Ain Shams Engineering Journal (2015) **xxx**, xxx–xxx

Ain Shams University
Ain Shams Engineering Journal
www.elsevier.com/locate/asej
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MECHANICAL ENGINEERING

Effect of magnetic field on unsteady natural convective flow of a micropolar fluid between two vertical walls

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Received 28 March 2015; revised 2 August 2015; accepted 28 August 2015

KEYWORDS

Micropolar fluid;
Vertical walls;
Unsteady flow;
Magnetic field

Abstract We study theoretically the boundary layer flow of an incompressible micropolar fluid under uniform magnetic field and motion takes place due to the buoyancy force between vertical walls. The governing unsteady boundary layer momentum, angular momentum and energy equations of micropolar fluid are nondimensionalized and solved numerically. Analytic result for steady state case is also discussed. The effects of magnetic parameter (M), vortex viscosity parameter (R), Prandtl number (Pr) and material parameter (b) on velocity, micro-rotation and Temperature profiles are discussed through several figures.

1. Introduction

Recently, the study of micropolar fluid has attracted many scholars as Navier–Stokes equations of Newtonian fluids cannot effectively describe the characteristics of fluid with suspended particles. Erigen [1] first introduced the theory of micropolar fluid and this theory is useful in explaining the characteristics of certain fluids such as liquid crystals, suspensions and animal blood. Chamkha [2] and Abdulaziz [3], studied the fully developed free convection of a micropolar fluid in vertical channel. Si [4] and Bég [5] have studied homotopy analysis solution for micropolar fluid flow through a porous medium. Rashidi [6], has studied Heat Convection in Magnetized Micropolar Fluid by Using Modified Differential Transform Method. Sadri [7] has discussed about Semi Analytical Solution of Boundary-Layer Flow of a Micropolar Fluid through a Porous Channel. Siddangoudaa [8] studied Squeezing Film Characteristics for Micropolar Fluid between Porous Parallel Stepped Plates.

Convection flow arises in many physical situations such as in the cooling of nuclear reactors and environmental heat transfer processes amongst others. It is of three types namely free, mixed and force. Amongst them, the problems of magneto hydrodynamic free convective flow in a porous medium have drawn considerable attentions of several researchers in various scientific and technological applications such as

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Peer review under responsibility of Ain Shams University.

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http://dx.doi.org/10.1016/j.asej.2015.08.013
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Nomenclature

b	material parameter
g	acceleration due to gravity
L	distance between two vertical walls
m	temperature ratio
M	magnetic parameter
Pr	Prandtl number
R	vortex viscosity parameter
t	time in non-dimensional form
t'	time
T'_c	temperature of the wall at $y' = L$
T'_h	temperature of the wall at $y' = 0$
T'_m	initial temperature of the fluid
u	fluid velocity in non-dimensional form
u'	velocity of fluid
y	dimensionless co-ordinate perpendicular to the walls
y'	co-ordinate perpendicular of the walls
ω	dimensionless angular velocity
j	micro-inertia density
κ	vortex viscosity
μ	dynamic viscosity

Greek symbols

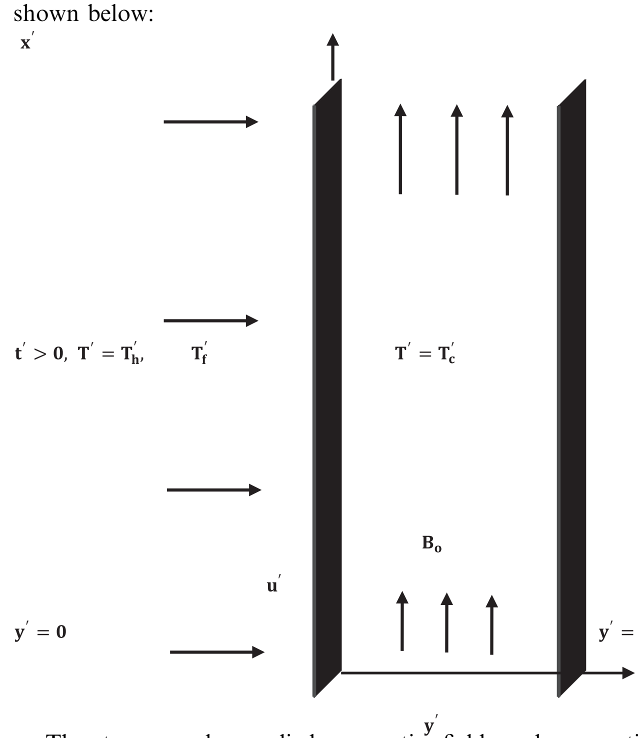
θ	temperature of the fluid in non-dimensional form
ν	kinematic viscosity of the fluid

pumps, flow meters, generators, accelerators, plasma jet engines, and magnetic control of molten iron flow in steel industry and industrial processes in metallurgy and material processing, in chemical industry, industrial power engineering and nuclear engineering. Special mention can be made, for instance, to the experiments on liquid metal flows in MHD channels performed by Hartmann [9]. Rashidi [10] has studied Unsteady Two-Dimensional and Axisymmetric Squeezing flows between Parallel Plates. Siddiqui [11] considered Homotopy Analysis Method to the Unsteady Squeezing Flow between Circular Plates. Singh [12] discussed on MHD Free Convective Flow Past a Semi-infinite Vertical Permeable wall. Hamad [13], Uddin [14] and Khan [15] have studied MHD natural convection flow on a nanofluid in different physical conditions then Khan [16] obtained the solution of unsteady two-dimensional and axisymmetric squeezing flow between parallel plates. Kataria [17] has studied induced magnetic field effects on mixed convection. Narayana [18], Oahimire [19], Olajuwon [20] and Prakash [21] have studied effect of hall current and radiation on MHD flow of a micropolar fluid. Mahmoud [22] has obtained the solution of MHD flow of a micropolar fluid over a stretching surface with heat generation and slip velocity. Bég [23] has studied nanofluid convection boundary layers from an isothermal spherical body in a permeable regime. Freidoonimehr [24] and Vendabai [25] have studied free convective MHD flow past a permeable stretching vertical surface in a nano-fluid then Borrelli [26] obtained the solution of Magneto convection of a micropolar fluid in a vertical channel.

