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European Journal of Operational Research 255 (2016) 911–921

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Innovative Applications of O.R.

Stochastic short-term mine production schedule accounting for fleet allocation, operational considerations and blending restrictions

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ARTICLE INFO

Article history:
Received 11 July 2015
Accepted 26 May 2016
Available online 16 June 2016

Keywords:
Stochastic programming
Mine production scheduling
Fleet management

ABSTRACT

A new short-term mine production scheduling formulation is developed herein based on stochastic integer programming. Unlike past approaches, the formulation simultaneously optimizes fleet and mining considerations, production extraction sequence and production constraints, while accounting for uncertainty in both orebody metal quantity and quality along with fleet parameters and equipment availability, all leading to a well-informed sequence of mining that is expected to have realistic as well as high performance during a mine's operation. To assess the latter performance and implementation intricacies of the proposed formulation, the formulation is applied at a multi-element iron mine and the resulting monthly schedules are assessed and compared to the conventional mine scheduling approach showing: lower cost, minable patterns, efficient fleet allocation ensuring higher and less variable utilization of the fleet.

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1. Introduction

Short-term mine production scheduling generates a sequence of extraction within an annual production plan. The production schedule is seen as the operational guide to meet the mine's long-term objectives developed under current operating conditions and constraints. It outlines extraction stages in terms of months, weeks or days. The optimization of short-term production scheduling is guided by the life-of-mine or long-term mine scheduling (Hustrulid & Kuchta, 1995) and it is typically optimized in two separate steps. The first step optimizes the physical sequence of extraction of materials. The second step optimizes the assignment of the mining equipment fleet based on equipment capacity, availability and hauling time. There are three limitations to the above mentioned separate optimization steps, which lead to non-optimal short-term production schedules, even if results are experimentally adopted to generate a combined final schedule.

First, the scheduling elements, material sequence of extraction and equipment utilization, are artificially separated when optimized so that they do not benefit from their simultaneous optimization. Second, neither of the optimization steps involved considers uncertainty in input parameters, nor do they account for the local variability of the characteristics of the materials being scheduled for extraction. Lastly, the optimization of the extraction sequence of material ignores operational considerations and fleet management, and thus can be unrealistic and become hostage to equipment availability. These limitations can have adverse effects on the performance of the production scheduling and this may lead to: (a) increased operating costs stemming from erroneous materials blending and decisions on material processing destinations; (b) uncertainty in equipment performance and sub-optimal equipment use; (c) inability to deliver expected material targets; and (d) infeasible mining patterns. This paper addresses these limitations

Several papers related to short-term production scheduling and fleet allocation are available in the technical literature; a first group outlines general concepts of short-term production scheduling optimization, while a second group of papers considers real-time fleet allocation. Early efforts in optimizing short-term mine production schedules focus on developing concepts and related formulations for deciding sequences of depletion based on mathematical programming (Fytas and Calder, 1986; Gershon, 1982; Kahle & Scheafter, 1979; Schleifer, 1996; Wilke & Reimer, 1977; Wilke & Woehrle, 1979). Accordingly, the outline of production progressions (extraction sequence) on a daily, weekly or monthly basis follows production targets set by the long-term mine production schedule. The optimization process considers the allocation of resources that match the available fleet capacity, the mine's layout

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http://dx.doi.org/10.1016/j.ejor.2016.05.050
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and operational issues, such as mining direction. While accounting for the above, the objective function of related formulations is typically set to minimize production deviations from the yearly production plan targets; if these targets are met, then the expected long-term targets and overall mine valuation will likely be met. Key physical constraints are considered and include the mobility of mining equipment and mineable extraction patterns, as well as quality constraints leading to blending of materials to extract so as to match quality feed targets for various ore processing streams. More recent work stays within the same context; for example, Vargas, Morales, Rubio, and Mora (2008) present a mathematical programming formulation accounting for quality and geometric constraints, mill and mine capacity. Similarly, Eivazy and Askari-Nasab (2012) account for multi-destinations, blending stockpiles and decisions on ramps while their objective function minimizes mining cost, processing cost, waste rehabilitation cost, re-handling cost and hauling cost. The latter two approaches have drawbacks, such as the use of aggregation of mining blocks prior to optimizing, leading to suboptimal solutions, as aggregation of materials ignores the practical selectivity of preferred ore types and cannot deal with the actual hauling process during the optimization process.

As noted earlier, all the above work does not integrate a key aspect of short-term planning, namely, the management and dispatching of mining equipment/fleet. The real-time fleet allocation for short-term production planning is presented in Alarie and Gamache (2002), and Souza, Coelho, Ribas, Santos, and Merschmann (2010). A fleet dispatching system considers different allocation strategies given that transportation may represent more than 50% of operating costs (Alarie & Gamache, 2002). The solution strategies used in truck dispatching systems aim to improve productivity and reduce operating costs, however, the extraction patterns to be mined are assumed to be available. A shortcoming of these algorithms is that the whole tonnage of every pit are seen as a single macro block where the short-scale variability of the grade is lost and the one hour production and dynamic allocation of the fleet is only related to the dispatch system. L'Heureux, Gamache, & Soumis (2013) present a deterministic mixed integer programming model for short-term planning in open-pit mines. The sequence of mining of this model considers operational activities, such as drilling, blasting, transportation, ore processing capacity, the availability and the locations of shovels and drills. Drawback of this formulation is that the mined blocks by day are aggregated regular blocks. The definition of sectors to mine is usually linked to irregular patterns because of the local scale grade variability of the orebody and quality requirements.

More recent work considers minimizing operating costs of trucks since they represent the largest portion of the fleet in open pit mines (Topal & Ramazan, 2010), and is formulated as an integer program. Maintenance costs not only are a significant proportion but also change non-linearly depending on the road conditions, truck age and truck types. The stochastic extension (Topal & Ramazan, 2012) of this model considers the uncertainty in truck maintenance costs for the available fleet when matching annual production targets. The approach provides a maintenance cost distribution of the optimized equipment schedule minimizing the cost. However, similarly to other aspects of short-term planning discussed above, this last work is done assuming a sequence of extraction.

The work herein presents a new, integrated approach to short-term mine production scheduling based on stochastic integer programming (SIP), aiming to contribute towards generating well-informed production sequences and improved performance during a mine's operation. The proposed SIP formulation simultaneously optimizes both fleet and production schedule, accounts for operational considerations, such as mining width and mining directions, and considers the possible fluctuation and uncertainty of the metal grade and ore quality, fleet parameters and availability. The approach formulated is based on previous developments in long-term mine planning (Boland, Dumitrescu, & Froyland, 2008; Ramazan & Dimitrakopoulos, 2013; Lamghari & Dimitrakopoulos, 2012). Note that grade and ore quality uncertainty and variability is modelled herein through the generation of stochastically simulated scenarios of the mineral deposit being mined (Goovaerts, 1997), based on minimum and maximum autocorrelation factors for multivariate ore bodies (Desbarats & Dimitrakopoulos, 2000).

In the following sections, the proposed stochastic mathematical programming formulation for short-term mine production scheduling is described first. Then, an application at an iron ore mine presents the pertinent aspects and related intricacies of the proposed method while assessing its performance. Finally, conclusions and recommendations are provided.

