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Radiation and mass transfer effects on MHD flow through porous medium past an exponentially accelerated inclined plate with variable temperature

Jyotsna Rani Pattnaik [a,*], Gouranga Charan Dash [b], Suprava Singh [a]

[a] *Department of Mathematics, Padmanava College of Engg., Rourkela, Odisha, India*
[b] *Department of Mathematics, I.T.E.R., S'O'A University, Bhubaneswar, Odisha, India*

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Abstract An analysis of unsteady MHD free convection flow, heat and mass transfer past an exponentially accelerated inclined plate embedded in a saturated porous medium with uniform permeability, variable temperature and concentration has been carried out. The novelty of the present study was to analyze the effect of angle of inclination on the flow phenomena in the presence of heat source/sink and destructive reaction. The Laplace transformation method has been used to solve the governing equations. The effects of the material parameters, magnetic field and the permeability of the porous medium are discussed. From the present analysis it is reported that the presence of magnetic field and porous medium prevents the flow reversal. Angle of inclination and heat source sustains a retarding effect on velocity. The present study has an immediate application in understanding the drag experienced at the heated/cooled and inclined surfaces in a seepage flow.

1. Introduction

The problem of free convection and mass transfer flow of an electrically conducting fluid past an inclined heated surface under the influence of magnetic field has attracted interest in view of its applications to geophysics, astrophysics and many engineering problems, such as cooling of nuclear reactors, boundary layer control in aerodynamics and cooling towers. Umemura and Law [1] have developed a generalized formulation for the natural convection boundary layer flow over a flat plate with arbitrary inclination. They have found that the flow characteristics depend not only on the extent of inclination but also on the distance from the leading edge. Alam et al. [2] have studied MHD free convective heat and mass transfer flow past an inclined surface with heat generation. They have not considered the effect of radiation and permeability of the medium in their study but in their method of solution they have made use of shooting iteration technique suggested by Nachtsheim and Swigert [3]. Unsteady free convection in a fluid flow past an infinite inclined plate immersed in a porous medium has been considered for viscous dissipative heat by Uddin and

* Corresponding author.
E-mail addresses: jrp_bapi@rediffmail.com (J.R. Pattnaik), gcdash45@gmail.com (G.C. Dash).

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Nomenclature

a^*	absorption coefficient
a	dimensionless accelerating parameter
t	dimensionless time
D	chemical molecular diffusivity
C'	dimensional concentration in the fluid
C'_w	concentration of the fluid near the plate
C'_∞	concentration of the fluid far away from the plate
q_r	radiative heat flux
T	dimensionless temperature of the fluid
T'_w	constant temperature of the plate
T'_∞	temperature of the fluid far away from the plate
S'	constant heat source
K_p	porosity parameter
K'_p	permeability of porous medium
K_c	chemical reaction parameter
K'_c	reaction rate constant
u'	velocity of the fluid in the x'-direction
u_0	characteristic velocity of the plate
U	dimensionless velocity
N_u	Nusselt number
S_c	Schmidt number
S	dimensionless heat source
M	magnetic field parameter
x', y'	co-ordinate axes along and perpendicular to the plate
a'	dimensional acceleration parameter
t'	dimensional time
C_p	specific heat at constant pressure
C	dimensionless fluid concentration
G_c	mass Grashof number
G_r	thermal Grashof number
g	acceleration due to gravity
P_r	Prandtl number
R	radiative parameter
T'	dimensional temperature
B_0	magnetic field strength

Greek symbols

μ	coefficient of viscosity
υ	kinematic coefficient of viscosity
β'	volumetric coefficient of expansion with concentration
β	volumetric coefficient of thermal expansion
ρ	density
σ	electric conductivity
τ	skin friction
α	thermal diffusivity
γ	inclination angle from the vertical direction
κ	thermal conductivity of the fluid

Kumar [4]. Convection effects on flow past an inclined plate with variable surface temperatures in water at 4 °C has been studied by Palani [5]. They have observed that as the angle of the plate from vertical direction increases the value of friction factor and heat transfer coefficient decreases. This is of interest to chemical engineers. Singh and Makinde [6] have analyzed MHD free convection flow along an inclined plate with Newtonian heating in the presence of exponentially decaying volumetric heat source.

Considering the importance of radiative heat and mass transfer of an electrically conducting fluid several studies have been made by many authors (Ogulu and Makinde [7], Makinde [8,9]) by formulating simple models. Radiation and mass transfer effects on MHD free convection flow through porous medium past an exponentially accelerated vertical plate with variable temperature has been studied by Pattnaik et al. [10]. Barik et al. [11] have studied the thermal radiation effect on an unsteady MHD flow past an inclined porous heated plate in the presence of chemical reaction and viscous dissipation.

The study of heat generation or absorption in moving fluids is important in problems dealing with chemical reactions dissociating fluids. Since some fluids can also emit and absorb thermal radiation, it is of interest to study the effects of magnetic field on the temperature distribution vis-à-vis heat transfer when the fluid is not only an electrical conductor but also it is capable of emitting and absorbing radiation. Makinde [12] has studied the computational modeling of MHD unsteady flow and heat transfer over a flat plate with Navier slip and Newtonian heating. Muthucumaraswamy and Kumar Senthil [13] have analyzed the thermal diffusion effects on moving infinite vertical plate in the presence of variable temperature and mass diffusion. Rahman and Sattar [14] have studied MHD convective flow of a micropolar fluid past a continuously moving vertical porous plate in the presence of heat generation or absorption. Recently, Mishra et al. [15] have studied the heat and mass transfer effect on MHD flow of a viscoelastic fluid through porous medium with oscillatory suction and heat source. Kumar and Varma [16] have studied radiation effects on MHD flow past an impulsively started exponentially accelerated vertical plate with variable temperature in the presence of heat generation. The cases of flow past vertical surfaces and along the horizontal surfaces are many, though unrealistic in many engineering applications. The most important application of MHD is the generation of electrical power with the flow of an electrically conducting fluid through a transverse magnetic field. Recently, experiments with ionized gases have been performed with the hopes of producing power on a large scale in stationary plants with large magnetic fields. Cryogenic and superconducting magnets are required to produce these very large magnetic fields. Generation of MHD power on a smaller scale is of interest for space applications.