In most of the research works, case of asymmetric or symmetric thermal condition is studied in the absence of magnetic field. In this paper, we have analysed effect of magnetic field on unsteady natural convective flow and micro rotation between infinitely long vertical walls for asymmetric/symmetric wall temperatures.

2. Basic equations and description of the problem

Consider the unsteady free-convective flow of an electric conductive micro-polar fluid between two insulated vertical walls separated by a distance L apart subjected to a uniform transverse magnetic field. The coordinate system is chosen such that x' measures the distance along the walls and y' measures the distance normal to it. Initially, the temperatures of walls and the fluid are same says T'_f. When time $t' > 0$, the temperature of the walls at $y' = 0$ and $y' = L$ is instantaneously raised and lowered to T'_h and T'_c respectively such that $T'_h > T'_c$ which is there after maintained constant. A constant uniformly distributed transverse magnetic field of strength B_0 is applied in the y'-direction. Physical model and coordinate system are shown below:



The transversely applied magnetic field and magnetic Reynolds number are very small and hence the induce magnetic field is negligible. No electrical field is assumed to exist and both viscous and magnetic dissipations are neglected. The hall effects, the viscous dissipation and the joule heating terms are also neglected. Under above assumptions and taking into account the Boussinesq and boundary layer approximations, momentum, angular momentum and energy equations of micropolar fluid can be expressed as follows:

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Table 1 Numerical and analytic values of steady state velocity, micro rotation and temperature profile.

M	Pr	m	R	b	y	Analytic solution u velocity	Numerical solution u velocity	Analytic solution w micro-rotation	Numerical solution w micro rotation	Analytic solution θ temperature	Numerical solution θ temperature
0.1	1.0	0	0.5	0.1	0.2	0.0320	0.0319	−0.0000783	−0.0000784	0.8000	0.8000
					0.6	0.0373	0.0373	0.0001167	0.0001164	0.4000	0.4000
5	1.0	0	0.5	0.1	0.2	0.0143	0.0144	−0.00001497	−0.00001517	0.8000	0.8000
					0.6	0.0127	0.0127	0.00006198	0.00006189	0.4000	0.4000
0.1	1.0	1	0.5	0.1	0.2	0.0533	0.0533	−0.0002128	−0.0002128	1	1
					0.6	0.0800	0.0800	0.0001064	0.0001064	1	1
5	1.0	1	0.5	0.1	0.2	0.0211	0.0211	−0.00007158	−0.00007175	1	1
					0.6	0.0289	0.0290	0.00003370	0.00003377	1	1
5	0.71	0	0.5	0.1	0.2	0.0143	0.0144	−0.00001497	−0.00001518	0.8000	0.8000
					0.6	0.0127	0.0127	0.00006198	0.00006189	0.4000	0.4000
5	2	0	0.5	0.1	0.2	0.0143	0.0142	−0.00001497	−0.00001516	0.8000	0.7999
					0.6	0.0127	0.0127	0.00006198	0.00006189	0.4000	0.3993
5	0.71	1	0.5	0.1	0.2	0.0211	0.0211	−0.00007158	−0.00007176	1	1
					0.6	0.0289	0.0290	0.00003370	0.00003377	1	1
5	2	1	0.5	0.1	0.2	0.0211	0.0211	−0.00007158	−0.00007172	1	0.9999
					0.6	0.0289	0.0289	0.00003370	0.00003376	1	0.9999
5	1	0	0.5	0.5	0.2	0.0143	0.0144	−0.0000774	−0.0000784	0.8000	0.8000
					0.6	0.0127	0.0127	0.0003028	0.0003024	0.4000	0.4000
5	1	0	0.5	1.5	0.2	0.0144	0.0144	−0.0002489	−0.0002518	0.8000	0.8000
					0.6	0.0127	0.0127	0.0008599	0.0008589	0.4000	0.4000
5	1	1	0.5	0.5	0.2	0.0211	0.0211	−0.0003553	−0.0003562	1	1
					0.6	0.0289	0.0290	0.0001670	0.0001674	1	1
5	1	1	0.5	1.5	0.2	0.0211	0.0212	−0.0010	−0.0011	1	1
					0.6	0.0290	0.0290	0.0005	0.0005	1	1
5	1	0	0.4	0.1	0.2	0.0148	0.0149	−0.00001211	−0.00001227	0.8000	0.8000
					0.6	0.0129	0.0129	0.00005379	0.00005371	0.4000	0.4000
5	1	0	1.2	0.1	0.2	0.0117	0.0131	−0.00003013	−0.00002278	0.8000	0.8000
					0.6	0.0111	0.0120	0.00009133	0.00007909	0.4000	0.4000
5	1	1	0.4	0.1	0.2	0.0217	0.0218	−0.00006079	−0.00006092	1	1
					0.6	0.0296	0.0296	0.00002851	0.00002857	1	1
5	1	1	1.2	0.1	0.2	0.0177	0.0177	−0.0001177	−0.0001179	1	1
					0.6	0.0249	0.0249	0.0000564	0.0000565	1	1

