) for $Q \to C$. The former represents a cosmology with a future big rip singularity at $t = 0$, where ϕ and V are obtained from (28), (29) making the substitution $\phi \to i\phi$. The latter describes a non-singular cosmology with $\dot{H} > 0$.

$C \to k$: From Eqs. (8), (9), (14), and (16), (17), we have

$$UF = -\frac{1}{6}(\dot{\psi} + H\psi)^2 - \frac{1}{3}\lambda\psi^4 + \mathcal{V}_0, \tag{34}$$

$$-2UxF_x = \frac{2}{3}(\dot{\psi} + H\psi)^2 + \frac{4}{3}\lambda\psi^4. \tag{35}$$

These equations lead to $U = \mathcal{V}_0$ and $F = 1 + mx^2$, which along with Eqs. (1) and (15) give the general solution (12), (13). For $m < 0$, the singular solution (12) of this purely kinematics k-essence model interpolates between radiation and de Sitter phases. For $m > 0$ the non-singular solution (13) bounces at $t = 0$.

For $C \to k$ the dual transformations (16), (18) gives

$$UF = \frac{7}{6}(\dot{\psi} + H\psi)^2 + \frac{7}{3}\lambda\psi^4 + \mathcal{V}_0, \tag{36}$$

$$-2UxF_x = -\frac{2}{3}(\dot{\psi} + H\psi)^2 - \frac{4}{3}\lambda\psi^4, \tag{37}$$

with $U = \mathcal{V}_0$ and $F = 1 + mx^{2/7}$. In this case the solution of Eq. (1) becomes Eqs. (12), (13) and the duality is expressed by Eqs. (32), (33).

$C \to C$: We complete this section investigating the duality between conformally coupled quintessences. Duality among $Q \to Q$ or $k \to k$ were studied in Refs. [3–15]. From, Eqs. (8), (9), (16) and (18), we have

$$\frac{1}{2}(\dot{\bar{\psi}} + \bar{H}\bar{\psi})^2 + \bar{\lambda}\bar{\psi}^4 = -\frac{1}{2}(\dot{\psi} + H\psi)^2 - \lambda\psi^4, \tag{38}$$

$$\bar{\mathcal{V}}_0 = \mathcal{V}_0 + (\dot{\psi} + H\psi)^2 + 2\lambda\psi^4. \tag{39}$$

Comparing Eqs. (11) and (38) we get $\bar{b}/\bar{a}^4 = -b/a^4 = \bar{b}a^4$, so $\bar{b} = b = 0$ and $\bar{\mathcal{V}}_0 = \mathcal{V}_0$ by Eq. (38). Eq. (1) turns into $3H^2 = \mathcal{V}_0$ and $a = e^{H_0 t}$, with $H_0 = \pm\sqrt{\mathcal{V}_0/3}$. Integrating Eq. (39) we obtain the conformal fields

$$\psi = \frac{H_0}{\sqrt{-2\lambda}[1 - e^{H_0 t}]}, \qquad \bar{\psi} = -\frac{H_0}{\sqrt{-2\bar{\lambda}}[1 - e^{-H_0 t}]}, \tag{40}$$

along with their transformation rule

$$\bar{\psi} = \frac{\sqrt{-2\lambda}}{\sqrt{-2\bar{\lambda}}}\left[\psi - \frac{H_0}{\sqrt{-2\lambda}}\right]. \tag{41}$$

So, the duality induces a linear transformation group acting on ψ and there is no phantom sector because $\dot{H} = 0$.

4. Conclusions

We have found a symmetry group generated by the additivity of the stress-energy tensor and shown that the different forms of summing two interacting fluid components induces two discrete symmetries in the Einstein equations. They correspond to cosmologies with *identical* geometry or to the *duality* between expanding and contracting backgrounds. The duality between contracting and superaccelerated expanding scenarios connects standard and *phantom* cosmologies.

The identical transformation relates two different cosmologies, both with the same scale factor, so the choice of a particular model to describe the evolution of the universe is not unique because there are a variety of ways of combining two interacting fluids with the same geometry. We highlight that in spatially flat FRW cosmologies, the dual transformation gives rise to linear relations between the components of the Einstein tensor, which read $\bar{G}^0_0 = G^0_0$, $\bar{G}^1_1 = 2G^0_0 - G^1_1$ and so on for the remaining spatial components. Consequently, the conservation laws preserve their form, i.e., $\bar{\nabla}_i\bar{G}^i_k = \nabla_i G^i_k = 0$. This analysis reveals that no matter how accurately future experiments may come to determine the cosmological observables there will always be a fundamental uncertainty about which of the possible models leading to the observed set of values has been chosen by Nature. This uncertainty could be avoided by considering other kind of frameworks as superstring field theory, M-theory or some specific interaction between the parts in which the total fluid was divided. For instance, in Ref. [17] it was shown that a particular interaction between both fluids reduces the linear transformations (16) to the identity restricting the form invariance symmetry.

We have shown that k-essence cosmologies generated by a linear kinetic function and quintessence ones share the same scale factor.

The conformally coupled quintessence model we have studied is equivalent to a purely kinematics k-essence model generated by a quadratic kinetic term. It describes a universe that begins to evolve as if it were *radiation* dominated at early times and ends in a constant *vacuum energy* dominated phase.

For $Q \to k$, $C \to Q$ and $C \to k$, we have found a phantom symmetry between contracting universes ending in a big crunch and expanding universes ending in a big rip. In these examples, the transformation rule for the scalar fields we have obtained is different than Wick rotation. In the case $C \to C$, the dual transformation is so restrictive allowing only the duality between expanding and contracting de Sitter geometries and inducing a linear relation among the conformal scalar fields.

Finally, the linear transformations we have found can be extended to the case of fluids with non-constant barotropic indexes and we think this is an interesting subject to be investigated in the future.

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