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Coulomb interaction from the interplay between confinement and screening

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Abstract

It has been noticed that confinement effects can be described by the addition of a $\sqrt{-F^a_{\mu\nu}F^{a\mu\nu}}$ term in the Lagrangian density. We now study the combined effect of such "confinement term" and that of a mass term. The surprising result is that the interplay between these two terms gives rise to a Coulomb interaction. Our picture has a certain correspondence with the quasiconfinement picture described by Giles, Jaffe and de Rujula for QCD with symmetry breaking.

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1. Introduction

It is well known that one of the long standing problems in physics is understanding the confinement physics from first principles. Hence the challenge is to develop analytical approaches which provide valuable insight and theoretical guidance. According to this viewpoint, an effective theory in which confining potentials are obtained as a consequence of spontaneous symmetry breaking of scale invariance has been developed [1]. In particular, it was shown that a such theory relies on a scale-invariant Lagrangian of the type [2]

$$\mathcal{L} = \frac{1}{4}w^2 - \frac{1}{2}w\sqrt{-F^a_{\mu\nu}F^{a\mu\nu}}, \tag{1}$$

where $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc}A^b_\mu A^c_\nu$, and w is not a fundamental field but rather is a function of 4-index field strength, that is,

$$w = \varepsilon^{\mu\nu\alpha\beta}\partial_\mu A_{\nu\alpha\beta}. \tag{2}$$

The $A_{\nu\alpha\beta}$ equation of motion leads to

$$\varepsilon^{\mu\nu\alpha\beta}\partial_\beta\left(w - \sqrt{-F^a_{\gamma\delta}F^{a\gamma\delta}}\right) = 0, \tag{3}$$

which is then integrated to

$$w = \sqrt{-F^a_{\mu\nu}F^{a\mu\nu}} + M. \tag{4}$$

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It is easy to verify that the A_a^μ equation of motion leads us to

$$\nabla_\mu\left(F^{a\mu\nu}+M\frac{F^{a\mu\nu}}{\sqrt{-F^b_{\alpha\beta}F^{b\alpha\beta}}}\right)=0. \tag{5}$$

It is worth stressing at this stage that the above equation can be obtained from the effective Lagrangian

$$\mathcal{L}_{\mathrm{eff}}=-\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu}+\frac{M}{2}\sqrt{-F^a_{\mu\nu}F^{a\mu\nu}}. \tag{6}$$

Spherically symmetric solutions of Eq. (5) display, even in the Abelian case, a Coulomb piece and a confining part. Also, the quantum theory calculation of the static energy between two charges displays the same behavior [1]. It is well known that the square root part describes string like solutions [3,4].

Within this framework the aim of the present Letter is to extend further the previous analysis by considering the effect of a mass term. To this end we will compute the static potential of this theory. In fact, we will show that the static potential for the new theory gives rise to an effective Coulomb interaction. We recall in passing that the static potential between a heavy quark and antiquark is a tool of considerable theoretical interest which is expected to provide the foundation for understanding confinement. According to our approach, the interaction potential between two charges is obtained once a suitable identification of the physical degrees of freedom is made. This methodology has been used previously in many examples for studying features of screening and confinement in gauge theories [6,7].

2. The interplay between confinement and mass terms

Some time ago, Giles et al. [8] proposed that in the presence of spontaneous breaking of gauge symmetry confinement in QCD may become an approximate effect and there could be in this case high mass states of unconfined quarks and gluons. Their analysis was done in the context of the MIT bag model [9].

Subsequently, this research was criticized by Georgi [10], who argued that the confinement properties of QCD will present an obstacle for the s.s.b. of gauge symmetry.

Here we want to show that even if s.s.b. of gauge symmetry is not in question and that there is indeed a mass term induced in the action, then the dynamics of a theory which is governed by a confining term (explained in the previous section) and a mass term presents highly unexpected features.