Table 2 Velocity, micro rotation and temperature profile for different step size at $M = 0.1$, $R = 0.5$, $b = 0.1$, $Pr = 1$, $m = 0$ and $t = 0.5$.

y	Velocity profile for step size 31 on [0 1]	Velocity profile for step size 21 on [0 1]	Velocity profile for step size 11 on [0 1]	Micro rotation profile for step size 31 on [0 1]	Micro rotation profile for step size 21 on [0 1]	Micro rotation profile for step size 11 on [0 1]	Temperature profile for step size 31 on [0 1]	Temperature profile for step size 21 on [0 1]	Temperature profile for step size 11 on [0 1]
0.2	0.0315	0.0315	0.0315	−0.0000757	−0.0000759	−0.0000772	0.7972	0.7972	0.7972
0.4	0.0418	0.0418	0.0418	0.0000114	0.0000110	0.0000091	0.4953	0.5955	0.5954
0.6	0.0365	0.0365	0.0365	0.0001147	0.0001143	0.0001123	0.3955	0.3955	0.3954
0.8	0.0208	0.0208	0.0208	0.0001313	0.0001311	0.0001297	0.1972	0.1972	0.1972

$$\rho\frac{\partial u'}{\partial t'} = (\mu + k)\frac{\partial^2 u'}{\partial y'^2} + k\frac{\partial \omega'}{\partial y'} + \rho g\beta(T' - T'_m) - B_0^2\sigma u', \tag{1}$$

$$\rho j\frac{\partial \omega'}{\partial t'} = (\mu + 0.5k)j\frac{\partial^2 \omega'}{\partial y'^2} - k\left(2\omega' + \frac{\partial u'}{\partial y'}\right), \tag{2}$$

$$\frac{\partial T'}{\partial t'} = \alpha\frac{\partial^2 T'}{\partial y'^2}, \tag{3}$$

Initial and boundary conditions are

$$\begin{aligned} &t \leqslant 0 \quad u' = \omega' = 0, \quad T' = T'_f, \quad 0 \leqslant y' \leqslant 1; \\ &t > 0 \quad u' = \omega' = 0, \quad T' = T'_h \quad y' = 0; \\ &\qquad\quad u' = \omega' = 0, \quad T' = T'_c \quad y' = L. \end{aligned} \tag{4}$$

Introducing the following similarity transformations in Eqs. (1)–(3),

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Table 3 Velocity, micro rotation and temperature profile for different step size at $M = 0.1$, $R = 0.5$, $b = 0.1$, $Pr = 1$, $m = 1$ and $t = 0.5$.

y	Velocity profile for step size 31 on [0 1]	Velocity profile for step size 21 on [0 1]	Velocity profile for step size 11 on [0 1]	Micro rotation profile for step size 31 on [0 1]	Micro rotation profile for step size 21 on [0 1]	Micro rotation profile for step size 11 on [0 1]	Temperature profile for step size 31 on [0 1]	Temperature profile for step size 21 on [0 1]	Temperature profile for step size 11 on [0 1]
0.2	0.0523	0.0522	0.0522	−0.0002070	−0.0002069	−0.0002069	0.9944	0.9944	0.9942
0.4	0.0783	0.0782	0.0782	−0.0001032	−0.0001032	−0.0001032	0.9910	0.9907	0.9907
0.6	0.0783	0.0782	0.0782	0.0001032	0.0001032	0.0001032	0.9910	0.9907	0.9907
0.8	0.0523	0.0522	0.0522	0.0002070	0.0002069	0.0002069	0.9944	0.9944	0.9942

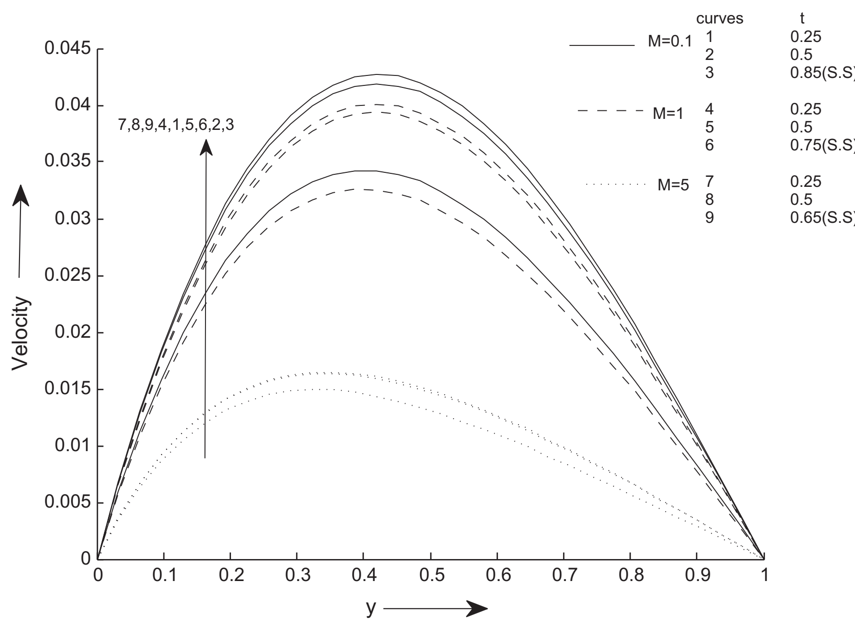

Figure 1 Velocity profile u for different values of y at $m = 0$, $R = 0.5$, $b = 0.1$ and $Pr = 1$.