2. Formulation

Short-term mine production scheduling is formulated as a stochastic integer programming model with recourse (Birge & Louveaux, 1997) and aims to minimize the total mining cost along with deviations from production targets, considers operational aspects such as mining direction and minimum width, and maximizes fleet utilization. In the formulation presented herein, the first-stage decisions are made before the uncertainty is revealed, then the second-stage decisions or recourse actions are made after uncertainty is considered. The notation used to formulate short-term scheduling follows. Note that indexes relate to the set of trucks, shovels, sectors, blocks, periods and realizations of uncertain parameters.

j: a sector or bench, where $j=1,\dots,J$
i: an shovel, where $i=1,\dots,I$
k: a block at sector j, where $k=1,\dots,K(j)$
l: a truck model, where $l=1,\dots,L$
p: a period of a production schedule, where $p=1,\dots,P$
ε: an element grade of k block that have economical value, where $\varepsilon=1,\dots,E$
δ: a deleterious element grade of k block, where $\delta=1,\dots,D$
s: simulated grade realization or scenario, where $s=1,\dots,S$
α: realization of shovel mechanical availability given historical data, where $\alpha=1,\dots,A$
r: truck cycle time and mechanical availability realization, where $r=1,\dots,R$

The parameters used at the fleet allocation, cost and penalties at objective function, production target and multi-element quality and tonnage are explained as follows:

h_{fleet}: fleet operation hours by period p
ι: maximum number of shovels allowed by sector
Q_i^{sh}: hourly production of shovel i.
$\omega_i(\mu_i,\sigma_i)$: mean and standard deviation of historical mechanical availability by shovel i
$a_{ij'}^{p-1}$: binary parameter, if shovel i is or not allocated to sector j' at previous period $p-1$
$c_{j'j}^{ExcM}$: cost of moving shovel from $p-1$ allocation sector j' to new allocation sector j
$c^{prodExc-}$: penalty cost for tonnage not produced regarding to the expected productivity
Q_l^{trk}: capacity of truck l

$\phi_{jl}(\mu_{jl}, \sigma_{jl})$: mean and standard deviation of cycle time by truck l at sector j
$\psi_l(\mu_l, \sigma_l)$: mean and standard deviation of historical mechanical availability by truck l
c^{ϕ}: time cycle cost per ϕ units
c^{m-}, c^{m+}: penalty cost for shortage and surplus total mining tonnage respect to the targets
c^{o-}, c^{o+}: penalty cost for shortage and surplus ore mining tonnage respect to the targets
$c^{\varepsilon-}, c^{\varepsilon+}, c^{\delta+}, c^{\delta-}$: penalty cost for deviation from main elements and contaminants limits
$M^{\min}, M^{\max}$: minimum and maximum mining tonnage target
$O^{\min}, O^{\max}$: minimum and maximum ore tonnage target
$G^{\varepsilon-}, G^{\varepsilon+}, G^{\delta-}, G^{\delta+}$: quality or grade requirements for ore tonnage produced
$\%Tol_{o-}, \%Tol_{o+}, \%Tol_{m-}, \%Tol_{m+}$, $\%Tol_{\varepsilon-}, \%Tol_{\varepsilon+}, \%Tol_{\delta-}, \%Tol_{\delta+}$: allowed percentage of tonnage and grade deviation from targets.
B_{jk}: block tonnage k at sector j
bh, ddh: Ore control data and exploration data at mined sector A
BH, DDH: Ore control data and exploration data at not mined sector B
c^m: mining cost by B_{jk} unit
$g^{\varepsilon}_{jks}, g^{\delta}_{jks}$: grade block k of main elements and deleterious in scenario s at sector j
O_{jks}: binary parameter flagging the block k at j sector for scenario s that has the minimum quality to be used at the blending process; otherwise, the block is flagged as waste.
ϕ_{jlr}: truck cycle time r of truck l at sector j given cycle time distribution
θ_{jlr}: maximum number of trips of truck l at sector j for cycle hauling realization r and mechanical availability realization r.

$$\theta_{jlr} = \frac{\psi_{lr} \times h_{fleet}}{\phi_{jlr}}, \quad \forall r = 1,\ldots,R, \quad \forall j = 1,\ldots,J, \quad \forall l = 1,\ldots,L$$

$Q^{sh}_{i\alpha}$: maximum production rate of shovel i per mechanic availability realization α and each realization $\omega_{i\alpha}$ is drawn from the available mechanical availability distribution, and it is

$$Q^{sh}_{i\alpha} = \omega_{i\alpha} \times h_{fleet} \times Q^{sh}_{i}, \quad \forall \alpha = 1,\ldots,A, \quad \forall i = 1,\ldots,I \tag{2.1}$$

The decision variables used are as follows:
x^p_{jk}: binary variable, if block k at sector j is mined or not at period p
e^p_{ji}: binary variable, if shovel i is or not allocated to sector j at period p
n^p_{jilr}: number of trips of truck l to sector j, shovel i at period p for cycle time realization and mechanical availability realization r
$f^p_{ji\alpha}$: deviation of shovel i at sector j from expected shovel production $Q^{sh}_{i\alpha}$
y^p_{jk}: number of blocks that were not scheduled at period p to mine block k at sector j to match mining width requirements.
d^{m-}_p, d^{m+}_p: shortage tonnage to match lower production limit and surplus tonnage to match upper production limit at period p
d^{o-}_{sp}, d^{o+}_{sp}: shortage of ore mining to match lower bound and the surplus to match upper bound at period p accounting for grade scenario s
$d^{\varepsilon-}_{sp}, d^{\varepsilon+}_{sp}$: deviation from ε grade targets at period p for grade scenario s
$d^{\delta-}_{sp}, d^{\delta+}_{sp}$: deviation from δ deleterious grade targets at period p for grade scenario s

2.1. Objective function

Decision variables x^p_{jk}, y^p_{jk} and e^p_{ji} are related with the first-stage and remaining decision variables are related with the second-stage. The first-stage decisions include minimizing the costs of extraction of materials, movement of shovels, production shortage, and lacking matching mining width. In the second-stage, these costs are minimized over a range of possibilities of a recourse cost associated with deviations from ore production and quality targets, hauling cost, and lack of mining with maximum shovel productivity. The objective function of the proposed mathematical model is:

