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Adaptive control of a synchronous motor via a sliding mode decomposition technique

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Abstract This paper presents a decoupled control strategy using time-varying sliding surface-based sliding-mode controller for speed control of permanent magnet synchronous motor (PMSM). The decoupled method provides a simple way to achieve asymptotic stability for a PMSM by dividing the system into two subsystems electrical and mechanical systems. The simulation results for PMSM are presented to demonstrate the effectiveness and robustness of the method. Comparing this controller with pulse width modulation (PWM) controller for the same motor.

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1. Introduction

In recent years, sliding-mode control (SMC) has been suggested as an approach for the control of systems with nonlinearities, uncertain dynamics and bounded input disturbances.

The most distinguished features of the SMC technique are: (i) insensitivity to parameter variations, (ii) external disturbance rejection and (iii) fast dynamic responses. However, there is undesirable chattering in the control effort and bounds on the uncertainties required in the design of the SMC. The uncertainties usually include unmodel dynamics, parameter variations and external disturbances [1–4]. If the actual bounds of the uncertainties exceed the assumed values designed in the controller, stability of the system is not guaranteed.

Like other conventional control structures [5–7], the design of sliding-mode controllers needs the knowledge of the mathematical model of the plant, which decreases the performance in some applications where the mathematical modeling of the system is very hard and where the system has a large range of parameter variation together with unexpected and sudden external disturbances [8–11].

In this paper, a decoupled sliding-mode control (DSMC) design strategy is used to control the speed of PMSM. The motor system is divided into two subsystems with different switching surfaces to achieve the desired speed.
2. PMSM mathematical model

PMSM drives are becoming more popular and replace classical motors in industrial applications, machine tools and residential applications. In a PMSM the excitation is provided by means of using permanent magnets mounted on the rotor. PMSMs present numerous advantages like high efficiency, high torque to inertia ratio, high power density, reliability and long life.

For control unit design the synchronous motor is modeled in rotating rotor coordinates \( \{d, q\} \). Unlike in stator coordinates \( \{a, b, c\} \), where the signals have to be modulated on sine waves in order to propel the machine, the waveform of all variables in rotor coordinates is unconstrained and the modulation is carried out implicitly during transformation from rotor to stator coordinate.

Eq. (1) shows the transformation matrix between 3 phase currents \( (I_{dab}) \) and \( dq \)-currents \( (I_{dq}) \):

\[
I_{dq} = \sqrt{2} \begin{bmatrix}
\cos(\theta) & \cos(\theta - \frac{\pi}{2}) & \cos(\theta + \frac{\pi}{2}) \\
\sin(\theta) & \sin(\theta - \frac{\pi}{2}) & \sin(\theta + \frac{\pi}{2})
\end{bmatrix} \begin{bmatrix}
I_a \\
I_b \\
I_c
\end{bmatrix}
\]

The PMSM model is given by the following differential equations as [12]:

\[
\begin{align*}
\dot{L}_d &= -R_d i_d + \rho o L_q i_q + u_d \\
\dot{L}_q &= -R_q i_q - \rho o L_d i_d - 2\rho o o + u_q \\
\dot{T}_e &= 1.5p(\lambda_d + (L_d - L_q)i_d) \\
\dot{\omega} &= T_e - \mu o - T_L \\
\dot{\theta} &= \omega,
\end{align*}
\]

where \( u_d, u_q \): the rotor voltages in \( \{d, q\} \) coordinates \( (V) \); \( i_d, i_q \): rotor currents in \( \{d, q\} \) coordinates \( (A) \); \( \theta \): the electrical rotor position \( (rad) \); \( \omega \): the angular velocity of the motor shaft in electrical \( (rad/s) \); \( R_d, R_q \): the winding resistance of \( d \) and \( q \) axis \( (\Omega) \); \( L_d, L_q \): the inductance of \( d \) and \( q \) axis \( (H) \); \( \lambda \): rotor magnet flux linkage \( (Wb) \); \( J \): the rotor and shaft inertia \( (kg m^2) \); \( \mu \): the coefficient of friction \( (N.m/s) \); \( p \): the number of permanent magnet pole pairs; \( T_L \): the disturbing external torque \( (N.m) \) and \( T_e \): the motor torque \( (N.m) \).

For \( L_d = L_q = L \) then the motor torque will be

\[
T_e = 1.5p\lambda_d i_q = Kl_q,
\]

where \( k \) is the motor torque constant.