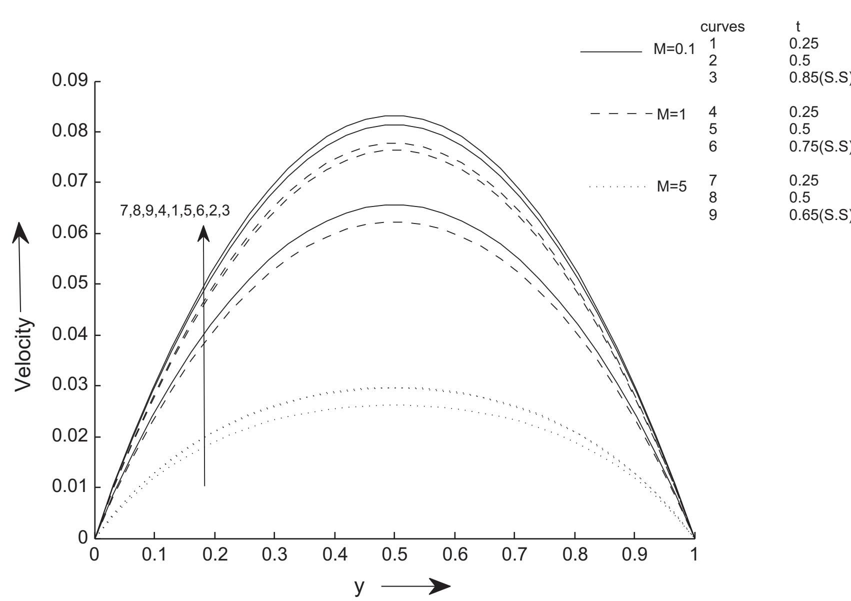

Figure 2 Velocity profile u for different values of y at $m = 1$, $R = 0.5$, $b = 0.1$ and $Pr = 1$.

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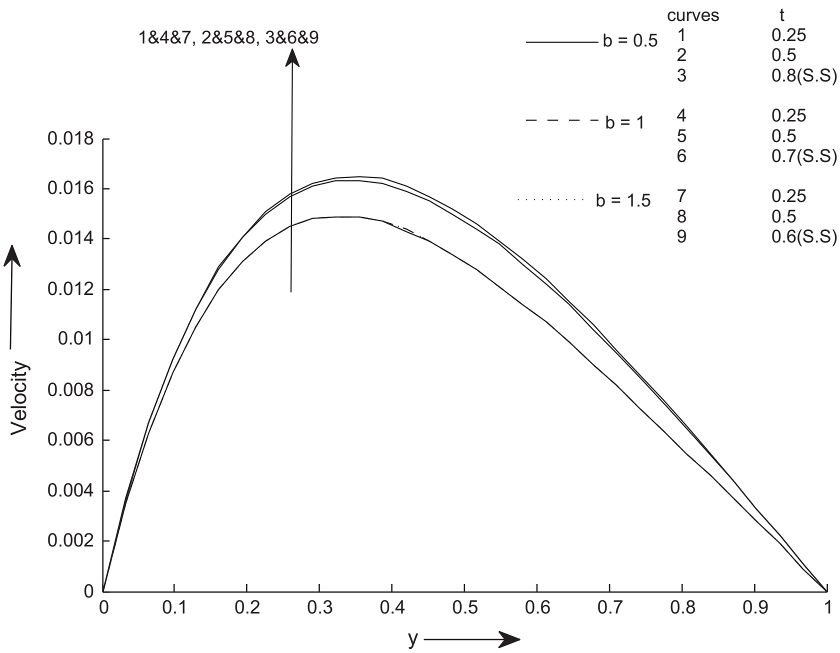

Figure 3 Velocity profile u for different values of y at $m = 0$, $R = 0.5$, $M = 5$ and $Pr = 1$.

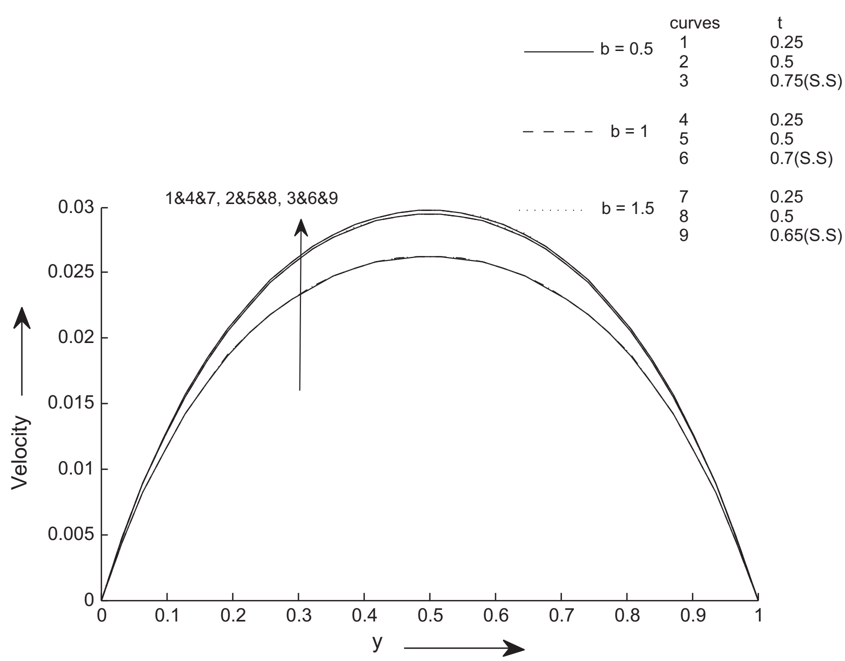

Figure 4 Velocity profile u for different values of y at $m = 1$, $R = 0.5$, $M = 5$ and $Pr = 1$.

