Research problems

The purpose of the research problems section is the presentation of unsolved problems in discrete mathematics. Older problems are acceptable if they are not as widely known as they deserve. Problems should be submitted using the format as they appear in the journal and sent to

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Readers wishing to make comments dealing with technical matters about a problem that has appeared should write to the correspondent for that particular problem. Comments of a general nature about previous problems should be sent to Professor Alspach.

Problems presented at the Twente Workshop on Hamiltonian Graph Theory, 6–10 April 1992

Problem 254. Posed by C. Hoede.
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The majority of results in hamiltonian graph theory concern sufficient conditions of a numerical nature. The typical example is Dirac’s theorem. Hardly any results are known about sufficient conditions for non-hamiltonicity that are of a structural nature. As an example, consider the following lemma, which is quite obvious.

If a graph $G$ contains the graph $H$ as an induced subgraph, then $G$ is non-hamiltonian, where $H$ is the graph $K_{1,3}$ with subdivided edges and the subdividing vertices have degree 2 in $G$. 
The problem is the study of induced substructures like $H$ that make a graph non-hamiltonian.

Reference


Problems 255 and 256. Posed by C. Hoede.

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The concept of closure as introduced by Bondy and Chvátal is of a numerical nature and defined for undirected graphs.

Problem 255. Is there a useful closure concept of a numerical nature for directed graphs?

Problem 256. Is there a useful closure concept of a structural nature for graphs or directed graphs?

References

[1] H.J. Broersma, A note on $K_4$-closures in hamiltonian graph theory, Memorandum no. 903, Department of Applied Mathematics, University of Twente, 1990.

Problem 257. Posed by H. Li.
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**Research Problems**

**Δ PPDC Conjecture.** Every simple graph admits a perfect path double cover \( \mathcal{P} \) such that the length of every path in \( \mathcal{P} \) is at most \( \Delta \), where \( \Delta \) is the maximum degree of the graph.

**Remarks.** (1) Li has shown [1] that every simple graph admits a perfect path double cover.

(2) The truth of the \( \Delta PPDC \) conjecture would imply the truth of Bondy’s regular perfect path double cover conjecture [2].

**References**


**Problems 258–261.** Posed by H.-J. Voss.

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In [1] I determined all 2-connected cubic graphs of given even circumference with the maximum number of vertices.

**Problem 258.** Determine all bipartite graphs with the same properties.

**Definition.** Let \( T \) be a tree with root \( W \) such that for each end vertex \( E \) of \( T \) the distance \( d(W, E) = s \) and each inner vertex has degree 3. Take two vertex-disjoint copies \( T', T'' \) of \( G \) and let \( E'_1, E'_2, \ldots \) and \( E''_1, E''_2, \ldots \) be the end vertices of \( T' \) and \( T'' \), respectively, where \( E'_i, E''_i \) correspond to the same end vertex \( E_i \) of \( T \). Take vertex-disjoint copies \( C_1, C_2, \ldots \) of a \( K_4 \) with one edge missing and identify one vertex of \( C_i \) of valency 2 with \( E_i \) and the other vertex of \( C_i \) of valency 2 with \( E'_i \) for all \( i \).

The graph \( D(4s + 6) \) obtained is 2-connected, cubic, and has circumference \( 4s + 6 \).

**Theorem.** Each 2-connected cubic graph of circumference \( 4s + 6 \), \( s \geq 1 \), with the maximum number of vertices is isomorphic with \( D(4s + 6) \).

The result for circumference \( 4s + 8 \), \( s \geq 1 \), can be found in [1].

The graph \( D^*(4s + 10) \) is obtained by using for \( C_1, C_2, \ldots \) copies of \( K_3,3 \) with one edge missing. The graph \( D^*(4s + 10) \) is bipartite, 2-connected, cubic, and has circumference \( 4s + 10 \).
Problem 259. Is the following conjecture true? 
"Each bipartite 2-connected cubic graph of circumference $4s + 10$, $s \geq 1$, with the maximum number of vertices is isomorphic with $D^*(4s + 10)$.

The conjecture for circumference $4s + 12$, $s \geq 1$, can be derived from the graph $D(4s + 8)$ from [1].
Following are two related problems.

Problem 260. Let $2s + 1$, $s \geq 1$, be fixed. Determine all 2-connected cubic graphs with the maximum number of vertices whose longest odd length cycles have length $2s + 1$.

Since in these graphs the circumference is at most $2((2s + 1) - 1) = 4s$ these graphs exists.

Problem 261. Let $2s$, $s \geq 1$, be fixed. Determine all 2-connected cubic graphs with the maximum number of vertices whose longest even length cycles have length $2s$.

In this case the existence of such graphs is unknown. In [1] a class of 2-connected graphs of minimum degree at least 3 has been constructed with fixed length of the longest even cycles having arbitrarily large odd cycles. But these graphs are not cubic.

Reference