Substitution in Eq. (2), so the model can be written as:

\[
\begin{align*}
\dot{L}_d &= -R_d i_d + \rho o L_q i_q + u_d \\
\dot{L}_q &= -R_q i_q - \rho o L_d i_d - \frac{3}{2}k\omega + u_q \\
\dot{\omega} &= T_e - \mu o - T_L \\
\dot{\theta} &= \omega,
\end{align*}
\]

3. Motor model in state space equations

Let

\[
\begin{align*}
x_1 &= i_d \\
x_2 &= i_q \\
x_3 &= \omega,
\end{align*}
\]

then the model can be written as

\[
\begin{align*}
\dot{x}_1 &= -\frac{R_d}{L_d} x_1 + px_3 x_2 + \frac{1}{L_d} u_d \\
\dot{x}_2 &= -\frac{R_q}{L_d} x_2 - px_3 x_1 + \frac{2}{L_d} k x_3 + \frac{1}{L_d} u_q \\
\dot{x}_3 &= -\frac{p}{L_d} x_3 + \frac{1}{L_d} k x_2 - \frac{1}{L_d} T_l,
\end{align*}
\]

so the model can be written as:

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3
\end{bmatrix} =
\begin{bmatrix}
\frac{1}{L_d} & px_3 & 0 \\
px_3 & -\frac{R_d}{L_d} & -\frac{2}{L_d} k \\
0 & \frac{k}{L_d} & -\frac{1}{L_d}
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix} +
\begin{bmatrix}
\frac{1}{L_d} \\
0 \\
0
\end{bmatrix} u_d
\]

From Eqs. (6) and (7), it is obvious that the dynamic model of PMSM is highly nonlinear because of the coupling between the speed and the electrical currents, in addition to the saturation effect of the magnetic circuit and the existing viscous friction.

The PMSM parameters are: 1.1 KW, 3000 RPM, \( R = 2.875 \Omega \), \( L_d = L_q = 8.5 \text{ mH} \), \( P = 4 \text{ pair of poles} \) (8 poles), \( J = 0.8 \times 10^{-7} \text{ kg m}^2 \), \( \mu = 1 \text{ N.m.s} \).

4. Sliding mode control and decoupling

There is currently a large interest in sliding mode control algorithms due to their robustness properties and possibilities to decouple a high dimensional design problem into a set of lower dimensional independent sub-problems.

The switching function: Consider a general type of system represented by the state equation,

\[
\dot{x} = f(x, u, t).
\]

The control \( u(x, t) \) with its respective entry \( u_t(x, t) \) has the form

\[
u_t(x, t) = \begin{cases}
u_t^0(x, t) & \text{if } s_t(x) > 0 \\
u_t^1(x, t) & \text{if } s_t(x) < 0
\end{cases}
\]

where \( u_t^0(x, t), u_t^1(x, t) \) and \( s_t(x) \) are continuous functions. \( s_t(x) \) is an \( (n-1) \) dimensional switching function. Since \( u_t(x, t) \) undergoes discontinuity on the surface \( s_t(x) = 0, s_t(x) = 0 \) is called a switching surface or switching hyperplane as in Fig. 1.
With sliding condition

\[
\lim_{x \to 0} \dot{S} < 0, \quad \lim_{x \to 0} \dot{S} > 0.
\]  

(10)

Since it is required to produce a control input \( u(x, t) \) that applied to the system to obtain the desired output, i.e., it is required to determine the PMSM wave currents \((i_d, i_q)\) and voltages \((u_d, u_q)\) in order to obtain the desired output speed \(\omega\).

Due to the nonlinearity and the complexity of the motor model, it is needed to decouple the system into two subsystems that are the electrical and the mechanical systems that shown in Fig. 2.

4.1. Electrical subsystem model

\[
\begin{bmatrix}
  i_d \\
  i_q
\end{bmatrix} =
\begin{bmatrix}
  \dot{x}_1 \\
  \dot{x}_2
\end{bmatrix} =
\begin{bmatrix}
  -\frac{p}{T} & P x_3 \\
  -p x_3 & -\frac{e}{T}
\end{bmatrix}
\begin{bmatrix}
  x_1 \\
  x_2
\end{bmatrix} +
\begin{bmatrix}
  \frac{1}{T} & 0 \\
  0 & \frac{1}{T}
\end{bmatrix}
\begin{bmatrix}
  u_d \\
  u_q
\end{bmatrix} 
\]

\( T_e = \begin{bmatrix} 0 & K \end{bmatrix} \begin{bmatrix} x_1 \\
  x_2 \end{bmatrix}. \)  

(11)

In the electrical system, the object is to produce output currents for driving the mechanical parts of the motor by adjusting the input voltages waveforms with knowing motor velocity.