$$\begin{aligned} y &= y'/\mathrm{L}, \quad t = \nu t'/L^2, \\ u &= \mathrm{u}'\nu/\beta \mathrm{g} \mathrm{L}^2(\mathrm{T}'_\mathrm{h} - \mathrm{T}'_\mathrm{m}), \quad \theta = (\mathrm{T}' - \mathrm{T}'_m)/(\mathrm{T}'_h - \mathrm{T}'_m), \\ \omega &= \omega'\nu/\beta g L(T'_h - T'_m), \quad Pr = \nu/\alpha, \\ m &= (T'_c - T'_m)/(T'_h - T'_m), \quad b = L^2/j, \\ \mathrm{R} &= \mathrm{k}/\mu, \quad M = \sigma B_0^2 L^2/\mu. \end{aligned} \tag{5}$$

we get the following linear system of differential equations:

$$\frac{\partial u}{\partial t} = (1+R)\frac{\partial^2 u}{\partial y^2} + \theta + R\frac{\partial \omega}{\partial y} - M^2 u, \tag{6}$$

$$\frac{\partial \omega}{\partial t} = (1+0.5R)\frac{\partial^2 \omega}{\partial y^2} - Rb\left(\frac{\partial u}{\partial y} + 2\omega\right), \tag{7}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial y^2}, \tag{8}$$

The corresponding initial boundary conditions (4) to the considered model are reduced as follows:

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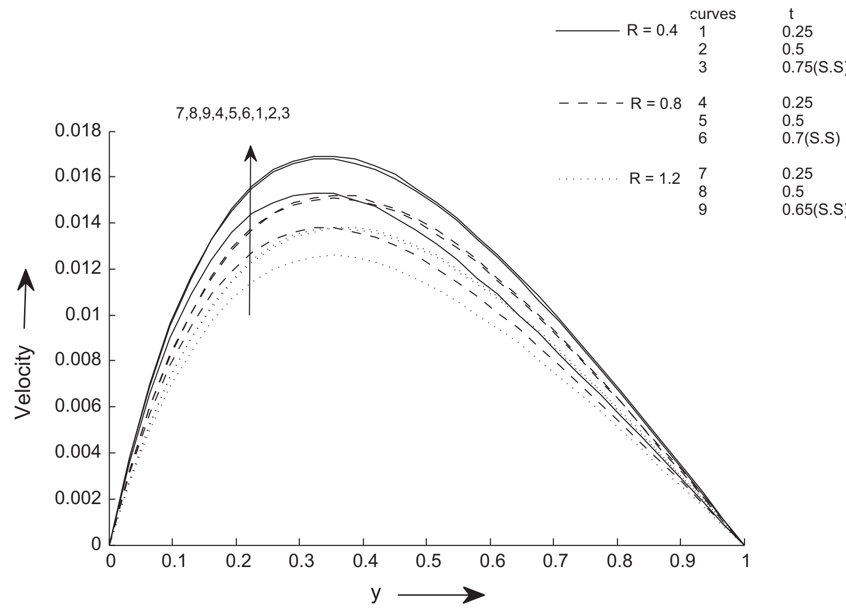

Figure 5 Velocity profile u for different values of y at $m = 0$, $b = 0.1$, $Pr = 1$ and $M = 5$.

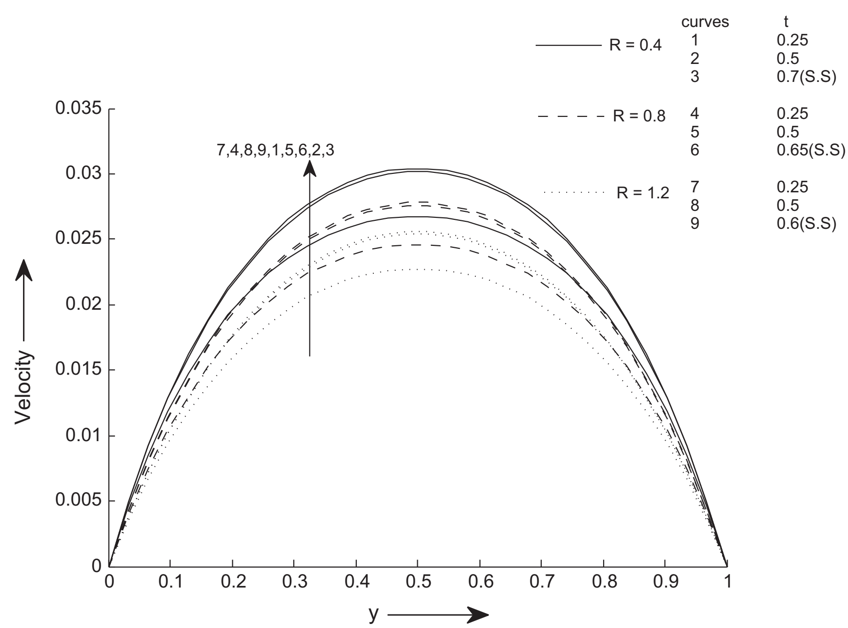

Figure 6 Velocity profile u for different values of y at $m = 1$, $b = 0.1$, $Pr = 1$ and $M = 5$.

$$t \leqslant 0 \quad u = \omega = \theta = 0, \qquad 0 \leqslant y \leqslant 1;$$
$$t > 0 \quad u = \omega = 0 \quad \theta = 1, \quad y = 0;$$
$$u = \omega = 0, \quad \theta = m \quad y = 1. \tag{9}$$

The physical quantities used in the above equations are defined in the nomenclature.