Let us study an effective action of the form

$$\mathcal{L}_{\mathrm{eff}}=-\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu}+\frac{M}{2}\sqrt{-F^a_{\mu\nu}F^{a\mu\nu}}-\frac{\mu^2}{2}A^a_\mu A^{a\mu}, \tag{7}$$

and let us study for simplicity the Abelian case. Then, equation for the spherically symmetric case is

$$\nabla\cdot\left(\mathbf{E}+\frac{M}{\sqrt{2}}\hat{\mathbf{r}}\right)=-\mu^2\phi. \tag{8}$$

Looking for static solutions where also we set $\mathbf{A}=0$, that is, $\mathbf{E}=-\nabla\phi$, we find that Eq. (8) becomes

$$\frac{1}{r}\frac{d^2}{dr^2}(r\phi)-\frac{M}{\sqrt{2}}\frac{1}{r}-\mu^2\phi=0, \tag{9}$$

which for $\mu^2=0$, has as solution [1]

$$\phi=\frac{C}{r}+\frac{M}{\sqrt{2}}r, \tag{10}$$

displaying a confinement (M) part and a Coulomb part. Notice that for $\mu^2\neq 0$ the nature of the solutions is totally different, being of the form

$$\phi=C\frac{e^{-\mu r}}{r}-\left(\frac{M}{\sqrt{2}\mu^2}\right)\frac{1}{r}. \tag{11}$$

From Eq. (11) we can appreciate the interesting phenomenon of the appearance of an effective Coulomb term, which depends on both the confining term (M dependence) and on the screening or mass term (μ^2 dependence). The confining term in Eq. (10) has disappeared and is being replaced by a Coulomb term, even for μ arbitrarily small. As $\mu^2\to 0$ instead of confinement one has an arbitrarily strong Coulomb term. These general arguments can be put in a more solid ground by the use of the full quantum mechanical gauge-invariant variables formalism.

3. Interaction energy

As already mentioned, our immediate objective is to compute explicitly the interaction energy between static pointlike sources for the mode under consideration. The starting point is the two-dimensional space–time Lagrangian obtained from (7) in the Abelian case and considering only r, t dependence, a sort of mini-superspace approach [5].

$$\mathcal{L} = 4\pi r^2\left\{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{M}{2\sqrt{2}}\varepsilon_{\mu\nu}F^{\mu\nu} - \frac{\mu^2}{2}A_\mu A^\mu\right\} - A_0 J^0, \tag{12}$$

where J^0 is the external current, and μ is the mass for the gauge fields. Here $\mu, \nu = 0, 1$, where $x^1 \equiv r \equiv |\mathbf{x}|$ and $\varepsilon^{01} = 1$. We have used that in a two-dimensional space (t, r),

$$\sqrt{-F^{\mu\nu}F_{\mu\nu}} = \frac{\varepsilon_{\mu\nu}F^{\mu\nu}}{\sqrt{2}}.$$

It is worthwhile sketching at this point the canonical quantization of this theory from the Hamiltonian analysis point of view. The canonical momenta read

$$\Pi^\mu = -4\pi x^2\left(F^{0\mu} + \frac{M}{\sqrt{2}}\varepsilon^{0\mu}\right),$$

which results in the usual primary constraint $\Pi^0 = 0$, and

$$\Pi^i = -4\pi x^2\left(F^{0i} + \frac{M}{\sqrt{2}}\varepsilon^{0i}\right).$$

The canonical Hamiltonian following from the above Lagrangian is:

$$H_0 = \int dx\left\{\Pi_1\partial^1 A^0 - \frac{1}{8\pi x^2}\Pi_1\Pi^1 - \frac{M}{\sqrt{2}}\varepsilon^{01}\Pi_1 + \pi x^2 M^2 + 2\pi x^2\mu^2\left(A_0A^0 + A_1A^1\right) + A_0J^0\right\}. \tag{13}$$

Requiring the primary constraint $\Pi_0 = 0$ to be preserved in time yields the following secondary constraint

$$\Gamma(x) \equiv \partial_1\Pi^1 - 4\pi x^2\mu^2 A^0 - J^0 = 0. \tag{14}$$

It is straightforward to see that both constraints are second class. This result is not surprising, it explicitly reflects the breaking of the gauge invariance of the theory under consideration. Thus, special care has to be exercised since it is the gauge invariance that generally establish unitarity and renormalizability in most quantum field theoretical models. To convert the second class system into first class we will adopt the procedure described in Refs. [11,12]. In this way the new system still has the basic features of the original one and has reobtained the gauge symmetry. As was explained in Refs. [11,12], we enlarge the original phase space by introducing a canonical pair of fields θ and Π_θ. Then a new set of first class constraints can be defined in this extended space:

$$\Lambda_1 \equiv \Pi_0 + 4\pi x^2\mu^2\theta = 0, \tag{15}$$

$$\Lambda_2 \equiv \Gamma + \Pi_\theta = 0. \tag{16}$$

It is easy to verify that the new constraints are first class and in this way restore the gauge symmetry of the theory under consideration. It is worthwhile remarking at this point that the θ fields only enlarge the unphysical sector of the total Hilbert space, not affecting the structure of the physical subspace [11]. Therefore, the new effective Lagrangian after integrating out the θ fields reads

$$\mathcal{L} = 4\pi r^2\left\{-\frac{1}{4}F_{\mu\nu}\left(1 + \frac{\mu^2}{\Box}\right)F^{\mu\nu} - \frac{M}{2\sqrt{2}}\varepsilon_{\mu\nu}F^{\mu\nu}\right\} - A_0J^0. \tag{17}$$