Further, Chen [17] has performed an analysis to study the natural flow over a permeable inclined surface with variable wall temperature and concentration. He has found that increasing the angle of inclination decreases the effect of buoyancy force. Recently, Ali et al. [18] have studied effects of heat and mass transfer on MHD free convection flow over an inclined plate embedded in a porous medium. They have considered impulsively started plate with variable temperature and mass transfer. Further, Ismail et al. [19] have studied rotation effects on coupled heat and mass transfer on unsteady MHD

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free convection flow in a porous medium past an infinite inclined plate. They have obtained exact solutions using Laplace transform. Influence of thermal radiation on unsteady free convection MHD flow of Brinkman type fluid in a porous medium with Newtonian heating has been studied by Ali et al. [20]. They have considered four different types of plate motion such as (i) flow induced by an impulsively motion of the plate, (ii) flow due to uniform acceleration of the plate, (iii) flow due to nonuniform acceleration of the plate, and (iv) flow due to highly nonuniform acceleration of the plate. The interesting part of the study is to obtain an analytical solutions using Laplace transform, which can be used as a benchmark by numerical analysts.

Very recently, a research article entitled effects of wall shear stress on MHD conjugate flow over an inclined plate in porous medium with ramped wall temperature is reported by Khan et al. [21]. They have found exact solutions for velocity and temperature in case of both ramped and constant wall temperatures. Further, Ismail et al. [22] have reported the effects of magnetohydrodynamic and radiation on flow of second grade fluid past an infinite inclined plate in porous medium in AIP conference proceedings. Moreover, Ismail et al. [23] have considered rotating MHD unsteady free convection flow of second grade fluid in a porous medium. Another study related to rotation and heat absorption on unsteady MHD free convection flow through porous medium past an infinite inclined plate with ramped wall temperature has been reported in "Recent Advance in Mathematics" by Ismail et al. [24]. Not much work related to the radiative, oscillatory flow past inclined surfaces embedded in porous medium is reported in the literature though it is of frequent occurrence in the industrial applications.

The present study focuses on a few physical situations, particularly in seepage flow, in which the entire flow domain may not be exactly vertical/horizontal. Further, electrically conducting fluids are of usual occurrence due to contamination and industrial waste. The oscillatory motion is usual in industries when the oscillating surfaces are embedded in a porous medium. Here, we have accounted for the effect of permeability of the medium through a linear Darcy model. The consideration of thermal radiation in energy equation and inclusion of first order chemical reaction in the solutal equation contribute to enhance the number of parameters in flow model without compromising the possible occurrence in physical situation. Further, the consideration of time dependent variation of temperature and concentration, which are common in practice, makes the study more realistic (Re < 1). Besides, the flow phenomenon, our objective is to find out heat transfer rate to a conducting flow under the influence of magnetic field. The application of this field of heat transfer is needed in many systems involving high temperature plasmas, liquid–metal and MHD power generation systems (Yadav et al. [25]). The analytical solutions of the governing equations are obtained through Laplace transformation. The exclusion of the nonlinear terms of the inertial forces, i.e. convected acceleration terms, renders the problem linear representing slow motion with very small Reynolds number. In such flows, the frictional forces are much larger than the inertial forces. The approximate solutions of system are supposed to hold true for the systems with Reynolds number less than one [26]. The exclusion of the nonlinear terms makes easier to apply Laplace transformation but it represents a specific flow condition. Even after inclusion of nonlinear terms, the system can be solved by Laplace transformation applying an iterative procedure. In the first iteration, the nonlinear part of the equation is to be omitted. Then Laplace transformation method is applied for the linear part. After inversion, the function is determined. Now, consider the entire equation including the nonlinear part and apply Laplace transform. Repeat the process until a stable solution is obtained.

The free convective flow of an electrically conducting as well as radiating viscous fluid past an exponentially accelerated inclined plate embedded in a porous medium with variable surface temperature and concentration in the presence of transverse magnetic field, heat source and chemical reaction has been studied. The objective of the present work is to record the effects of pertinent parameters governing the flow and to discuss the work of Kumar and Varma [16] as a particular case. They have considered in their study, the fluid is gray, absorbing/emitting radiation but a non-scattering medium. They have not considered the effect of free convection, permeability of the porous medium and mass transfer due to difference in concentration of diffusing species. They have observed that there is a fall in velocity in the presence of high radiation. In the computation they have taken $P_r = 0.71$, $M = 3$ and $R = 15$ in a specific velocity profile. Further, Rajput and Kumar [27] have studied radiation effects on MHD flow with variable heat and mass transfer. Muthucumaraswamy et al. [28] have also studied radiative heat and mass transfer on moving isothermal vertical plate in the presence of chemical reaction. The above studies are related to thermal radiation. The authors have taken $P_r = 0.71$ in their computation.

2. Formulation and solution of the problem

In this problem we consider an unsteady uniform MHD free convective flow of a viscous, incompressible and radiating fluid past an exponentially accelerated inclined infinite plate with variable temperature embedded in a saturated porous medium. The x'-axis is taken along the plate and y'-axis is normal to the plate. Magnetic field of intensity B_0 is applied in the direction perpendicular to the plate. The plate is inclined to vertical direction by an angle γ. The induced magnetic field is neglected as the magnetic Reynolds number of the flow is very small. Initially, it is assumed that the plate and the surrounding fluid are at the same temperature T'_∞ and concentration C'_∞. At time $t' > 0$, the plate is exponentially accelerated with a velocity $u' = u_0 \exp(a't')$ in its own plane. At the same time the temperature and concentration level are also raised or lowered linearly with time t. The physical model is represented in Fig. 1. Following Bansal [29], Schlichting and Gersten [30], Kumar and Varma [16] the boundary layer equations of flow, heat and mass transfer past an exponentially accelerated inclined plate are given by

$$\frac{\partial u'}{\partial t'} = g\beta(T' - T'_\infty)\cos\gamma + g\beta'(C' - C'_\infty)\cos\gamma + \upsilon\frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_o^2 u'}{\rho} - \frac{\upsilon u'}{K'_p} \tag{1}$$

$$\rho C_p \frac{\partial T'}{\partial t'} = \kappa\frac{\partial^2 T'}{\partial y'^2} - \frac{\partial q_r}{\partial y'} + S'(T' - T'_\infty) \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y'^2} - K'_c(C' - C'_\infty) \tag{3}$$