3. Numerical solution

The governing linear parabolic partial differential Eqs. (6)–(8) with initial and boundary conditions are solved numerically by using Matlab software (finite difference method). We have taken increment step along t as 0.05 and y directions as 0.0323 in entire numerical computations. In present problem, the cost and the accuracy of the solution depend strongly on length of the vector y.This attentive problem requests the solution on mesh produced by spaced points from the spatial interval 31 values of y from the space interval $[0, 1]$ and 40 values of t from the time interval $[0, 2]$.

4. Steady state analytic solution

In order to check the accuracy of the numerical solution obtained with Matlab software, we compare the steady-state numerical solution with the analytic solution of the corre-

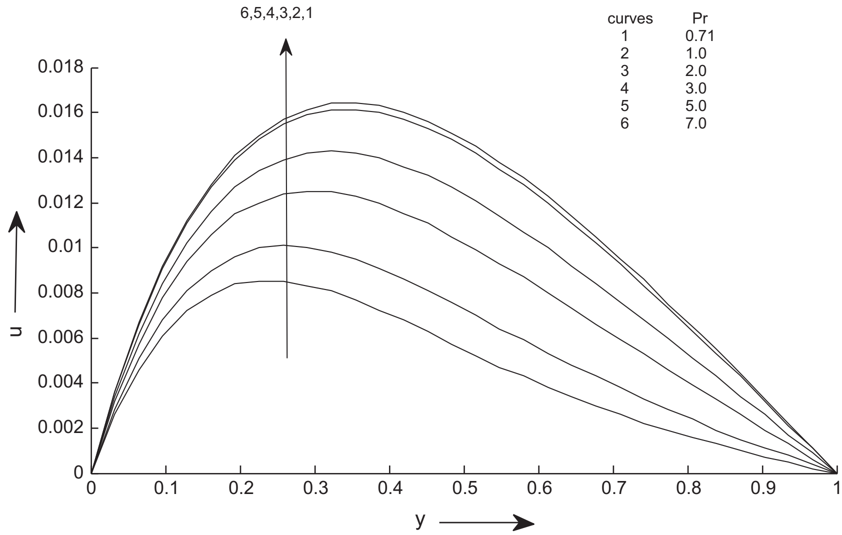

Figure 7 Velocity profile u for different values of y at $m = 0$, $b = 0.1$, $R = 0.5$ and $M = 5$.

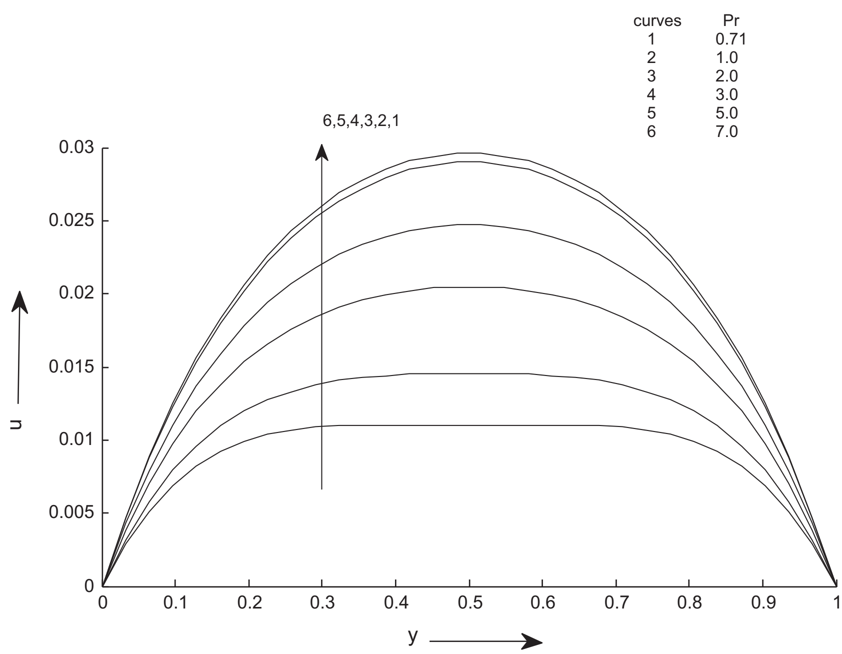

Figure 8 Velocity profile u for different values of y at $m = 1$, $b = 0.1$, $R = 0.5$ and $M = 5$.

sponding study flow. As it is steady state solution, left hand side of Eqs. (6)–(8) becomes zero. Then there is only one independent variable and thus problem is a system of ordinary differential equation.