$$\begin{aligned}
Minimize = & \overbrace{\sum_{p=1}^{P}\sum_{j=1}^{J}\sum_{k=1}^{K(j)} c^m B_{jk} x^p_{jk}}^{1st} \\
& + \overbrace{\frac{1}{R}\sum_{p=1}^{P}\sum_{j=1}^{J}\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{r=1}^{R} \phi_{jlr} c^{\phi} n^p_{jilr}}^{2nd} \\
& + \overbrace{\sum_{p=1}^{P}\sum_{j'=1}^{J}\sum_{j=1}^{J}\sum_{i=1}^{I} \left(c^{ExcM}_{j'j} e^p_{ji} a^{p-1}_{j'i}\right)}^{3rd} + \overbrace{\frac{1}{A}\sum_{p=1}^{P}\sum_{j=1}^{J}\sum_{i=1}^{I}\sum_{\alpha=1}^{A} \left(c^{prodExc-} f^p_{ji\alpha}\right)}^{4th} \\
& + \overbrace{\sum_{p=1}^{P}\sum_{j=1}^{J}\sum_{k=1}^{K(j)} \left(c^{smoth-} y^p_{jk}\right)}^{5th} \\
& + \overbrace{\frac{1}{S}\left\{\sum_{s=1}^{S}\sum_{p=1}^{P}\sum_{\varepsilon=1}^{E} \left(c^{\varepsilon-} d^{\varepsilon-}_{sp} + c^{\varepsilon+} d^{\varepsilon+}_{sp}\right) + \sum_{s=1}^{S}\sum_{p=1}^{P}\sum_{\delta=1}^{D} \left(c^{\delta+} d^{\delta+}_{sp} + c^{\delta-} d^{\delta-}_{sp}\right)\right\}}^{6th} \\
& + \overbrace{\frac{1}{S}\sum_{s=1}^{S}\sum_{p=1}^{P} \left(c^{o+} d^{o+}_{sp} + c^{o-} d^{o-}_{sp}\right)}^{7th} + \overbrace{\sum_{p=1}^{P} \left(c^{m-} d^{m-}_{p} + c^{m+} d^{m+}_{p}\right)}^{8th}
\end{aligned} \tag{1}$$

The first component of the objective function is associated with the cost of extracting material from the mine. The second component corresponds to minimizing the hauling cost given the uncertainty in the trucks' hauling time and mechanical availability so as to ensure both optimal allocation and maximum truck utilization. The third component is the minimization of cost of the shovel movements among sectors. The fourth component minimizes the lack of production per shovel given uncertainty in its mechanical availability, so as to maximize shovel utilization. The fifth term ensures that the operational considerations are respected by penalizing the lack of mining blocks that match the required mining width. The sixth, seventh and eighth components deal with the minimization of geological risk with respect to the quality and the quantity of ore production, and penalize deviations from production targets, respectively. Each component is linked to its respective cost and all of them must have the same units to minimize overall cost. Note that the first, second, fourth, sixth and seventh components are stochastic and contain decision variables that change, given the corresponding realizations of the fleet parameters or element quality.

The decision variables are present in the objective function and there are several constraints linking the fleet allocation decision variables with mined block decision variables. This ensures that the formulation herein delivers a short-term production schedule that account for both fleet allocation and production targets.

2.1.1. Constraints for production and fleet allocation

The constraints below link the fleet allocation decision variables with mined block decision variables, to guarantee that the short-term production schedule accounts for fleet allocations and production targets.

$$\sum_{p=1}^{P} x_{jk}^{p} \leq 1, \quad \forall j = 1, \ldots, J, \quad \forall k = 1, \ldots, K(j) \tag{2}$$

Constraint (2) ensures that a block of material may be mined once at any period. The block is a selective mining unit that may be mined in one period assuming that the time period may be from weeks to months.

$$\sum_{i=1}^{I} e_{ji}^{p} \leq \iota, \quad \forall p = 1, \ldots, P, \quad \forall j = 1, \ldots, J \tag{3}$$

$$\sum_{j=1}^{J} e_{ji}^{p} \leq 1, \quad \forall p = 1, \ldots, P, \quad \forall i = 1, \ldots, I \tag{4}$$

$$x_{jk}^{p} - \sum_{i=1}^{I} e_{ji}^{p} \leq 0, \quad \forall p = 1, \ldots, P, \quad \forall j = 1, \ldots, J, \quad \forall k = 1, \ldots, K(j) \tag{5}$$

$$\sum_{j=1}^{J} \sum_{i=1}^{I} \phi_{jlr} \times n_{jilr}^{p} \leq h_{fleet} \times \psi_{lr}, \quad \forall p = 1, \ldots, P, \quad \forall l = 1, \ldots, L, \quad \forall r = 1, \ldots, R \tag{6}$$

$$n_{jilr}^{p} - \theta_{jlr} e_{ji}^{p} \leq 0, \quad \forall p = 1, \ldots, P, \quad \forall j = 1, \ldots, J, \quad \forall i = 1, \ldots, I, \quad \forall l = 1, \ldots, L, \quad \forall r = 1, \ldots, R \tag{7}$$

$$\sum_{l=1}^{L} \left(Q_{l}^{truck} \times n_{jilr}^{p} \right) - Q_{i\alpha}^{sh} \times e_{ji}^{p} + f_{ji\alpha}^{p} = 0 \quad \forall p = 1, \ldots, P, \quad \forall j = 1, \ldots, J, \quad \forall i = 1, \ldots, I, \quad \forall \alpha = 1, \ldots, A, \quad \forall r = 1, \ldots, R \tag{8}$$

$$\sum_{i=1}^{I} \sum_{l=1}^{L} \left(Q_{l}^{truck} \times n_{jilr}^{p} \right) - \sum_{k=1}^{K(j)} \left(B_{jk} \times x_{jk}^{p} \right) = 0, \quad \forall p = 1, \ldots, P, \quad \forall j = 1, \ldots, J, \quad \forall r = 1, \ldots, R \tag{9}$$

The mining equipment can be placed in a given number of locations. A possible path of the locations of each piece of equipment is provided as part of short-term plan. Shovels are allocated to available sectors or remain in the current sector previously allocated. A sector must be mined at some period and a shovel must be allocated to the sector that has a lower cost of hauling and provides the material to match quality requirements. Constraints (3) ensure that each sector is allocated with less equal than ι shovels at sector j per period p. The parameter ι is the maximum number of shovels that can be allocated in each sector. Constraint (4) ensures that each shovel i may be assigned to one sector while the cost of movement is minimized in the objective function to prevent excessive shovel movement among sectors. Inequality constraints are used for the fleet allocation because not all the available shovels or trucks are allocated in scenarios where there are more equipment than the production requires in accounting for hauling distance. Constraint (5) guarantees that a mining block in sector j is mined only if a shovel is allocated to sector j.

Variable n_{jilr}^{p} decides the optimal number of trips for truck l to sector j and shovel i per period p, thus accounting for fluctuations of truck cycle time and mechanical availability. The number of trips decision variable n_{jilr}^{p} also supports in the allocation of each truck l to shovel i to sector j for mechanical availability and hauling realization r per period p. The formulation considers that a truck can be allocated to more than one shovel at the same sector j or different sectors. Constraint (6) limits the number of trips of a truck to its scheduled time per period as the operation progresses by extracting minerals and continuously extending the access. Indeed, the roads change dynamically. This implies uncertainty in the hauling time. The trip cycle time ϕ_{jlr} of truck l to sector j is drawn from distribution R times.

The decision variable n_{jilr}^{p} is also subject to the maximum number of trips that a truck l can haul from each sector j. The maximum number of trips θ_{jlr} per truck l is a preprocessed parameter because its components are not decision variables. Then, the number of total trips to each sector is restricted to a maximum number of trips times the e_{ji}^{p} binary decision variable. The decision variable e_{ji}^{p} is relevant in the constraints (7) because not all the sectors will be allocated with a shovel and a sector without a shovel cannot have number of trips. Decision variables n_{jilr}^{p} and e_{ji}^{p} are linked. The inequality constraint (7) also ensures that only an allocated sector with a shovel is assigned with trucks, and not all trucks are allocated at some scenarios. The link of truck l, shovel i and sectors j in the constraints ensure that all assignment possibilities for the trucks, shovel and sectors are taken into account.