The initial and boundary conditions are

$$\left.\begin{array}{l} t' \leqslant 0: \quad u' = 0, \quad T' = T'_\infty, \quad C' = C'_\infty \quad for\ all\ y' \\ t' > 0: \quad u' = u_0 \exp(a't'), \quad T' = T'_\infty + \frac{(T'_w - T'_\infty)u_0^2 t'}{\upsilon}, \\ \qquad C' = C'_\infty + \frac{(C'_w - C'_\infty)u_0^2 t'}{\upsilon} \qquad at\ y' = 0 \\ \qquad and\ u' \to 0, \quad T' \to T'_\infty, \quad C' \to C'_\infty \quad as \quad y' \to \infty \end{array}\right\} \tag{4}$$

The boundary conditions for the temperature at the plate impose a linearity relation between temperature and time with a residual temperature T'_∞ and having a constant slope u_0^2/υ, which depends upon square of the characteristic velocity and material property. Similar explanation holds for concentration at the plate. The fluid considered here is a gray, absorbing/emitting radiation but a non-scattering medium. The local gradient for the case of an optically thin gray gas (England and Emery [31], Pattnaik et al. [10]) is expressed by

$$\frac{\partial q_r}{\partial y'} = -4a^*\sigma(T'^4_\infty - T'^4) \tag{5}$$

We assumed that the temperature differences within the flow are sufficiently small such that T'^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T'^4 in a Taylor series about T'_∞ and neglecting the higher order terms, we get

$$T'^4 \cong 4T'^3_\infty T' - 3T'^4_\infty \tag{6}$$

Using Eqs. (5) and (6) in (2) we have,

$$\rho C_p \frac{\partial T'}{\partial t'} = \kappa \frac{\partial^2 T'}{\partial y'^2} - 16a^*\sigma T'^3_\infty (T' - T'_\infty) + S'(T' - T'_\infty) \tag{7}$$

On introducing the following non-dimensional quantities

$$\left.\begin{array}{l} y = \frac{y'u_0}{\upsilon},\ U = \frac{u'}{u_0},\ t = \frac{t'u_0^2}{\upsilon},\ a = \frac{a'\upsilon}{u_0^2},\ T = \frac{(T' - T'_\infty)}{(T'_w - T'_\infty)},\ C = \frac{(C' - C'_\infty)}{(C'_w - C'_\infty)} \\ G_r = \frac{g\beta\upsilon(T'_w - T'_\infty)}{u_0^3}, \Pr = \frac{\mu C_p}{\kappa}, G_c = \frac{g\beta'\upsilon(C'_w - C'_\infty)}{u_0^3}, M = \frac{\sigma B_0^2 \upsilon}{\rho u_0^2}, \\ K_p = \frac{u_0^2 K'_p}{\upsilon^2},\ K_c = \frac{\upsilon K'_c}{u_0^2},\ R = \frac{16a^*\upsilon^2\sigma T'^3_\infty}{\kappa u_0^2},\ S = \frac{S'\upsilon}{\rho C_p u_0^2},\ S_c = \frac{\upsilon}{D}, \end{array}\right\} \tag{8}$$

in Eqs. (1), (3), (7) and (4) we get

$$\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial y^2} - MU - \frac{U}{K_p} + G_r T\cos\gamma + G_c C\cos\gamma \tag{9}$$

$$\frac{\partial T}{\partial t} = \frac{1}{P_r}\frac{\partial^2 T}{\partial y^2} - \frac{RT}{P_r} + ST \tag{10}$$

$$\frac{\partial C}{\partial t} = \frac{1}{S_c}\frac{\partial^2 C}{\partial y^2} - K_c C \tag{11}$$

The initial and boundary conditions in dimensionless form are

$$\left.\begin{array}{l} U = 0,\ T = 0,\ C = 0 \quad \forall y, \quad t \leqslant 0 \\ \left.\begin{array}{l} U = \exp(at),\ T = t,\ C = t\ at\ y = 0 \\ U \to 0,\ T \to 0,\ C \to 0\ as\ y \to \infty \end{array}\right\} \quad t > 0 \end{array}\right\} \tag{12}$$

Using Laplace transform and inverse Laplace transform to Eqs. (9)–(11) with initial and boundary conditions (12) we obtain the followings:

$$\begin{aligned} U = {} & \left(\frac{\exp(at)}{2}\right)\Big[\exp\left(y\sqrt{\lambda + a}\right) erfc\left(\frac{y}{2\sqrt{t}} + \sqrt{(\lambda + a)t}\right) \\ & + \exp(-y\sqrt{\lambda + a}) erfc\left(\frac{y}{2\sqrt{t}} - \sqrt{(\lambda + a)t}\right)\Big] \\ & + \left(\frac{G_r\cos\gamma}{2(P_r - 1)}\right)\Big[\frac{\exp(\alpha_1 t)}{\alpha_1^2}\Big\{\exp\left(y\sqrt{\lambda + \alpha_1}\right) \\ & \times erfc\left(\frac{y}{2\sqrt{t}} + \sqrt{(\lambda + \alpha_1)t}\right) \\ & + \exp\left(-y\sqrt{\lambda + \alpha_1}\right) erfc\left(\frac{y}{2\sqrt{t}} - \sqrt{(\lambda + \alpha_1)t}\right) \\ & - \exp\left(y\sqrt{(d + P_r\alpha_1)}\right) erfc\left(\frac{y}{2}\sqrt{\frac{P_r}{t}} + \sqrt{\left(\frac{d}{P_r} + \alpha_1\right)t}\right) \\ & - \exp\left(-y\sqrt{(d + P_r\alpha_1)}\right) erfc\left(\frac{y}{2}\sqrt{\frac{P_r}{t}} - \sqrt{\left(\frac{d}{P_r} + \alpha_1\right)t}\right)\Big\} \\ & - \frac{1}{\alpha_1}\left(t + \frac{1}{\alpha_1} + \frac{y}{2\sqrt{\lambda}}\right)\exp\left(y\sqrt{\lambda}\right) erfc\left(\frac{y}{2\sqrt{t}} + \sqrt{\lambda t}\right) \\ & - \frac{1}{\alpha_1}\left(t + \frac{1}{\alpha_1} - \frac{y}{2\sqrt{\lambda}}\right)\exp\left(-y\sqrt{\lambda}\right) erfc\left(\frac{y}{2\sqrt{t}} - \sqrt{\lambda t}\right) \\ & + \frac{1}{\alpha_1}\left(t + \frac{1}{\alpha_1} + \frac{yP_r}{2\sqrt{d}}\right)\exp\left(y\sqrt{d}\right) erfc\left(\frac{y}{2}\sqrt{\frac{P_r}{t}} + \sqrt{\frac{dt}{P_r}}\right) \\ & + \frac{1}{\alpha_1}\left(t + \frac{1}{\alpha_1} - \frac{yP_r}{2\sqrt{d}}\right)\exp\left(-y\sqrt{d}\right) erfc\left(\frac{y}{2}\sqrt{\frac{P_r}{t}} - \sqrt{\frac{dt}{P_r}}\right)\Big] \\ & + \left(\frac{G_c\cos\gamma}{2(S_c - 1)}\right)\Big[\frac{\exp(\alpha_2 t)}{\alpha_2^2}\Big\{\exp\left(y\sqrt{\lambda + \alpha_2}\right) \\ & \times erfc\left(\frac{y}{2\sqrt{t}} + \sqrt{(\lambda + \alpha_2)t}\right) \\ & + \exp\left(-y\sqrt{(\lambda + \alpha_2)t}\right) erfc\left(\frac{y}{2\sqrt{t}} - \sqrt{(\lambda + \alpha_2)t}\right) \\ & - \exp\left(y\sqrt{S_c(K_c + \alpha_2)}\right) erfc\left(\frac{y}{2}\sqrt{\frac{S_c}{t}} + \sqrt{(K_c + \alpha_2)t}\right) \\ & - \exp\left(-y\sqrt{S_c(K_c + \alpha_2)}\right) erfc\left(\frac{y}{2}\sqrt{\frac{S_c}{t}} - \sqrt{(K_c + \alpha_2)t}\right)\Big\} \\ & - \frac{1}{\alpha_2}\left(t + \frac{1}{\alpha_2} + \frac{y}{2\sqrt{\lambda}}\right)\exp\left(y\sqrt{\lambda}\right) erfc\left(\frac{y}{2\sqrt{t}} + \sqrt{\lambda t}\right) \\ & - \frac{1}{\alpha_2}\left(t + \frac{1}{\alpha_2} - \frac{y}{2\sqrt{\lambda}}\right)\exp\left(-y\sqrt{\lambda}\right) erfc\left(\frac{y}{2\sqrt{t}} - \sqrt{\lambda t}\right) \\ & + \frac{1}{\alpha_2}\left(t + \frac{1}{\alpha_2} + \frac{y}{2}\sqrt{\frac{S_c}{K_c}}\right)\exp\left(y\sqrt{S_c K_c}\right) erfc\left(\frac{y}{2}\sqrt{\frac{S_c}{t}} + \sqrt{K_c t}\right) \\ & + \frac{1}{\alpha_2}\left(t + \frac{1}{\alpha_2} - \frac{y}{2}\sqrt{\frac{S_c}{K_c}}\right)\exp\left(-y\sqrt{S_c K_c}\right) erfc\left(\frac{y}{2}\sqrt{\frac{S_c}{t}} - \sqrt{K_c t}\right)\Big] \end{aligned} \tag{13}$$

$$\begin{aligned} T = \frac{1}{2}\Big[& \left(t + \frac{yP_r}{(2\sqrt{d})}\right)\exp\left(y\sqrt{d}\right) erfc\left(\left(\frac{y}{2}\right)\sqrt{\frac{P_r}{t}} + \sqrt{\frac{dt}{P_r}}\right) \\ & + \left(t - \frac{yP_r}{(2\sqrt{d})}\right)\exp\left(-y\sqrt{d}\right) erfc\left(\left(\frac{y}{2}\right)\sqrt{\frac{P_r}{t}} - \sqrt{\frac{dt}{P_r}}\right)\Big] \end{aligned} \tag{14}$$

$$\begin{aligned} C = \frac{1}{2}\Big[& \left(t + \left(\frac{y}{2}\right)\sqrt{\frac{S_c}{K_c}}\right)\exp\left(y\sqrt{S_c K_c}\right) erfc\left(\left(\frac{y}{2}\right)\sqrt{\frac{S_c}{t}} + \sqrt{K_c t}\right) \\ & + \left(t - \left(\frac{y}{2}\right)\sqrt{\frac{S_c}{K_c}}\right)\exp\left(-y\sqrt{S_c K_c}\right) erfc\left(\left(\frac{y}{2}\right)\sqrt{\frac{S_c}{t}} - \sqrt{K_c t}\right)\Big] \end{aligned} \tag{15}$$

Now, the expression for skin friction (τ) which measures shear stress at the plate, Nusselt number (Nu) and Sherwood number (Sh) is presented in the following form

Skin friction:

$$
\begin{aligned}
\tau = & -\frac{\partial U}{\partial y}\Big|_{y=0} \\
= & \exp(at)\Big[\sqrt{\lambda+a}\Big(1-erfc\sqrt{(\lambda+a)t}\Big)+\frac{1}{\sqrt{\pi t}}\exp(-t(\lambda+a))\Big] \\
& +\left(\frac{G_r\cos\gamma}{(1-P_r)}\right)\left[\frac{\exp(\alpha_1 t)}{\alpha_1^2}\Big\{\sqrt{\lambda+\alpha_1}\Big(erfc\sqrt{(\lambda+\alpha_1)t}-1\Big)\right. \\
& -\frac{1}{\sqrt{\pi t}}\exp\left(-t(\lambda+\alpha_1)\right) \\
& -\sqrt{d+P_r\alpha_1}\left(erfc\sqrt{\left(\frac{d}{P_r}+\alpha_1\right)t}-1\right) \\
& +\sqrt{\frac{P_r}{\pi t}}\exp\left(-t\left(\frac{d}{P_r}+\alpha_1\right)\right)\Big\}+\frac{1}{2\alpha_1\sqrt{\lambda}}\Big(1-erfc\sqrt{\lambda t}\Big) \\
& +\frac{1}{\alpha_1}\left(t+\frac{1}{\alpha_1}\right)\Big\{\sqrt{\lambda}\Big(1-erfc\sqrt{\lambda t}\Big)+\frac{1}{\sqrt{\pi t}}\exp\left(-\lambda t\right)\Big\} \\
& -\frac{P_r}{2\alpha_1\sqrt{d}}\left(1-erfc\sqrt{\frac{dt}{P_r}}\right)-\frac{1}{\alpha_1}\left(t+\frac{1}{\alpha_1}\right) \\
& \left.\times\left\{\sqrt{d}\left(1-erfc\sqrt{\frac{dt}{P_r}}\right)+\sqrt{\frac{P_r}{\pi t}}\exp\left(-\frac{dt}{P_r}\right)\right\}\right] \\
& +\left(\frac{G_c\cos\gamma}{(1-S_c)}\right)\left[\frac{\exp(\alpha_2 t)}{\alpha_2^2}\Big\{\sqrt{\lambda+\alpha_2}\Big(erfc\sqrt{(\lambda+\alpha_2)t}-1\Big)\right. \\
& -\frac{1}{\sqrt{\pi t}}\exp\left(-t(\lambda+\alpha_2)\right) \\
& -\sqrt{S_c(K_c+\alpha_2)}\Big(erfc\sqrt{(K_c+\alpha_2)t}-1\Big) \\
& +\sqrt{\frac{S_c}{\pi t}}\exp\left(-t(K_c+\alpha_2)\right)\Big\}+\frac{1}{2\alpha_2\sqrt{\lambda}}\Big(1-erfc\sqrt{\lambda t}\Big) \\
& +\frac{1}{\alpha_2}\left(t+\frac{1}{\alpha_2}\right)\Big\{\sqrt{\lambda}\Big(1-erfc\sqrt{\lambda t}\Big)+\frac{1}{\sqrt{\pi t}}\exp\left(-\lambda t\right)\Big\} \\
& -\frac{1}{2\alpha_2}\sqrt{\frac{S_c}{K_c}}\Big(1-erfc\sqrt{K_c t}\Big) \\
& \left.-\frac{1}{\alpha_2}\left(t+\frac{1}{\alpha_2}\right)\left\{\sqrt{S_cK_c}\Big(1-erfc\sqrt{K_c t}\Big)+\sqrt{\frac{S_c}{\pi t}}\exp\left(-K_c t\right)\right\}\right]
\end{aligned} \tag{16}
$$

Nusselt number:

$$
\begin{aligned}
Nu = & -\frac{\partial T}{\partial y}\Big|_{y=0} \\
= & \left(\frac{P_r}{2\sqrt{d}}+t\sqrt{d}\right)\left(1-erfc\left(\sqrt{\frac{dt}{P_r}}\right)\right)+\sqrt{\frac{tP_r}{\pi}}\exp\left(-\frac{dt}{P_r}\right)
\end{aligned} \tag{17}
$$

Sherwood number:

$$
\begin{aligned}
Sh = & -\frac{\partial C}{\partial y}\Big|_{y=0} \\
= & \left(\frac{1}{2}\sqrt{\frac{S_c}{K_c}}+t\sqrt{S_cK_c}\right)\Big(1-erfc\sqrt{K_c t}\Big)+\sqrt{\frac{tS_c}{\pi}}\exp\left(-K_c t\right)
\end{aligned} \tag{18}
$$

where, $\lambda = M+\frac{1}{K_p}$, $d = R-SP_r$, $\alpha_1 = \frac{(R-\lambda-SP_r)}{1-P_r}$, $\alpha_2 = \frac{(S_cK_c-\lambda)}{1-S_c}$.

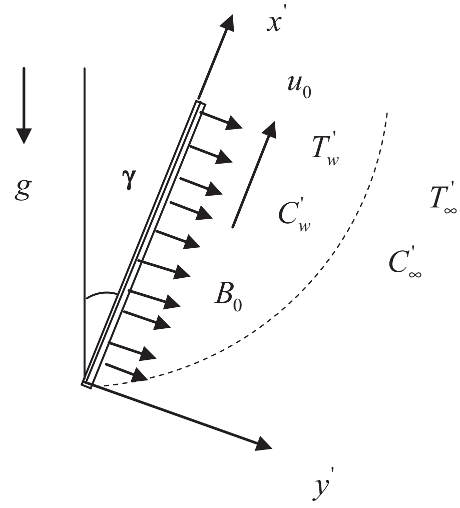

Figure 1 Flow geometry.

The analytical results (13)–(18) are simulated numerically for various values of emerging parameters and the numerical results are exhibited through graphs and tables.

3. Results and discussion

The analysis of the graphical representation of flow, heat and mass transfer phenomena brings out the effects of various parameters governing the flow. The effect of inclined plate on flow characteristics has been also discussed. Moreover, assigning zero to the angle of inclination, the case of vertical plate can be derived as a particular case. Further, for $a = 0$ in boundary condition (12), the plate is set to a constant motion. It is also evident from boundary condition that elapse of time induces higher start-up for $a > 0$ in velocity, temperature and concentration distribution.