We now restrict our attention to the Hamiltonian framework of this theory. The canonical momenta read

$$\Pi^\mu = -4\pi x^2\left[\left(1 + \frac{\mu^2}{\Box}\right)F^{0\mu} + \frac{M}{\sqrt{2}}\varepsilon^{0\mu}\right].$$

This yields the usual primary constraint $\Pi^0 = 0$, and

$$\Pi^i = -4\pi x^2\left[\left(1 + \frac{\mu^2}{\Box}\right)F^{0i} + \frac{M}{\sqrt{2}}\varepsilon^{0i}\right].$$

Therefore, the canonical Hamiltonian takes the form

$$H_C = \int dx\left\{-A_0\left(\partial_1\Pi^1 - J^0\right) - \frac{1}{8\pi x^2}\Pi_1\left(1 + \frac{\mu^2}{\Box}\right)^{-1}\Pi^1 - \frac{M}{\sqrt{2}}\left(1 + \frac{\mu^2}{\Box}\right)^{-1}\varepsilon^{01}\Pi_1\right\} + \int dx\left\{\pi M^2\left(1 + \frac{\mu^2}{\Box}\right)^{-1}x^2\right\}. \tag{18}$$

Temporal conservation of the primary constraint Π_0 leads to the secondary constraint $\Gamma_1(x) \equiv \partial_1\Pi^1 -$

$J^0 = 0$. It is straightforward to check that there are no further constraints in the theory. The extended Hamiltonian that generates translations in time then reads

$$H = H_C + \int dx \left(c_0(x)\Pi_0(x) + c_1(x)\Gamma_1(x) \right),$$

where $c_0(x)$ and $c_1(x)$ are the Lagrange multipliers. Moreover, it follows from this Hamiltonian that $\dot{A}_0(x) = [A_0(x), H] = c_0(x)$, which is an arbitrary function. Since $\Pi_0 = 0$, neither A^0 nor Π^0 are of interest in describing the system and may be discarded from the theory. As a result, the Hamiltonian becomes

$$H = \int dx \left\{ -\frac{1}{8\pi x^2}\Pi_1 \left(1 + \frac{\mu^2}{\Box}\right)^{-1} \Pi^1 - \frac{M}{\sqrt{2}}\left(1 + \frac{\mu^2}{\Box}\right)^{-1} \varepsilon^{01}\Pi_{01} + c'\left(\partial_1 \Pi^1 - J^0\right) \right\}, \tag{19}$$

where $c'(x) = c_1(x) - A_0(x)$.

According to the usual procedure we introduce a supplementary condition on the vector potential such that the full set of constraints becomes second class. A convenient choice is found to be [1,6,7]

$$\Gamma_2(x) \equiv \int_{C_{\xi x}} dz^\nu A_\nu(z) \equiv \int_0^1 d\lambda\, x^1 A_1(\lambda x) = 0, \tag{20}$$

where λ $(0 \leqslant \lambda \leqslant 1)$ is the parameter describing the spacelike straight path $x^1 = \xi^1 + \lambda(x - \xi)^1$, and ξ is a fixed point (reference point). There is no essential loss of generality if we restrict our considerations to $\xi^1 = 0$. In this case, the only nontrivial Dirac bracket is

$$\left\{A_1(x), \Pi^1(y)\right\}^* = \delta^{(1)}(x - y) - \partial_1^x \int_0^1 d\lambda\, x^1 \delta^{(1)}(\lambda x - y). \tag{21}$$

We are now equipped to compute the interaction energy between pointlike sources in the model under consideration, where a fermion is localized at the origin $\mathbf{0}$ and an antifermion at $\mathbf{y}$. In order to accomplish this purpose, we will calculate the expectation value of the energy operator H in the physical state $|\Phi\rangle$. From our above discussion, we see that $\langle H\rangle_\Phi$ reads

$$\langle H\rangle_\Phi = \langle\Phi| \int dx \left(-\frac{1}{8\pi x^2}\Pi_1\left(1 + \frac{\mu^2}{\Box}\right)^{-1}\Pi^1 - \frac{M}{\sqrt{2}}\left(1 + \frac{\mu^2}{\Box}\right)^{-1}\varepsilon^{01}\Pi_{01}\right)|\Phi\rangle. \tag{22}$$