There are capacity limits for each truck Q_{l}^{trk} and shovel Q_{i}^{sh}. The available fleet and their respective capacity are included in the formulation. The production of each shovel assigned to sector j is constrained to the maximum production of each shovel $Q_{i\alpha}^{sh}$. The e_{ji}^{p} binary decision variable helps to formulate the shovel capacity constraints (8) because not all of the shovels may be allocated. The lack of expected production by each shovel is stored by the decision variable $f_{ji\alpha}^{p}$, which is minimized at the objective function.

There are J sectors and each sector has $K(j)$ blocks to be evaluated. The tonnage of block k is B_{jk} and each block may be hauled from an in-situ location to a blending area or waste dump taking into account the fleet capacity constraints. Note that the model assumes the blending area and the location of the waste dump are nearby and considers that a truck cycle time distribution is used for each sector and truck, and can be extended to multiple blending and waste dumps as well as complex waste management at any location. The decision variables at operational and production constraints are linked to fleet allocation constraints. Indeed, constraint (9) links number of trips n_{jilr}^{p} of truck l from sector j and shovel i given mechanic availability and hauling time realization r with the mined block decision variable x_{jk}^{p}. The hauling tonnage by the trucks from sector j for mechanical availability and hauling time realization r must be equal to scheduled blocks tonnage at sector j.

$$\sum_{j=1}^{J} \sum_{i=1}^{I} \sum_{l=1}^{L} Q_{l}^{trk} \times n_{jilr}^{p} \geq M^{\min} \quad \forall p = 1, \ldots, P, \quad \forall r = 1, \ldots, R \tag{10}$$

$$0 \le d_p^{m-} \le \%Tol_{m-} \times M^{\min} \quad \forall p = 1,\ldots,P \tag{11}$$

$$\sum_{j=1}^{J}\sum_{i=1}^{I}\sum_{l=1}^{L} Q_l^{trk} \times n_{jilr}^{p} - d_p^{m+} \le M^{\max} \quad \forall p = 1,\ldots,P, \quad \forall r = 1,\ldots,R \tag{12}$$

$$0 \le d_p^{m+} \le \%Tol_{m+} \times M^{\max} \quad \forall p = 1,\ldots,P \tag{13}$$

$$\sum_{j=1}^{J}\sum_{k=1}^{K(j)} \left(O_{jks} \times B_{jk} \times x_{jk}^{p}\right) + d_{sp}^{o-} \ge O^{\min} \quad \forall p = 1,\ldots,P, \quad \forall s = 1,\ldots,S \tag{14}$$

$$\sum_{j=1}^{J}\sum_{k=1}^{K(j)} \left(O_{jks} \times B_{jk} \times x_{jk}^{p}\right) - d_{sp}^{o+} \le O^{\max} \quad \forall p = 1,\ldots,P, \quad \forall s = 1,\ldots,S \tag{15}$$

$$0 \le d_{sp}^{o-} \le \%Tol_{o-} \times O^{\min} \quad \forall p = 1,\ldots,P, \quad \forall s = 1,\ldots,S \tag{16}$$

$$0 \le d_{sp}^{o+} \le \%Tol_{o+} \times O^{\max} \quad \forall p = 1,\ldots,P, \quad \forall s = 1,\ldots,S \tag{17}$$

$$\sum_{j=1}^{J}\sum_{k=1}^{K(j)} \left(B_{jk} \times \left(g_{jks}^{\varepsilon} - G^{\varepsilon-}\right) \times O_{jks} \times x_{jk}^{p}\right) + d_{ps}^{\varepsilon-} \ge 0 \quad \forall p = 1,\ldots,P, \quad \forall \varepsilon = 1,\ldots,E, \quad \forall s = 1,\ldots,S \tag{18}$$

$$\sum_{j=1}^{J}\sum_{k=1}^{K(j)} \left(B_{jk} \times \left(g_{jks}^{\varepsilon} - G^{\varepsilon+}\right) \times O_{jks} \times x_{jk}^{p}\right) - d_{ps}^{\varepsilon+} \le 0 \quad \forall p = 1,\ldots,P, \quad \forall \varepsilon = 1,\ldots,E, \quad \forall s = 1,\ldots,S \tag{19}$$

$$0 \le d_{sp}^{\varepsilon-} \le \%Tol_{\varepsilon-} \times O^{\min} \times G^{\varepsilon-} \quad \forall p = 1,\ldots,P, \quad \forall \varepsilon = 1,\ldots,E, \quad \forall s = 1,\ldots,S \tag{20}$$

$$0 \le d_{sp}^{\varepsilon+} \le \%Tol_{\varepsilon+} \times O^{\max} \times G^{\varepsilon+} \quad \forall p = 1,\ldots,P, \quad \forall \varepsilon = 1,\ldots,E, \quad \forall s = 1,\ldots,S \tag{21}$$

$$\sum_{j=1}^{J}\sum_{k=1}^{K(j)} \left(B_{jk} \times \left(g_{jks}^{\delta} - G^{\delta+}\right) \times O_{jks} \times x_{jk}^{p}\right) - d_{sp}^{\delta+} \le 0 \quad \forall p = 1,\ldots,P, \quad \forall \delta = 1,\ldots,D, \quad \forall s = 1,\ldots,S \tag{22}$$

$$\sum_{j=1}^{J}\sum_{k=1}^{K(j)} \left(B_{jk} \times \left(g_{jks}^{\delta} - G^{\delta-}\right) \times O_{jks} \times x_{jk}^{p}\right) + d_{sp}^{\delta-} \ge 0 \quad \forall p = 1,\ldots,P, \quad \forall \delta = 1,\ldots,D, \quad \forall s = 1,\ldots,S \tag{23}$$

$$0 \le d_{sp}^{\delta+} \le \%Tol_{\delta+} \times O^{\max} \times G^{\delta+} \quad \forall p = 1,\ldots,P, \quad \forall \delta = 1,\ldots,D, \quad \forall s = 1,\ldots,S \tag{24}$$

$$0 \le d_{sp}^{\delta-} \le \%Tol_{\delta-} \times O^{\min} \times G^{\delta-} \quad \forall p = 1,\ldots,P, \quad \forall \delta = 1,\ldots,D, \quad \forall s = 1,\ldots,S \tag{25}$$

Production per time period p is constrained to the production targets (10). The production includes ore tones plus the waste tones. The ore tonnage is the material that has positive economic value meanwhile the waste tonnage is the material without positive economic value that needs to be extracted to allow access to ore and ensure continuity of ore production in the following periods. The number of trip decision variables and truck capacities are used to calculate the total tonnage extracted per period. The proposed model considers strict constraints for early periods and can be relaxed for the latest periods. To relax the production constraints, the shortage d_p^{m-} with respect to the target planned is considered, along with their respective tolerance of deviation (11). Traditionally, an upper bound is not used in production formulation because the cost of mining will limit overproduction; however, at the current formulation the production must be limited because the capacity shovel constraints maximize the production by sector to increase the utilization of the shovel (12). The upper bound limits this maximization to keep close to the production targets. The deviation $d_p^{m+}$with respect to the upper bound total production is penalized in the objective function and their tolerance is considered (13).