Fig. 2 exhibits the velocity profile when the plate is subjected to cooling. It is seen that sudden decrease in velocity is observed near the plate in the absence of magnetic field and porous medium. It is also seen that their presence reduces

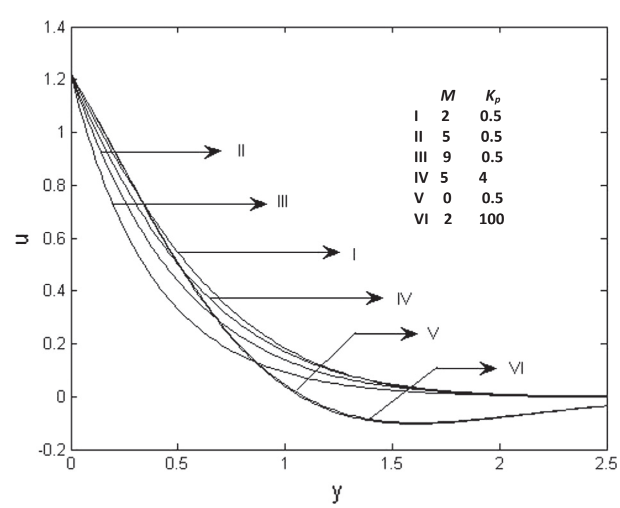

Figure 2 Velocity profile when $G_r = 10$, $G_c = 5$, $K_c = 0.2$, $R = 4$, $t = 0.4$, $P_r = 0.71$, $S_c = 0.6$, $\gamma = \pi/6$, $S = 2$, $a = 0.5$.

the velocity field at all points (curve-*I*). Thus, it is concluded that the effect of magnetic field in the presence of porous matrix, sustains a retarding effect on the velocity distribution. Uniform fall is indicated for $M = 5.0$ and 9.0 in the presence of porous media ($K_p = 0.5$). The curves-*V* and *VI* represent the absence of magnetic field and porous matrix respectively and both the curves coincide and indicate back flow when $y > 1.1$. The most striking feature of the profile is indicated through curves-*V* and *VI*, where the velocity profiles suffer a flow reversal which asserts that to avoid the flow reversal, the presence of magnetic field and porous medium is warranted. The non-occurrence of flow reversal in the neighborhood of the plate is due to overriding effect of plate oscillation which compensates the absence of both magnetic interaction and permeability of the medium.

Fig. 3 shows the effect of angle of inclination of the plate. The case of Kumar and Varma [16], has been derived as a particular case for $\gamma = 0.0$ and the exact coincidence of the curve-*I* is a validity check for our result. It is further remarked that an increase in angle of inclination reduces the velocity at all points, as the forcing forces are depleted due to the factor $\cos\gamma$.

Fig. 4 presents the effect of heat source parameter (S) and radiation parameter (R). It is observed that in the presence of

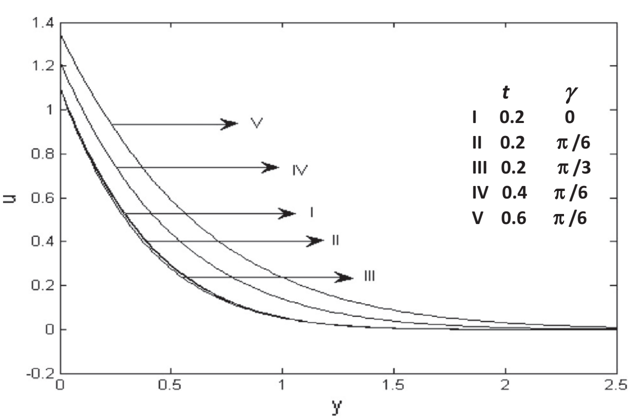

Figure 3 Velocity profile when $G_r = 10$, $G_c = 5$, $K_c = 0.2$, $R = 4$, $P_r = 0.71$, $M = 5$, $K_p = 0.5$, $S_c = 0.6$, $S = 2$, $a = 0.5$.

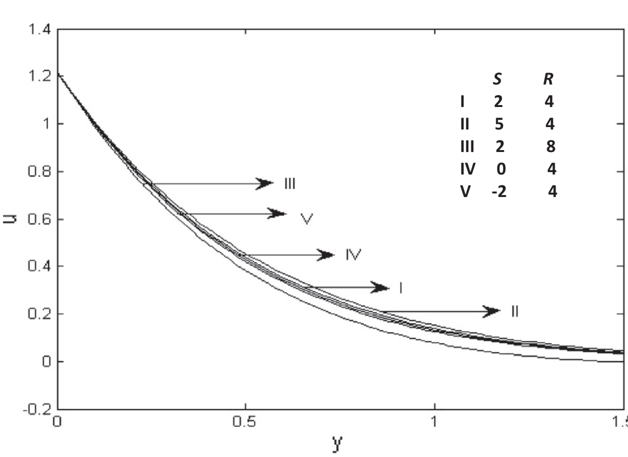

Figure 4 Velocity profile when $G_r = 10$, $G_c = 5$, $K_c = 0.2$, $M = 5$, $K_p = 0.5$, $P_r = 0.71$, $S_c = 0.6$, $\gamma = \pi/6$, $a = 0.5$, $t = 0.4$.

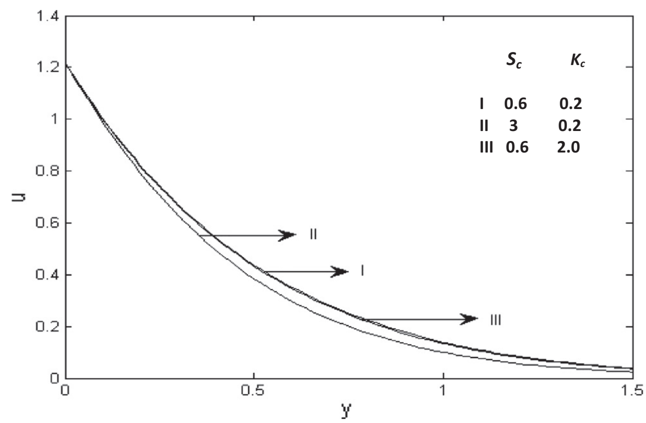

Figure 5 Velocity profile when $G_r = 10$, $G_c = 5$, $R = 4$, $M = 5$, $K_p = 0.5$, $P_r = 0.71$, $t = 0.4$, $S = 2$, $a = 0.5$, $\gamma = \pi/6$.

