Robust Power Control for Cognitive Radio Networks with Proportional Rate Fairness

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Abstract
This paper studies the power control problem in cognitive radio networks where a primary user and multiple secondary users (SUs) coexist. Imperfect channel state information is considered. The objective is to maximize the SUs' sum rate while guaranteeing the proportional rate fairness among SUs. The problem under consideration is non-convex. By doing a transformation, it is equivalently changed to a second-order cone programming problem, which can be efficiently solved by existing standard methods. Simulations have been done to verify the network performance under different channel uncertainty conditions.

Index Terms: Cognitive radio networks, Robust power control, Imperfect channel state information, Proportional rate fairness

1. Introduction

As the rapid development of advanced technologies on wireless communications, a lot of high transmission rate services and applications have emerged, which increases the demand for spectrum. On the other hand, experimental results have shown that traditional fixed spectrum allocation schemes yield inefficient spectrum utilization [1]. To improve the spectrum utilization and provide high quality of services (QoS), cognitive radio networks (CRNs) that allow the unlicensed secondary users (SUs) share the licensed spectrum with the licensed primary users (PUs) have been proposed.

Spectrum allocation problem in CRNs has drawn large attention in recent years [2-7]. In most of these works, it is assumed that perfect channel state information (CSI) is known [2-4]. However, in practice perfect CSI, especially the channel gain from the SUs to PUs, cannot be obtained due to the lack of cooperation among PUs and SUs. Therefore, this motivates the research on resource allocation problem in CRNs with imperfect CSI [4-6]. Mitliagkas et al. investigated the joint power control and admission control problem in CRNs with imperfect CSI [5]. Kim et al. in [6] studied the sum rate maximization problem under the total power and interference power constraints. Parsaeefard et al. in [7] worked on the social utility of SUs while satisfying each SU’s signal to noise ratio requirement and interference power constraint. However, all those works do not explicitly consider SUs’ different transmission rate requirements and fairness issue, thus they are not suitable for a situation where different SUs have different transmission rate requirements. To flexibly allocate transmission rates to each SU and guarantee fairness among SUs, we will investigate the resource allocation problem with proportional rate fairness requirements in CRNs under imperfect CSI.

In this paper, we will investigate the power control problem in CRNs, where imperfect CSI from secondary BS to the primary user is considered. The objective is to maximize the SUs’ sum rate subject to the proportional rate fairness constraint among SUs, the total power constraint at secondary BS, and the interference power constraint to the PU. The problem is formulated as a non-convex optimization problem. By doing a transformation, the problem is changed to an equivalent second-order cone programming (SOCP) problem, which can be efficiently solved by existing standard methods. Simulations have been done to demonstrate the network performance under different channel uncertainty conditions.
2. System Model and Problem Formulation

Consider a network setting where a PU and \( K \) SUs coexist. Downlink transmission from the secondary base station (BS) to SUs is considered. The SUs can adopt the available channels that are licensed to the PU for its own data transmission. It is assumed that the total available bandwidth is divided into multiple non-overlapping channels. And each SU is allocated one such channel for its own data transmission.

The channel gain from the secondary BS to SU \( k \), \( \forall k \in \{1, 2, \cdots, K\} \) is denoted by \( h_k \). \( \sigma_k \) is the variance of the additive white Gaussian noise in that channel. For notational brevity, let \( h_k = h_k / \sigma_k \). The data rate for SU \( k \) is denoted by

\[
R_k = 0.5 \log_2 \left( 1 + h_k P_k \right), \tag{1}
\]

where \( P_k \) is the transmission power for SU \( k \).

To protect the PU’s QoS, the interference to the PU should not be greater than the given threshold \( T_{th} \). This can be expressed by

\[
\sum_{k=1}^{K} P_k d_k \leq T_{th}, \tag{2}
\]

where \( d_k \) is the channel gain from the secondary BS to the primary user. In practice, imperfect channel information cannot be obtained, especially the channel gain from the secondary users to the primary users. Because generally there is a lack of cooperation between primary user and SUs, and thus the primary user will not feedback the CSI to the SUs. Ellipsoidal uncertainty will be adopted to model the uncertainty of channel gain \( d_k \). Let us define vector \( d = [d_1, d_2, \cdots, d_K]^T \). Adopting the ellipsoidal uncertainty \cite{5}, the uncertainty region of \( d \) can be expressed by

\[
d \in \Omega = \{ d + Du : \|u\|_2 \leq 1 \}, \tag{3}
\]

where \( \bar{d} \) is the nominal value of \( d \), \( D \) is a \( K \times K \) matrix, and \( u \) is a \( K \) dimensional vector. To facilitate the following analysis, let us define a vector \( \mathbf{P}_s = \begin{bmatrix} P_1 & P_2 & \cdots & P_K \end{bmatrix}^T \), and then (2) can be rewritten as

\[
d^T \mathbf{P}_s \leq T_{th}. \tag{4}
\]