As a production constraint, the ore tonnage should match the target ore production given by long-term production schedules (14, 15). The shortage d_{sp}^{o-} respects to the target planned and the surplus d_{sp}^{o+}, respects the upper bound ore processing and are penalized in the objective function. The deviations are limited by a percentage of ore production *%Tol* (16, 17). The upper bound is directly related to the ore tonnage scheduled plus the maximum capacity pile of ore next to the delivering location. The exceeding material from the upper bound may be considered as material that go to stockpile, and its tonnage are penalized by the corresponding re-handled cost.

Ore production must match certain quality constraints, that is, the expected grades or quality of the material at the end of the week or month must fit into specific ranges and this range depends on long-term production schedule specifications. To meet this demand, a block x_{jk}^{p} is mined only if their grade helps to satisfy the required quality given the available fleet. Assuming that the study case has E elements that have economic value and D elements as deleterious elements, $2(E+D)$ quality constraints are needed to meet quality conditions. The grade of the main commodity for ore tonnage should satisfy the constraints (18, 19) and the quality deviations have tolerance (20, 21) to ensure a production schedule with low variable average quality. Ore production cannot have more than the required limits of contaminants because this contains D deleterious elements. These deleterious elements influence the physical and chemical properties of the ore product, thus the performance of the process that the ore product will be used for. The constraints (22, 23) ensure that the ore delivered by period given S scenarios of the grades have average grades less than $G^{\delta+}$ and more than $G^{\delta-}$ for deleterious element $\delta = 1,\ldots,D$. The quality deviations related with contaminants are also constrained to tolerance (24, 25) to ensure production schedule with low variable average quality.

Blending of ore from sectors is carried out based on cutoffs that define the minimum quality that a block k must have to be included in the blending process. If a block k has the chance of being used for blending $O_{jks} = 1$; otherwise, the block k is allocated to the waste dump directly $O_{jks} = 0$. The quality constraints are satisfied when the total ore production meets the required quality conditions set as targets.

2.1.2. Constraints for operational considerations

Operational considerations relate to the size of the equipment and accessibility restrictions that may require feasible, in a mining sense, production schedule patterns that allow the available equipment to work efficiently and streamline movements for safety reasons. The first operation consideration is the mining direction that facilitates access to the sectors to be mined and it is:

$$x_{jk}^{p} - \sum_{\tau=1}^{p} x_{jk'}^{\tau} \le 0, \quad \forall p = 1,\ldots,P, \quad \forall j = 1,\ldots,J, \quad \forall k = 1,\ldots,K(j), k' \in \Omega_{k'} \tag{26}$$

where $\Omega_{k'}$ is the set of indexes representing blocks that are horizontal predecessors which must be mined before block k to match

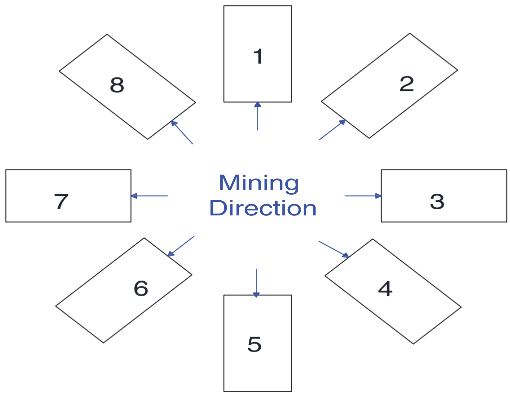

Fig. 1. Eight mining directions considered by the formulation.

the mining direction. A sector could be mined following eight directions, as shown in Fig. 1.

The second operational consideration is the mining width and relates to the minimum width the patterns of a short-term schedule period has that permits fleet access to the orebody and materials that need be extracted. Production schedules without accounting for mining width may deliver schedule patterns with singular blocks of early periods surrounded by blocks from later period, as shown in Fig. 2. This production scheduling cannot be implemented as the blocks scheduled for period 1, (blue squares) cannot be mined before some blocks belonging to period 2 (orange squares) are extracted.

The following mining width constraints account for feasible extraction patterns and may force the mining of some blocks before a given block k as shown in Fig. 2.

$$-2 \times x^{p}_{jk'} - x^{p}_{jk''} + (2 \times \nu + \upsilon) \times x^{p}_{jk} - y^{p}_{jk} \leq 0, \quad \forall p = 1, \ldots, P,$$
$$\forall j = 1, \ldots, J, \quad \forall k = 1, \ldots, K(j) \quad k' \in \Psi_{k'}, k'' \in \Psi_{k''} \tag{27}$$

The mining width is discretized into υ blocks where $\Psi_{k''}$ is its set of indexes. To mine a block k, υ blocks may be mined at the same period or have been mined at previous periods. $\Psi_{k'}$ is the set of indexes representing the adjacent blocks and priority of mining adjacent blocks ν is considered to avoid single blocks from some periods being surrounded by blocks from different periods. Indeed, the blocks ν that surround block k must be mined with twice the priority than the second term at constraints (27) to avoid infeasible mining patterns. The adjacent ν blocks belong to the inner window and the υ blocks belong to the outer window in smooth constraints (Dimitrakopoulos & Ramazan, 2004). These smooth mining constraints are linked to mining width to provide feasible mining sequences that the fleet requires to operate efficiently. It is important to remark that υ number of blocks that match mining width are variable through the sector. The blocks that are located close to the border will require less υ blocks to be moved because some blocks were already mined or are 'air' (non-physically existing) blocks.

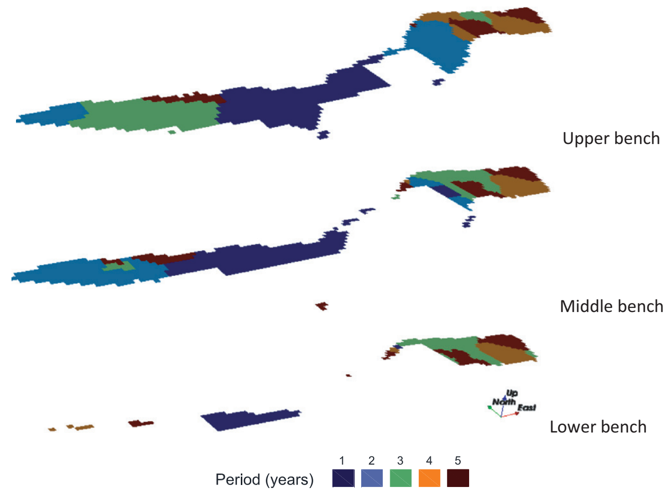

Fig. 3. Stochastic long-term mine production schedule, 5 periods located at three benches (upper middle and lower), modified from (Benndorf, 2005).

The mining width constraints are relaxed because at some locations feasible solutions will require to mine only some υ blocks. The discrete decision variable y^{p}_{jk} will store the lack of mining blocks that match the mining width considerations. This decision variable is penalized and minimized at the objective function.