4.2. Mechanical subsystem model

\[
\dot{\omega} = \dot{x}_3 = \left[ -\frac{J}{T} \right] x_3 + \left[ K \right] x_2 - \left[ \frac{1}{T} \right] T_i,
\]

(12)

In the mechanical system, from the controlled input currents the motor rotates till reaching the desired velocity.

5. Sliding mode control design

The main idea behind the decoupled strategy is to decouple a nonlinear system appearing in the form of Eq. (7) into two subsystems as electrical and mechanical in the form of Eqs. (11) and (12). The electrical subsystem is chosen as a primary target while the mechanical subsystem is used as a secondary target.

However, the selection of the primary and the secondary subsystems is problem dependent. Here, the control objective is to devise a control strategy that would move the states of both subsystems toward their sliding surfaces \(S_1 = 0\) and \(S_2 = 0\). The electrical subsystem involves knowledge from mechanical subsystem, and the mechanical subsystem is driven from the electrical subsystem.

Let the sliding surface function \(S_1\) be defined as

\[
S_1 = c_1 x_1 + c_2 x_2,
\]

(13)  

where \(c_1\) and \(c_2\) are the sliding surface \(S_1\) constants for dq-axis, for simplified calculations use current transform to convert from dq-axis to abc-axis, so Eq. (13) can be expressed as:

\[
S_1 = c_1 (i_a - i_d) + c_2 (i_b - i_q) + c_3 (i_c - i_d),
\]

(14)  

where \(c_1\), \(c_2\) and \(c_3\) are the sliding surface \(S_1\) constants for abc-axis \(i_a, i_b\) and \(i_c\) are the references three phases currents, \(i_d, i_q\) and \(i_c\) are the actual three phases currents. Let the sliding surface function \(S_2\) is defined as

\[
S_2 = c_4 (\omega_o - \omega),
\]

(15)  

where \(c_4\) is the sliding surface \(S_2\) constant, \(\omega_o\) is the reference motor speed, \(\omega\) is the actual motor speed.

In the design of decoupled sliding-mode controller, an equivalent control is first given so that the states can stay on sliding surface. Thus, in sliding motion, the system dynamic is independent of the original system and a stable equivalent control system is achieved. So the decoupled sliding-mode controller \(u\) can be divided into an equivalent control input and a hitting control input if it has the following control law:

\[
u = u_{eq} - M_{sat}(S_1),
\]

(16)  

where \(M\) is a positive constant, \(u_{eq}\) is the equivalent.

The sliding mode controller block diagram can be shown in Fig. 3 after determining the desired position and calculating the reference currents \(i_{a_ref}, i_{b_ref}, i_{c_ref}\). Sliding mode control produces on/off signals \(g\) for the switching inverter which in turn operates the synchronous machine. Observing the motor current position and angular velocity to adapt the sliding mode for reaching the sliding surfaces.

6. SMC controller

The SMC speed controller is shown in Fig. 4.

The model contains PI block which is used to calculate the reference signal needed in dq-axis. A dq-abc-transform block transfers the dq-currents to abc-currents format. The SMC2 starts to calculate the tracking error in speed Eq. (15), hence produces the reference currents needed for the motor. SMC1 calculates the tracking error in currents Eq. (14), then produces output voltages to force the system toward the sliding surface first (reaching), then SMCs try to keep the system slides on the switching surfaces till reaching the desired speed.

The SMC1 contents are shown in Fig. 5.

The contents of compare, compare2 and 3 are shown in Figs. 6–8, respectively.

The transfer function is a filter to reduce the harmonics with the measured current, then satisfy the sliding surface \((S_1)\) to obtain the control output. Selecting \(c_1 = c_2 = c_3\) because of the contribution of each phase is similar to the others. Large constants required for faster response. But not too large, otherwise the motor will suffer from torque ripples. When the \(S_1\) moves toward the origin (the reference currents are near the actual currents) and with motor current ranges from +1.72 to -1.72 A, larger constants \(c_1\), \(c_2\) and \(c_3\) are required specially with small current differences. If these constants are too large then \(S_1\) may change its sign and may induce torque ripples.

The SMC2 contents are shown in Fig. 9.

For reaching the desired speed with faster response and without speed oscillations, it should select \(c_4\) not small than 1 for achieving faster response and not large for reducing the speed oscillations i.e., \(c_4\) should be close to 1.
Figure 3 Sliding mode speed control block diagram.

Figure 4 PMSM speed control using SMC.