Since \( d \) satisfies (3), to guarantee (4) hold, it is equivalent to make sure the following inequality (5) holds,

\[
\sup_{d \in \Omega} \| \mathbf{d}^T \mathbf{P}_s \| \leq T_{th}. \tag{5}
\]

From (5), by invoking the Cauchy-Schwarz inequality, one gets that

\[
\sup_{d \in \Omega} \| \mathbf{d}^T \mathbf{P}_s \| = \bar{d}^T \mathbf{P}_s + \| \mathbf{u}^T D \mathbf{P}_s \| \leq \bar{d}^T \mathbf{P}_s + \| \mathbf{u} \| \| D \mathbf{P}_s \|_2 \leq \bar{d}^T \mathbf{P}_s + \| \mathbf{P}_s \| \| \mathbf{D} \| \leq T_{th}.
\]

We desire to study the power control problem to maximize the sum rate of SUs under several constraints. The problem under consideration can be formulated as follows,

\[
\max_{\{P_k, R_k\}} \sum_{k=1}^{K} R_k \quad \text{s.t.} \quad \begin{align*}
C1. & \sum_{k=1}^{K} P_k \leq P_{th} \\
C2. & 0 \leq P_k, \forall k \in \{1, 2, \cdots, K\} \\
C3. & (6) \\
C4. & R_1 : R_2 : \cdots : R_K = \gamma_1 : \gamma_2 : \cdots : \gamma_K \\
C5. & \geq 1, \forall k \in \{1, 2, \cdots, K\}
\end{align*} \tag{7}
\]

Where \( C1 \) represents the BS total power constraint, and \( P_{th} \) is the power threshold at the BS. \( C2 \) indicates that the consumed power for each SU at the BS should be non-negative. \( C3 \) is the interference power constraint to the primary user. \( C4 \) is the proportional rate fairness constraint; \( \gamma_1, \gamma_2, \cdots, \gamma_K \) are given constants, and they indicate the proportional rate requirements of SUs. \( C5 \) represents the SU’s transmission rate constraint.

3. Optimal Solution

Problem (7) is a non-convex optimization problem since the nonlinear equality constraint \( C5 \). To make the problem easy to solve, we will transform problem (7) into its equivalent form. By replacing the equality constraint in \( C5 \) by an inequality form

\[
R_k \leq 0.5 \log_2 (1 + h_k P_k), \tag{8}
\]

problem (7) becomes

\[
\max_{\{P_k, R_k\}} \sum_{k=1}^{K} R_k \quad \text{s.t.} \quad \begin{align*}
C1. & \sum_{k=1}^{K} P_k \leq P_{th} \\
C4. & R_1 : R_2 : \cdots : R_K = \gamma_1 : \gamma_2 : \cdots : \gamma_K \\
C5'. & R_k \leq 0.5 \log_2 (1 + h_k P_k), \forall k \in \{1, \cdots, K\}
\end{align*} \tag{9}
\]

Problem (9) is an SOCP problem, since its objective function is a linear function, its constraint set is a convex set, and \( C3 \) is a second-order cone constraint. A proposition will be given in the following to show that the optimal solution of problem (9) satisfies \( R_k = 0.5 \log_2 (1 + h_k P_k) \), and thus problem (9) is equivalent to problem (7). Hence, we can solve Problem (9) instead of Problem (7).

**Proposition 1.** The rates that optimize problem (9) satisfy that \( R_k = 0.5 \log_2 (1 + h_k P_k), \forall k \in \{1, 2, \cdots, K\} \).

**Proof.** Because the objective function of problem (9) is an increasing function with respect to \( R_k \), and \( R_k \) satisfies constraint \( C5' \). It is easy to see that when problem (9) admits its optimal solution \( R_k \) satisfies that \( R_k = 0.5 \log_2 (1 + h_k P_k), \forall k \in \{1, 2, \cdots, K\} \). \( \blacksquare \)
Problem (9) is an SOCP problem, existing standard methods such as interior-point methods can solve it efficiently. In Section 3, CVX toolbox [8] will be used to find the optimal solution of problem (9).

4. Numerical Results

In this Section, simulation results are presented to illustrate the network performance under different channel uncertainty conditions.

Consider a simulation model shown in Fig.1, where the secondary BS is located at (0, 0) and the primary user is located at (294 meter, 500 meter). There are eight SUs, which are randomly generated around the BS. The channel gain in any transmission pair contains a large-scale Rayleigh fading component and a large scale path loss component with path loss factor four. The uncertainty of channel gain in (6) is given by [3]

$$D(i, j) = \begin{cases} 
\alpha \theta_i d_i, & \text{if } i = j \\
0, & \text{otherwise}
\end{cases}$$

(10)

Where $D(i, j)$ indicates the element on the $i$th row and $j$th column of $D$, and $\theta, \theta_i \in [0, 1]$.