3. Application in an iron ore mine

The proposed stochastic short-term production schedule (SSTPS) formulation is applied at an iron deposit. Iron ore deposits are typical examples of a multi-element environment, where the main production objective is to satisfy the customer quality requirement at a lower cost by optimally blending the different sectors of a mine. More specifically, when the iron content is evaluated and must be within customer specified limits there are also specific restrictions on the content of the so-called deleterious elements, such as phosphorous (P), silica (SiO_2), alumina (Al_2O_3) and the water and organic content measured as "loss on ignition" (LOI). These deleterious elements influence the physical and chemical properties of the iron ore product, significantly varies from customer to customer and contractual agreement to be met, and the performance of the process it will be used for. For instance, phosphorous affects steel quality (added cost), high silica and high alumina affect furnace efficiency, and the LOI affect fuel use and water in a hot furnace for steel making.

As noted earlier, the stochastic long-term production scheduling (SLTPS) of a given mine provides the larger scale framework defining the targets production of the short-term production schedule. Fig. 3, for example shows the long-term production schedule of the iron mine in this case study and contains five periods (years). The first year (dark blue in the figure) is used herein for short-term production scheduling which is optimized over twelve periods (months).

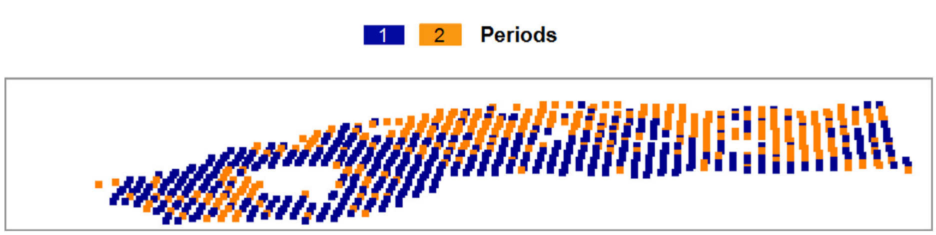

Fig. 2. Production scheduling without mining width constraints.

Table 1
First year production quantity and quality requirements.

Period	Ore Tonnage	Fe_2O_3(%)	P (%)	SiO_2 (%)	Al_2O_3 (%)	LOI (%)
1	14,000,000	57.1–59.4	0.032–0.038	4.6–5.2	0.9–1.05	9.5–11

Note: Ore/Waste cut-off grade is Fe>=56%.

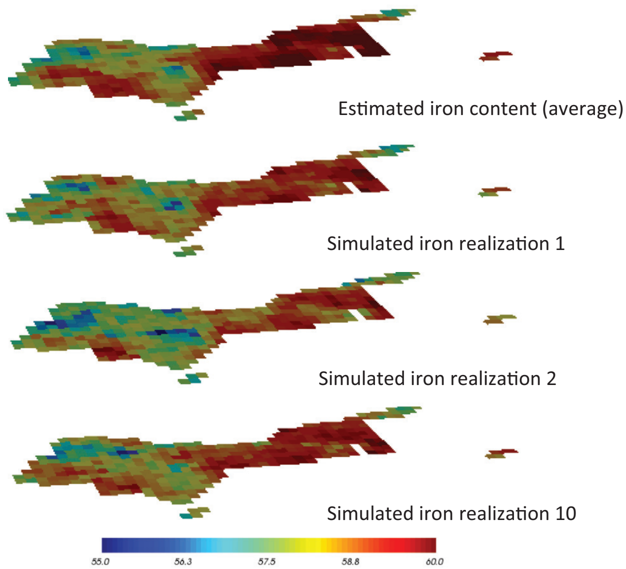

Fig. 4. Iron ore content within the sector to be mined in the first year of production (Fig. 3); 3 stochastically simulated realizations and the deterministic estimate for the upper bench (extraction units of $25 \times 25 \times 12$ meter^3).

Table 2
Shovel model and mechanical availability parameter distribution.

			Mechanical availability (%)	
Model	Shovel (i)	Production (Tonnes/hour)	Mean	Std. Dev.
HS6020	1	1180	83	4.5
HS6030	2	1400	83	4

Table 3
Truck model and mechanical availability parameter distribution.

			Mechanical availability (%)	
Model	Truck (l)	Tonnes	Mean	Std. Dev.
Cat785D_501	1	136	83	5
Cat785D_502	2	136	83	4
	:	:	:	:
Cat785D_510	8	136	83	4
Cat77G_511	9	100	83	5
Cat77G_512	10	100	83	5

Table 4
Trucks cycle time and parameter distribution (ϕ_{jlr}).

		Cycle time (minutes)	
Sector (j)	Truck (l)	Mean	Std.Dev.
1	1	32	2.8
1	:	:	:
1	10	32	3.3
2	1	25	2.6
2	:	:	:
2	1	25	3.1
3	1	20	2.5
3	:	:	:
3	10	20	3

The quality targets and tonnes for the SSTPS and for the first year of production considered herein are given see Table 1. From the first year tonnage in Table 1; the mine must produce iron ore of about 1.16 millions of Iron tonnes each month. The average grade of the related elements per month may be in the intervals of the first year long-term ore quality given; however, the spatial variability of these grades varies when monthly increments are considered and along the mining direction and operational mining width. Ore quality intervals correspond to the upper bound and lower bound per element over the total year.

The iron ore may be extracted from blocks of $25 \times 25 \times 12$ meter^3 located at three consecutive mining benches of 12 meter height. For this case study, ten equally probable scenarios of iron content, phosphorous, silica, aluminum and LOI are used to quantify the joint uncertainty in the characteristics of the iron ore deposit considered and are the input to the SSTPS formulation proposed in the previous section. The simulated scenarios available were provided and generated using the stochastic simulated technique detailed in Boucher and Dimitrakopoulos (2012). The area considered is bounded by the limits of the given volume of production in the long-term first year production schedule provided. Fig. 4 shows 3 scenarios of iron ore content as well as the corresponding conventional and single estimated (average) representation of iron content (Fe_2O_3%) for the upper bench. In total, 734 blocks from 3525 to 21,150 tonnes, with Fe_2O_3 from 54.59% to 60.63%, P from 0.02% to 0.04%, SiO_2 from 3.10% to 8.58%, Al_2O_3 from 0.53% to 1.88% and LOI from 8.75% to 11.75% are available.

In addition to the uncertainty of the materials being extracted addressed above, the parameters related to the mining fleet available are given, so as to allocate efficiently and maximize the utilization of this fleet. The fleet size, mechanical availability and hauling time from the orebody to the various destinations are parameters used to allocated shovels and trucks at the related mine sectors. For this case study two shovels and ten trucks are the available fleet. The hourly productivity of each shovel fluctuates between 1180 and 1400 tonnes. The shovel model, digging rate and mechanical availability parameter distributions are given (Table 2) along with the truck model, capacity and mechanical availability parameter distribution per truck (Table 3).

Short-term evaluation has the advantage of accounting for additional short-term information such as the hauling distance that is available at the short-term evaluation. This supports the allocation of trucks because the past records of speed per truck, truck hauling time per sector in a mine and blending pad location are available. Additionally, the parameter distribution of the time that spends l truck from the sector j to the destination is calculated as shown in Table 4. The cycle time ϕ_{jlr} from sector j to destination will be drawn r times from the respective distribution and the maximum trips are calculated given the mechanical availability per truck. The parameters used to implement the proposed SSTPS formulation proposed herein are given in Table 5.