Figure 5 The SMC1 controller.

Figure 6 Compare contents.
7. Simulation results

Figs. 10–14 summarize the simulation results for a three-phase motor rated 1.1 kW, 3000 rpm. The load torque applied to the machine’s shaft is originally set to its nominal value (3 N.m) and steps down to 1 N.m at \( t = 0.04 \) s. The desired speed is 300 rad/s.

Fig. 10 shows the sliding surfaces. It is noted SMC forces the states of the system to move toward the sliding surfaces, then keeping the states slide on these surfaces achieving the required performance.

Fig. 11 shows the PMSM speed, torque, currents and voltage using the SMC controller. These results can be summarized as the following:
- The reference speed is 300 rad/s, the motor speed stabilized at 0.02 s. Table 1 summarizes the results.
- The torque stabilized at the nominal torque 3 N.m till 0.04 s. The motor torque drops to −1 N.m at 0.02 s due to the torque reducing, then the SMC controller tries to overcome the sudden torque changes and stabilize the torque at 2 N.m.
Figure 10  The sliding surfaces.

Figure 11  SMC controller simulation curves of PMSM.
The motor starts with high current 30 A, then the currents are reduced and stabilized in sinusoidal waveform with max. 4 A. When the load torque is reduced to 2 N.m at 0.04 s, all currents are reduced to max. 1.72 A. Chopping the motor voltage to achieve the required currents for adjusting the speed.

8. Comparative analysis

Another technique can be used to control the speed of the PMSM. PWM (pulse width modulation) inverter is used to control the waveforms of currents and voltages that will be applied to the motor. The PWM controller is shown in Fig. 12.
Fig. 13 summarizes the simulation results for PWM controller with the same motor parameters. Fig. 13 shows the PMSM speed, torque, currents and voltage using PWM controller. These results can be summarized as the following:

- The reference speed is 300 rad/s, the motor speed stabilized at 0.02 s. Table 2 shows the results.

From Tables 1 and 2, it is noted that the SMC controller reaches the motor speed close to the reference speed than PWM controller. Also Fig. 14 shows the speed oscillations for a sample time for both controllers at steady state which concluded that the speed oscillations in the SMC controller is less than in PWM controller.

- The motor torque drops to \(-1.8\) N.m at 0.02 s due to the torque reducing. The compared results are shown in Table 3.

The torque in PWM controller switches between positive and negative torques which means high torque ripples than in the SMC controller.

**Table 1** Speed results using SMC.

<table>
<thead>
<tr>
<th>Time of reaching</th>
<th>0.02 s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max. speed deviation (steady state)</td>
<td>300.06 Rad/s</td>
</tr>
<tr>
<td>Min. speed deviation (steady state)</td>
<td>299.94 Rad/s</td>
</tr>
<tr>
<td>Speed oscillations (steady state)</td>
<td>Approx. ± 0.02%</td>
</tr>
</tbody>
</table>

Fig. 14  Steady state sample period speed of PMSM.

![Figure 14](image)

**Table 2** Speed results using PWM.

<table>
<thead>
<tr>
<th>Time of reaching</th>
<th>0.02 s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max. speed deviation (steady state)</td>
<td>300.13 Rad/s</td>
</tr>
<tr>
<td>Min. speed deviation (steady state)</td>
<td>299.8 Rad/s</td>
</tr>
<tr>
<td>Speed oscillations (steady state)</td>
<td>Approx. ± 0.06%</td>
</tr>
</tbody>
</table>

- The three phases currents have max. current 2 A at steady state in PWM controller, while they have max. 1.72 A in the SMC controller for the same motor.
- The voltage is chopped achieving the required motor currents and speed.

**Table 3** Compared torque results.

<table>
<thead>
<tr>
<th>Torque</th>
<th>PWM</th>
<th>SMC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Torque range</td>
<td>(-0.25),-(-2.25) N.m</td>
<td>0.1,-,2 N.m</td>
</tr>
<tr>
<td>Sudden torque change</td>
<td>(-1.8) N.m</td>
<td>(-1) N.m</td>
</tr>
</tbody>
</table>

9. Conclusion

This paper presents the design of control system for controlling the speed of permanent magnet synchronous motor using the decoupled sliding mode control algorithm. The decoupled technique divides the system into electrical and mechanical subsystems which can be separately modeled and controlled. The proposed controller assures its validity and effectiveness against the regular pulse width modulation controller. Simulation results were presented for both controllers and a comparative analysis with previous works is introduced to illustrate the effectiveness of the suggested controller.

**References**

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