Since the fermions are taken to be infinitely massive (static), we can substitute $\Box$ by $-\partial_1^2$ in Eq. (22). Here $-\partial_1^2$ refers to the radial part of the spherical Laplacian. In such a case we write

$$\langle H\rangle_\Phi = \langle\Phi| \int dx \left(-\frac{1}{8\pi x^2}\Pi_1\left(1 - \frac{\mu^2}{\partial_1^2}\right)^{-1}\Pi^1 - \frac{M}{\sqrt{2}}\left(1 - \frac{\mu^2}{\partial_1^2}\right)^{-1}\varepsilon^{01}\Pi_1\right)|\Phi\rangle. \tag{23}$$

Next, as was first established by Dirac [13], the physical state can be written as

$$|\Phi\rangle \equiv \left|\bar{\Psi}(\mathbf{y})\Psi(\mathbf{0})\right\rangle = \bar{\psi}(\mathbf{y}) \exp\left(ie \int_{\mathbf{0}}^{\mathbf{y}} dz^i A_i(z) \right) \psi(\mathbf{0})|0\rangle, \tag{24}$$

where $|0\rangle$ is the physical vacuum state and the line integral appearing in the above expression is along a spacelike path starting at $\mathbf{0}$ and ending $\mathbf{y}$, on a fixed time slice. From this we see that the fermion fields are now dressed by a cloud of gauge fields.

Taking into account the above Hamiltonian structure, we observe that

$$\Pi_1(x)\left|\bar{\Psi}(y)\Psi(0)\right\rangle = \bar{\Psi}(y)\Psi(0)\Pi_1(x)|0\rangle - e\int_0^y dz_1\, \delta^{(1)}(z_1 - x)|\Phi\rangle. \tag{25}$$

Inserting this back into (23), we get

$$\langle H\rangle_\Phi = \langle H\rangle_0 - \frac{e^2}{4\pi}\frac{e^{-\mu L}}{L} - \frac{Me}{\sqrt{2}\cdot 4\pi\mu^2}\frac{1}{L}, \tag{26}$$

where $\langle H\rangle_0 = \langle 0|H|0\rangle$ and with $|y| \equiv L$. Since the potential is given by the term of the energy which depends on the separation of the two fermions, from the expression (26) we obtain

$$V = -\frac{e^2}{4\pi}\frac{e^{-\mu L}}{L} - \frac{Me}{\sqrt{2}\cdot 4\pi\mu^2}\frac{1}{L}. \tag{27}$$

In this way the static interaction between fermions arises only because of the requirement that the $|\bar{\Psi}\Psi\rangle$ states be gauge invariant.

4. Final remarks

From our final expression for the heavy interquark potential we see that:

(a) For $\mu^2 = 0$ the theory describes an exactly confining phase.

(b) For $\mu^2 \neq 0$ but μ^2 very small, we observe that the linear potential is now replaced by a Coulomb potential which is, however, a very strong one. In this limit, states will be indeed bound, that is, confined due to the very strong Coulomb potential unless they correspond to very high excitations. Indeed, the "ionization energy" of this system goes to infinity as $\mu^2 \to 0$. However, the Coulomb potential is not exactly confining, therefore, even for small μ^2, the confining nature the potential is lost. In general, this picture agrees qualitatively with that of Giles, Jaffe and de Rujula of quasiconfinement for QCD with a small gauge symmetry breaking term [8].

One may question other issues concerning the model, for example, the question of renormalizability. In this respect, we can observe that in the term that gives rise to the confining behavior (that is, the square root term) we have introduced the coupling constant M which has dimensions of $(mass)^2$. From the naive criteria is a superrenormalizable interaction, since the coupling constant has positive dimensions of mass. In the mini-superspace example one can check explicitly that this term becomes the totally harmless term proportional to $x^2\varepsilon^{\mu\nu}F_{\mu\nu}$, an "almost topological" term and therefore totally ultraviolet-safe. It is hoped that such good ultraviolet properties will remain in the fully not truncated version of the theory. This, however, is a separate question and we do not intend to address it in this Letter. We expect to report on progress along these lines elsewhere.

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