The total tonnage to be mined after twelve months of production is approximately 14,400,000 iron ore tonnes, and given the ore cut-off >=56% Fe_2O_3 almost all the material will be mined as ore. The targets of production and actual ore production are quite similar. Note that a high penalty is applied to the lack of mining from the expected monthly production because all material scheduled for the twelve months must be mined to align short-term

Table 5
Target month production and parameters.

Production target	Parameter	Value	Unit	Penalty
	Max Production	1,210,000	Tonnes	160
	Min Production	1,100,000	Tonnes	160
	Max Ore Production	1,210,000	Tonnes	16
	Mine Ore Production	1,000,000	Tonnes	4
	Allowed deviation tolerance	<=10	%	
Quality Requirement	Iron Ore (Fe203)	57.0–59.4	%	1
	Phosphorous	0.032–0.038	%	10
	Silica	4.6–5.2	%	10
	Alumina	0.9–1.05	%	10
	Loss on ignition	9.5–11	%	1
	Allowed deviation tolerance	<=10	%	
Ore Definition	**Parameter**	**Value**	**Unit**	
	Fe203	>=56	%	
Economic Parameters	**Parameter**	**Value**	**Unit**	
	Mining Cost*	40	$/Tonne	
	Cycle time Cost	120	$/hour	
	Shovel Moving Cost	1000	$/100 meters	

* Not include hauling cost

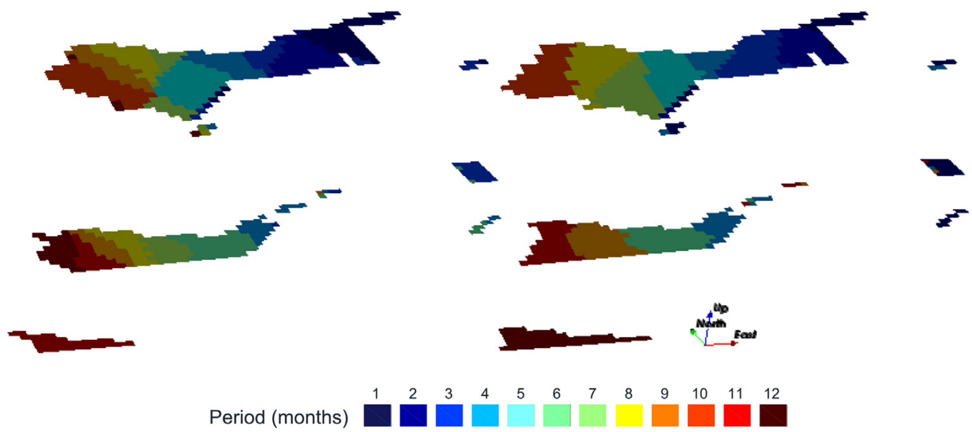

Fig. 5. The stochastic short-term schedule (left) and the deterministic schedule (right).

production with long-term planning targets. The shovel moving cost is computed from the unproductive time that a shovel may spend and the approximate cost of delaying the production.

The software used for solving the model is IBM ILOG CPLEX Optimization Studio v12.4 in a computer of a dual-core processor of 2.67 Gigahertz and 24GigaByte of RAM. The model formulation was written in C++ using the set of libraries Concert Technology (IBM, 2010). The optimization of the 12 periods demanded unreasonable computing time and sequential optimization with aggregated continuous periods is implemented to accelerate the solutions time. The computational time required to find the optimal solution was 292 seconds. The solution of 12 periods required a model with 47,228 constraints, 13,418 decision variables which include 5104 binary decision variables.

3.1. Short-term scheduling under uncertainty

The uncertainty in the iron grade and deleterious elements, mechanical fleet availability and hauling time are a major source of uncertainty that is incorporated into the production schedule formulation presented in Section 2.2. Fig. 5 (left) shows the SSTPS production schedule at the iron ore mine in this application. For reasons of comparison, Fig. 5 (right) shows the corresponding deterministic schedule generated from the deterministic equivalent of the SSTPS presented in Section 2.2, based on the average values for all related inputs. It is important to stress that the deterministically generated schedule may not be feasible in the actual presence of uncertainty that is not accounted for but is present.

Both production schedules in Fig. 5 consider the same operational considerations and allocate similar sectors of the iron ore deposit to be mined until the 5th month of production; then the effect of uncertainty becomes evident as not enough materials are located at the upper bench to match quality requirements and the fleet is moved to lower benches.

3.2. Utilization of the fleet

Maximum expected shovel production is planned given mechanical availability and scheduled time. The lack of matching this expected production is penalized to maximize the utilization of the shovels. The shovel utilization accounting for uncertainty at the SSTPS solution results in a higher and less variable than the shovel utilization of the deterministic STPS solution. From the STPS solution, the shovel with a historically high production was allocated preferentially to a sector that ensures its better utilization and the shovel with a historically low production to sectors with high production uncertainty. The risk profiles of the utilization cumulative distribution are given in term of P10, P50 and P90 with suffix S indicates stochastic solution.

The utilization of each shovel and the trucks are not exactly proportional because the trucks can be assigned to more than one shovel per period meanwhile the shovel is assigned to a sector and their movement between sectors are restricted by the cost associated. For example the small shovel HS6020 (Fig. 6, upper) is allocated to the sector that has less available material to be mined making its utilization low at the final periods; however, the

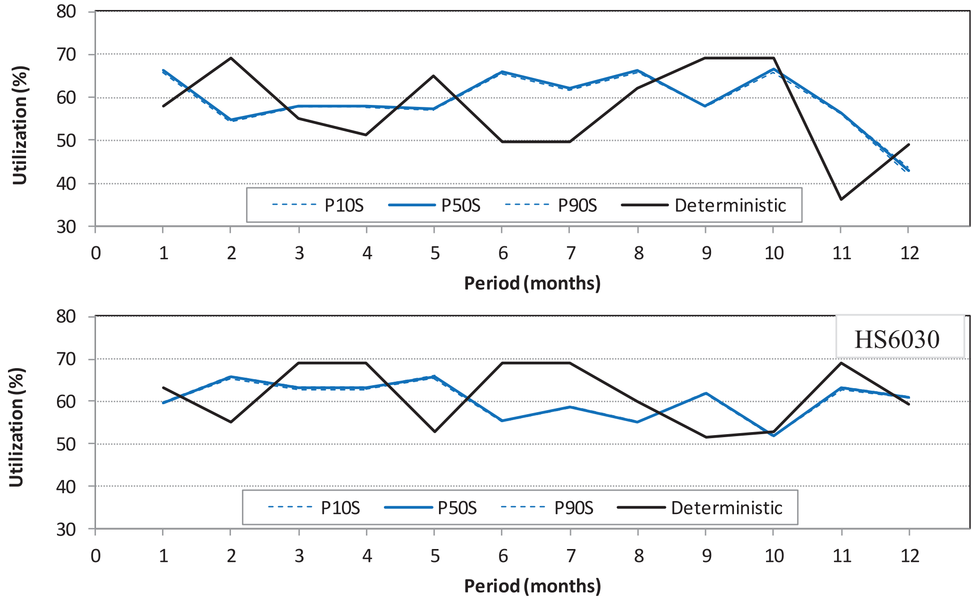

Fig. 6. Utilization risk profiles of shovels for SSTPS in blue lines; P10, P50 and P90 are percentiles and suffix S indicates stochastic solution. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article).