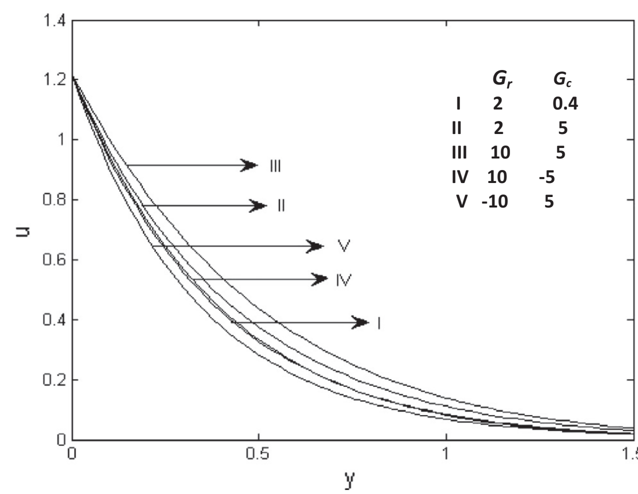

Figure 6 Velocity profile when $K_c = 0.2$, $S_c = 0.6$, $t = 0.4$, $M = 5$, $K_p = 0.5$, $P_r = 0.71$, $a = 0.5$, $\gamma = \pi/6$, $S = 2$, $R = 4$.

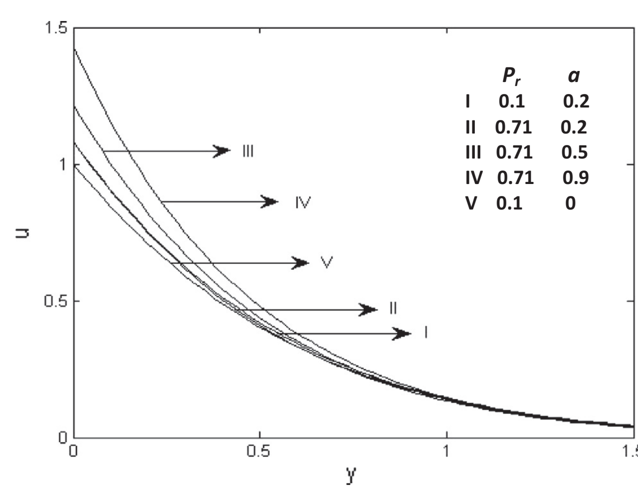

Figure 7 Velocity profile when $G_r = 10$, $G_c = 5$, $t = 0.4$, $M = 5$, $K_p = 0.5$, $S_c = 0.6$, $K_c = 0.2$, $\gamma = \pi/6$, $S = 2$, $R = 4$.

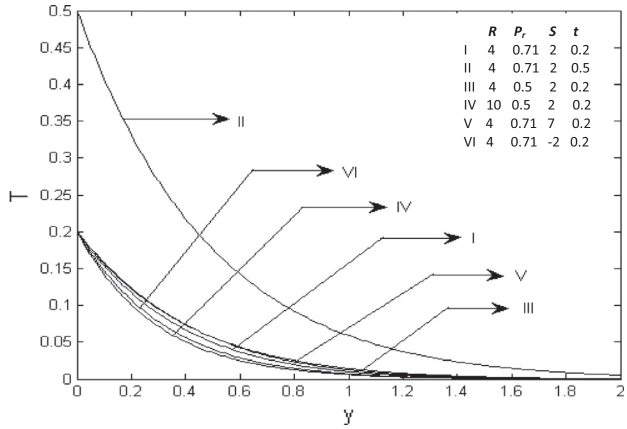

Figure 8 Temperature profile for different R, P_r, S and t.

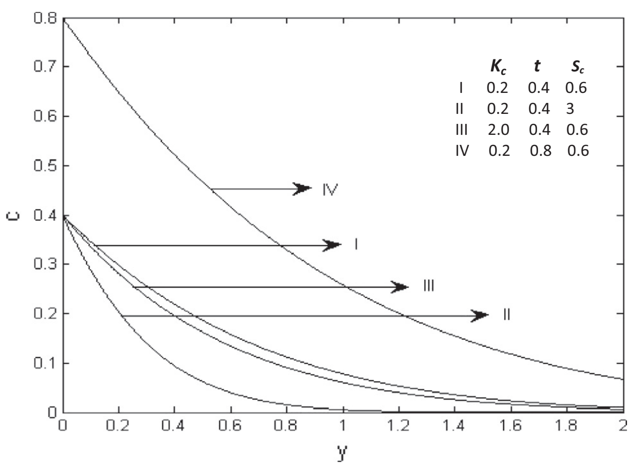

Figure 9 Concentration profile for different K_c, t, S_c.

constant radiation, an increase in heat source parameter leads to an increase in the velocity at all points (curves-*I*, *II*, *IV*) but reverse effect is observed in respect of radiation parameter. Thus, the presence of heat source is found to be favorable in enhancing the velocity.

Fig. 5 indicates that heavier diffusing species, i.e. with higher value of $S_c[S_c = 0.22$(Hydrogen), $S_c = 0.60$(Water vapor), $S_c = 0.78$(Ammonia)] and increasing rate of chemical reaction cause a reduction in velocity. Moreover, heavier species with destructive reaction causes a retardation in the velocity in case of cooled plate.

Fig. 6 shows that increase in thermal buoyancy and mass buoyancy, in case of cooling of the plate, leads to increase the velocity whereas, opposite effect is marked due to heating of the plate. Thus, the bounding surface heating and cooling have opposite buoyancy effect.

Fig. 7 exhibits the effect of acceleration parameter and Prandtl number on velocity. The increase in Prandtl number decreases the velocity and the reverse effect is observed in case of accelerating parameter. The curve-V ($a = 0$), represents the case of constant velocity of the plate and satisfies the boundary condition ($U = 1$) gives rise to a thinner boundary layer.

The smooth variation of temperature is marked throughout the flow domain. An increase in radiation parameter decreases the temperature. From the boundary condition it is evident that the plate temperature equals to the time. Therefore, an elapse of time leads to higher temperature on the plate. Further, high Prandtl number fluid (i.e. fluid with low thermal diffusivity) and with an increasing strength of heat source, decreases the temperature slightly at all points (Fig. 8). It is to note that the velocity decreases in the presence of sink (curve-VI).

Variation of concentration in the flow domain is depicted through Fig. 9. It is observed that high value of S_c (heavier species with low diffusivity) and higher rate of chemical reaction decrease the concentration at all points of the flow domain.

Table 1 Variation of skin friction.