Fig. 1 Simulation model

Fig. 2 Sum rate versus $T_{th}$

Fig. 3 SU’s transmission rate distribution

Fig. 2 shows the sum rate of the SUs versus the interference power threshold. The parameters are $P_{th} = 4W$ and $\gamma_1 : \gamma_2 : \cdots : \gamma_8 = 1:1:2:2:3:3:4:4$. The curves can be parted into two parts. In the first part, i.e., $T_{th} < 0.4$, the SU’s sum rate increases as the interference power threshold $T_{th}$ increases, that is because during this period the interference power constraint is the dominating constraint. As the increase of $T_{th}$, more transmission power can be used for SUs’ data transmission, and thus the sum rate increases. During this period, the sum rate obtained with a smaller $\alpha$ is always much higher than that achieved with a greater $\alpha$. That is because the uncertainty region of the channel gain with a greater $\alpha$ is much larger than that with a smaller $\alpha$. The algorithm needs to sacrifice much more sum rate to guarantee all the constraints are satisfied when the uncertainty region of the channel gain is much larger. In the second part, as the increase of $T_{th}$, the total power constraint gradually becomes the dominating constraint at some different points of $T_{th}$ for the three cases with different values of $\alpha$. After these points they will keep at their highest sum rate no matter how large $T_{th}$ becomes. Although the three cases achieve their highest sum rates at different values of $T_{th}$, the final sum rates are the same.

Fig. 3 shows each SU’s transmission rate distribution with different values of $\alpha$ when $T_{th} = 0.25$. The other simulation parameters are the same as those in Fig. 2. It is evident from Fig. 3 that the SU’s transmission rate satisfies the proportional fairness constraint. And each SU’s transmission rate with a smaller value of $\alpha$ is much higher than that with a greater value of $\alpha$. 

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5. Conclusion

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References

1. D. Chen, S. Yin, Q. Zhang, M. Liu, and S. Li, “Mining

spectrum usage data: a large-scale spectrum measurement

study,” in Proceedings of the 15th Annual ACM

International Conference on Mobile Computing and

Networking, pp.13–24, Beijing, China, September 2009.


and power allocation in OFDMA based cognitive

radio networks,” in Proceedings of IEEE International

Conference on Communications, pp. 1–5, Shanghai,


3. Z. Wang, L. Jiang, and C. He, “A novel pricebased power

control algorithm in cognitive radio networks,” IEEE


4. G. Bansal, M. Hossain, and V. Bhargava, “Optimal and

suboptimal power allocation schemes for OFDM-based

cognitive radio systems,” IEEE Transactions on Wireless


5. I. Mitliagkas, N. Sidiropoulos, and A. Swami, “Joint

power and admission control for adhoc and cognitive

underlay networks: Convexapproximation and distributed

implementation,” IEEE Trans. Wireless Communications,


Allocation for OFDMA cognitive radios under channel

uncertainty,” IEEE Trans. Wireless Communications,


7. S. Parsaeefard and A. Sharafat, “Robust distributed power

control in cognitive radio networks,” IEEE Transactions


8. CVX Research Inc., “CVX: Matlab software for disciplined

convex programming, version 2.0,” http://cvxr.com/cvx,

2012.

5. Conclusion

In this work, we have considered the power control problem

in CRNs with channel uncertainty. Our objective is to

maximize the sum rate of SUs while guaranteeing the

proportional rate fairness among SUs. The problem is

formulated as a non-convex optimization problem. By

doing a problem transformation, it becomes an SOCP

problem. And it can be efficiently solved by existing

methods. In the future, we will propose distributed

algorithms to solve this problem.

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Fig. 4 shows the sum rate changes with total power $P_{th}$. The

parameters are $\gamma_1: \gamma_2: \cdots: \gamma_8 = 1: 1: 2: 2: 3: 3: 4: 4$

and $T_{th} = 0.3W$. When $P_{th} \leq 2.6$, the figure shows that the

problems with different values of $\alpha$ obtain the same sum

rate, that is because when $P_{th} \leq 2.6$, the total power

constraint is the dominating constraint for the three cases,

thus all the curves achieve the same sum rate. As the

increase of $P_{th}$, the interference power constraint gradually

becomes the dominating constraint. The greater the value of

$\alpha$, the earlier the interference constraint becomes a

dominating constraint. When the interference constraint

becomes a dominating constraint, the sum rate will keep

constant even if $P_{th}$ changes.

Fig. 4 Sum rate versus total power

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