From Appendix A we obtain the steady state analytic result as follows:

Case-I

$\boldsymbol{a_1} \neq 0, \boldsymbol{a_2} \neq 0$

$k > 0, \quad k = p^2$

(i) $p_1 > 0$ & $p_2 > 0$

$$w = c_1 e^{\sqrt{p_1} y} + c_2 e^{-\sqrt{p_1} y} + c_3 e^{-\sqrt{p_2} y} + c_4 e^{-\sqrt{p_2} y} + A \tag{10}$$

$$u = b_{10} e^{p_1 y} - b_{11} e^{-p_1 y} + c_3 b_6 e^{p_2 y} - c_4 b_6 e^{-p_2 y} + A_6 y + A_7 \tag{11}$$

(ii) $p_1 > 0$ & $p_2 < 0$

$$w = c_5 e^{\sqrt{p_1} y} + c_6 e^{-\sqrt{p_1} y} + c_7 \cos \sqrt{p_2} y + c_8 \sin \sqrt{p_2} y + A \tag{12}$$

$$u = b_{15} e^{p_1 y} - b_{16} e^{-p_1 y} + c_7 b_{12} \sin p_2 y - c_8 b_{12} \times \cos p_2 y + A_6 y + A_7 \tag{13}$$

(iii) $p_1 < 0$ & $p_2 > 0$

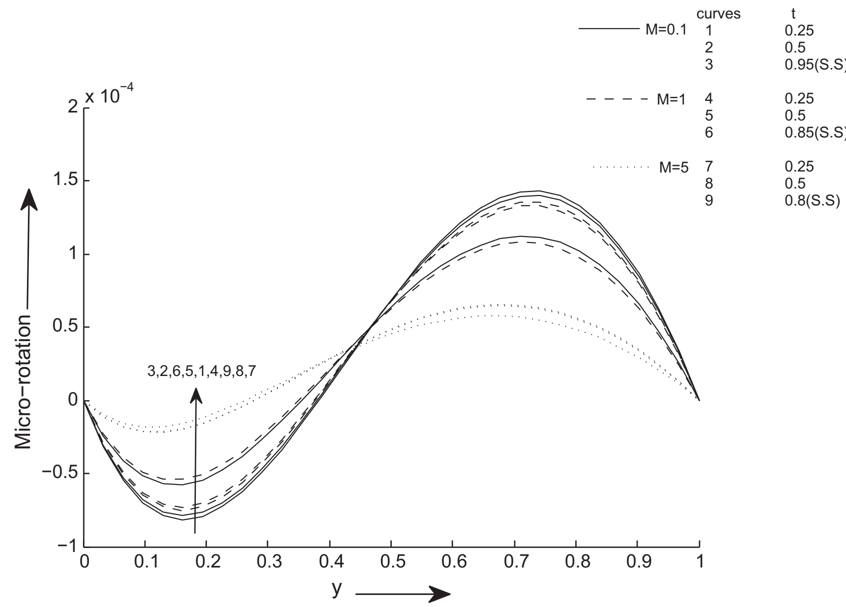

Figure 9 Micro-rotation profile w for different values of y at $m = 0$, $R = 0.5$, $b = 0.1$ and $Pr = 1$.

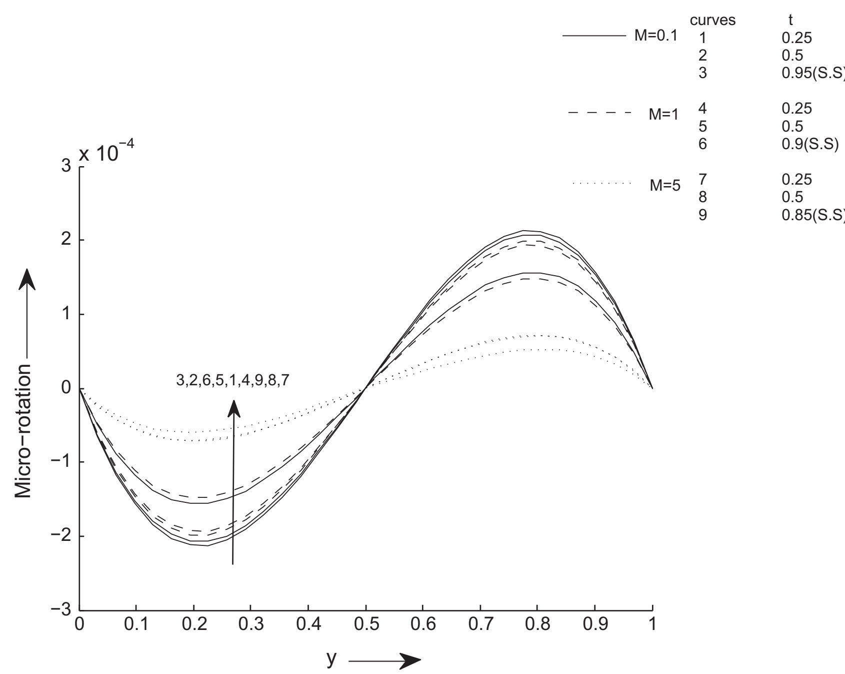

Figure 10 Micro-rotation profile w for different values of y at $m = 1$, $R = 0.5$, $b = 0.1$ and $Pr = 1$.

$$w = c_9 \cos\sqrt{p_1}y + c_{10}\sin\sqrt{p_1}y + c_{11}e^{-\sqrt{p_2}y} + c_{12}e^{-\sqrt{p_2}y} + A \quad (14)$$

$$u = b_{18}\sin p_1 y + b_{19}\cos p_1 y + c_{11}b_6 e^{p_2 y} - c_{12}b_6 e^{-p_2 y} + A_6 y + A_7 \quad (15)$$

(iv) $p_1 < 0$ & $p_2 < 0$

$$w = c_{13}\cos\sqrt{p_1}y + c_{14}\sin\sqrt{p_1}y + c_{15}\cos\sqrt{p_2}y + c_{16}\sin\sqrt{p_2}y + A \quad (16)$$

$$u = b_{20}\sin p_1 y + b_{21}\cos p_1 y + c_{15}b_{12}\sin p_2 y e^{p_2 y} - c_{16}b_{12}\cos p_2 y + A_6 y + A_7 \quad (17)$$