M	G_r	G_c	K_p	K_c	R	t	P_r	S_c	γ	S	a	τ
1	10	5	.5	.2	4	.4	.71	.6	$\pi/6$	2	.5	1.2389
5	10	5	.5	.2	4	.4	.71	.6	$\pi/6$	2	.5	2.3617
0	10	5	.5	.2	4	.4	.71	.6	$\pi/6$	2	.5	0.9254
1	4	5	.5	.2	4	.4	.71	.6	$\pi/6$	2	.5	1.6663
1	−10	5	.5	.2	4	.4	.71	.6	$\pi/6$	2	.5	2.6635
1	10	.4	.5	.2	4	.4	.71	.6	$\pi/6$	2	.5	1.5951
1	10	−5	.5	.2	4	.4	.71	.6	$\pi/6$	2	.5	2.0132
1	10	5	4	.2	4	.4	.71	.6	$\pi/6$	2	.5	0.7410
1	10	5	.5	2	4	.4	.71	.6	$\pi/6$	2	.5	1.2698
1	10	5	.5	.2	7	.4	.71	.6	$\pi/6$	2	.5	1.3072
1	10	5	.5	.2	4	1	.71	.6	$\pi/6$	2	.5	−0.5019
1	10	5	.5	.2	4	.4	.1	.6	$\pi/6$	2	.5	1.1909
1	10	5	.5	.2	4	.4	.71	3	$\pi/6$	2	.5	1.3503
1	10	5	.5	.2	4	.4	.71	.6	$\pi/3$	2	.5	1.7036
1	10	5	.5	.2	4	.4	.71	.6	$\pi/6$	9	.5	1.7614
1	10	5	.5	.2	4	.4	.71	.6	$\pi/6$	0	.5	1.2742
1	10	5	.5	.2	4	.4	.71	.6	$\pi/6$	−2	.5	1.3041
1	10	5	.5	.2	4	.4	.71	.6	$\pi/6$	−5	.5	1.3415
1	10	5	.5	.2	4	.4	.71	.6	$\pi/6$	2	.2	0.8947

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Table 2 Variation of Nusselt number.

R	P_r	S	t	Nu
4	0.71	2	0.2	0.5214
4	0.71	2	0.5	1.0208
4	0.5	2	0.2	0.4849
10	0.5	2	0.2	0.6832
4	0.71	7	0.2	0.4629
4	0.71	−2	0.2	0.6145

Table 3 Variation of Sherwood number.

S_c	t	K_c	Sh
0.6	0.4	0.2	0.5674
3	0.4	0.2	1.2688
0.6	0.4	2.0	0.6896
0.6	0.8	0.2	0.8228

Skin friction, Nusselt number and Sherwood number provide flow characteristics at the boundary surface which is vital for flow, thermal and solutal stability. On careful study of Table 1, it is observed that the skin friction decreases with an increase in the values of permeability parameter as well as thermal and mass buoyancy parameter, and in all other cases, it increases. Thus, it may be concluded that convection current and absence of porous matrix favors the reduction of the skin friction. One peculiarity is marked for larger time, $t = 1.0$, for which skin friction assumes negative values, which indicates that the flow instability is marked for larger time span.

From Table 2, it is seen that Nusselt number increases with an increase in P_r, t and R, whereas, it decreases with heat source parameter (S). Thus it is concluded that the fluid with low thermal diffusivity and higher radiative property favors higher rate of heat transfer at the surface.

From Table 3, it is observed that Sherwood number, which determines the rate of solutal concentration at the surface of the wall, increases with an increase in S_c, K_c and t. Thus, heavier species with higher rate of chemical reaction increases the rate of solutal concentration at the surface.

4. Conclusion

The forgoing discussion brings out following cases:

(i) Flow past a vertical plate (Fig. 3, curve-I).
(ii) Plate with constant velocity (Fig. 7, curve-V).
(iii) Flow in the absence of magnetic field (Fig. 2, curve-V) and permeability parameter (Fig. 2, curve-VI).

Particular cases confirm the work of previous authors [16] as discussed in the corresponding figures. Presence of transverse magnetic field and saturated porous medium offers a resistance to the fall of velocity distribution thereby acts as a controlling device for preventing backflow. Angle of inclination, chemical reaction and presence of heavier species also sustain a retarding effect on the velocity.

The combined effects of heat source ($S > 0$) and convection current due to cooled plate ($G_r > 0$) accelerates velocity, but care should be taken to limit the sink parameter to avoid flow instability.

An increase in radiation parameter decreases the temperature so as to reach the ambient state earlier. The low diffusing species with higher rate of chemical reaction deplete the concentration level at all points.

An increase in rate of chemical reaction gives rise to increasing skin friction and increase in permeability of the medium decreases the skin friction which is desirable. Absence of porous matrix and free convective current reduce the skin friction, whereas other parameters enhance it. The flow instability is marked due to larger time span.

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Jyotsna Rani Pattnaik is working as an Asst. Professor in the Department of Mathematics, Padmanava College of Engineering, Rourkela, Odisha, India. She did M.Sc. from National Institute of Technology, Rourkela, Odisha, India, in 2003. Now, she is pursuing Ph.D. from S'O'A University, Odisha, India. She has 11 years of teaching experience. Her area of interest is Fluid Dynamics.

Professor Gouranga Charan Dash is a senior professor in the Department of Mathematics, S'O'A University Bhubaneswar, Odisha, India. He has vast experience in teaching and administration and also in research. His area of research is Fluid Dynamics, Magneto hydrodynamics, Heat and Mass transfer, Porous media, Computational fluid dynamics. He published 150 papers in national and international journals. He presented several papers in various conferences at national and international levels. He also visited different Universities at Switzerland, Italy, USA. He is also Chairperson of National and International Conferences. He guided 32 students for Ph.D. and 4 M.Tech. He is the life member of various bodies. He is the member of Board of Studies of various Universities and Autonomous Institutions.

Dr. Suprava Singh is presently working as Asst. Prof. in Padmanava College of Engineering, Rourkela, Odisha, India. She has 15 years teaching experience in different colleges. She received Ph.D. from Utkal University, Odisha, in 2005 under the supervision of Prof. B.K. Ojha and Prof. G.C. Dash. She published seven papers in National and International Journals.

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