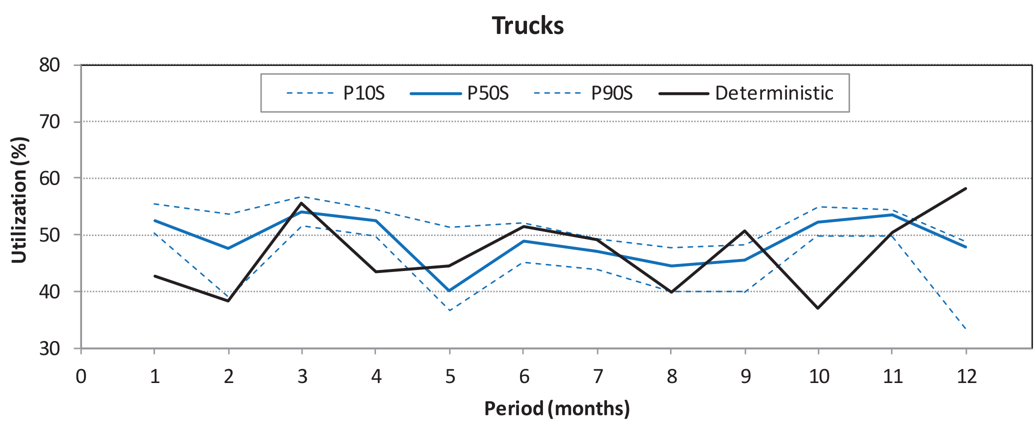

Fig. 7. Utilization risk profiles of trucks for SSTPS in blue lines; P10, P50 and P90 are the related percentiles and suffix S indicates stochastic solution. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article).

availability of the trucks are not affected because the trucks can be allocated to different sectors as shovels are allocated for each period. This ensures optimal utilization of the trucks each period, as shown in Fig. 7.

In some periods not all the trucks need to be allocated to match production targets. Considering that both schedules match production targets, the SSTPS shows a more efficient allocation or high utilization than the deterministic STPS because it allocates a lower number of trucks, as shown in Fig. 8, that is, the fleet allocation accounting for deterministic truck parameters is inefficient when compared to that accounting for possible fluctuations of mechanical availability and hauling time.

The stochastic formulation provides a well-informed schedule because it accounts for possible fluctuations of the grades and fleet parameters. From Figs. 6 and 7, the utilization of the fleet is shown as less variable through the periods when the uncertainty is considered.

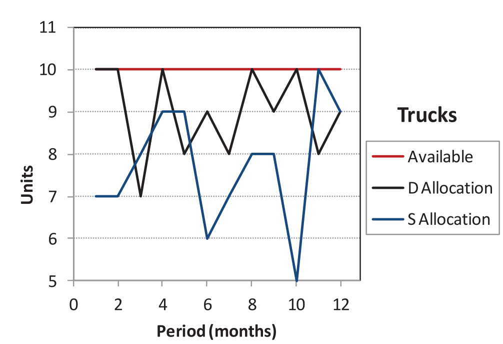

Fig. 8. Available trucks in red line, number of trucks allocated accounting for four source of uncertainty on blue line and without accounting for uncertainty on black line. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article).

3.3. Cost in the objective function

The objective function consider some terms associated with operating fleet cost, mining cost, and penalty cost to penalize deviation from production target, expected fleet utilization and mining width. The penalties cause some terms to have more priority than the others because the optimization preferably minimizes the components that have high value. The stochastic short-term production schedule solution shows less cost through the terms

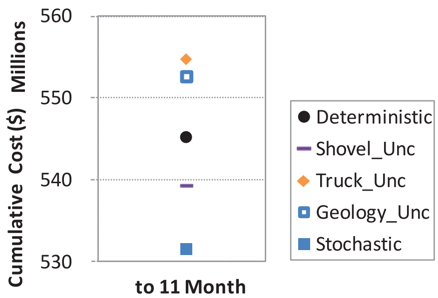

Fig. 9. Cumulative minimized objective cost.

in the objective function than the deterministic schedule solution. From the formulation, the minimized cost means that the plan guarantees the minimum deviation from the production targets, such as tonnage and quality, maximum utilization of the fleet, minimum cost of production extraction and a better match of the mining width considerations. Indeed, the best production schedule may be the one that obtains the lower minimized cost.

Fig. 9 shows that the mechanical availability shovel source of uncertainty has more influence in the stochastic solution than the uncertainty in the parameters of the trucks and orebody uncertainty. Also, the proposed stochastic formulation in an iron ore deposit provides an improvement cost of about fifteen million CAD dollars less than the deterministic or production schedule that ignores parameter uncertainty. The deterministic STPS formulation cannot minimize in the same range as the stochastic STPS does because the uncertainty in the parameter is not accounted for. The uncertainties in the mechanical availability, in the orebody model and in the hauling time give more feasible solutions in the solution space to choose the best solution.

The formulation proposed for stochastic short-term production scheduling obtains the solution with the lower cost in the application at a multi-element ore iron mine; however, the robustness of this formulation is based on the idea that their schedule is a well-informed plan because it accounts for operations considerations, possible fluctuations of the orebody metal quality and fluctuations of the fleet parameters to decide which sector to be mined per period.

4. Conclusions

A new formulation based on stochastic mixed integer programming is proposed herein to address short-term mine production scheduling in a single formulation, where mining considerations, production constraints, uncertainty in the orebody metal quantity as well as fleet parameters and availability are evaluated simultaneously. This allows to define a well-informed sequence of mining that has high performance at the mine operation. The quality of material scheduled to be extracted may influence also in the allocation of the fleet. The optimization process allocates the fleet to sectors that ensure the accomplishment of the production target, match the quality conditions, maximize fleet utilization, respect operational considerations, and accounts for uncertainty in the input parameters and information. The components of the objective function are expressed in terms of costs where the minimized total cost implies that the plan guarantees the minimum deviation from production target, maximum utilization of the fleet, minimum cost of production extraction and better match of mining width requirements. At the time of short-term production scheduling, additional information related to the operational restrictions, such as mining width and mining directions, are available. These additional physical constraints were implemented to deliver feasible production schedule patterns that will have better performance during operations.

It is anticipated that the ability to jointly optimize related elements, as detailed in this paper, entails more realistic thus better production planning. It is understood that operational flexibility and adaptation are part for any scheduling process; for example, in an operating mine additional sampling and grade control will lead to further adopting a short-term production schedule. The practical significance of the proposed optimization formulations is that it improves the overall production performance and minimizes the production scheduling changes needed, in reaction to operational aspects. Further research will address the dynamic simultaneous updating of short-term mine production scheduling with incoming information as production proceeds.

Acknowledgments

We thank the organizations and companies who funded this research: NSERC Collaborative Research and Development Grant CRDPJ 411270-10, entitled “Developing new global stochastic optimization and high-order stochastic models for optimizing mining complexes with uncertainty”; industry collaborators: AngloGold Ashanti, Barrick Gold, BHP Billiton, De Beers, Newmont Mining and Vale; and NSERC Discovery Grant 239019.

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