$k = 0$

(i) $a_1 > 0$

$$w = (c_{17} + c_{18}y)e^{\sqrt{\frac{a_1}{2}}y} + (c_{19} + c_{20}y)e^{-\sqrt{\frac{a_1}{2}}y} + A \quad (18)$$

$$u = (b_{29} + b_{30}y)e^{q_1 y} + \left(b''_{29} + b''_{30}y\right)e^{-q_1 y} + A_6 y + A_7 \quad (19)$$

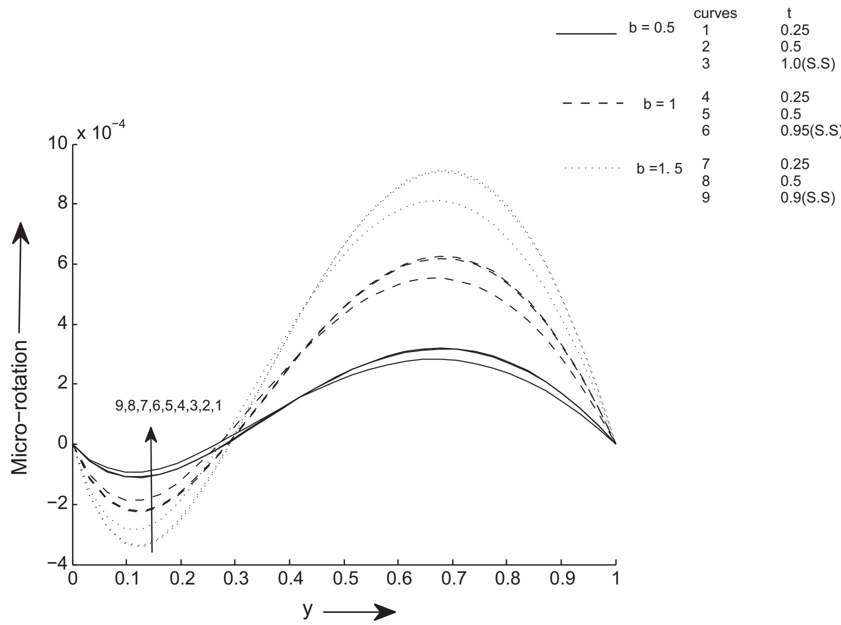

Figure 11 Micro-rotation profile w for different values of y at $m = 0$, $R = 0.5$, $M = 5$ and $Pr = 1$.

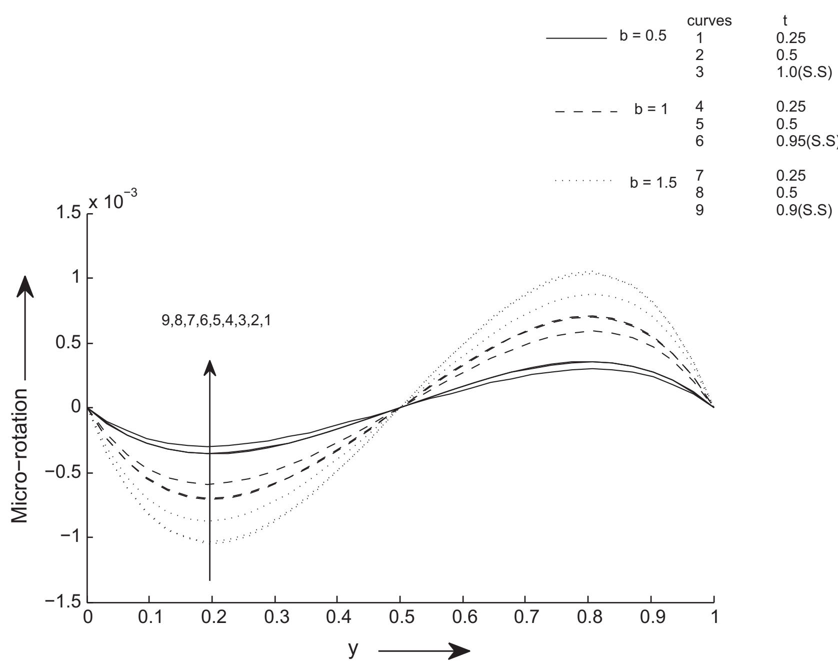

Figure 12 Micro-rotation profile w for different values of y at $m = 1$, $R = 0.5$, $M = 5$ and $Pr = 1$.

(ii) $a_1 < 0$

$$w = (c_{21} + c_{22}y)\cos\sqrt{\frac{a_1}{2}}y + (c_{23} + c_{24}y)\sin\sqrt{\frac{a_1}{2}}y + \frac{a_3}{a_2} \tag{20}$$

$$u = (b_{34} + b_{35}y)\sin q_1 y + (b_{36} + b_{37}y)\cos q_1 y + A_6 y + A_7 \tag{21}$$

$k < 0, \quad k = -p^2$

$$w = e^{v_2 y}((c_{25} + c_{26}y)\cos v_1 y + (c_{27} + c_{28}y)\sin v_1 y) + \frac{a_3}{a_2} \tag{22}$$

$$u = (b_{72} + b_{73}y)e^{v_2 y}\cos v_1 y + (b_{36} + b_{37}y)e^{v_2 y} \times \sin v_1 y + A_6 y + A_7 \tag{23}$$

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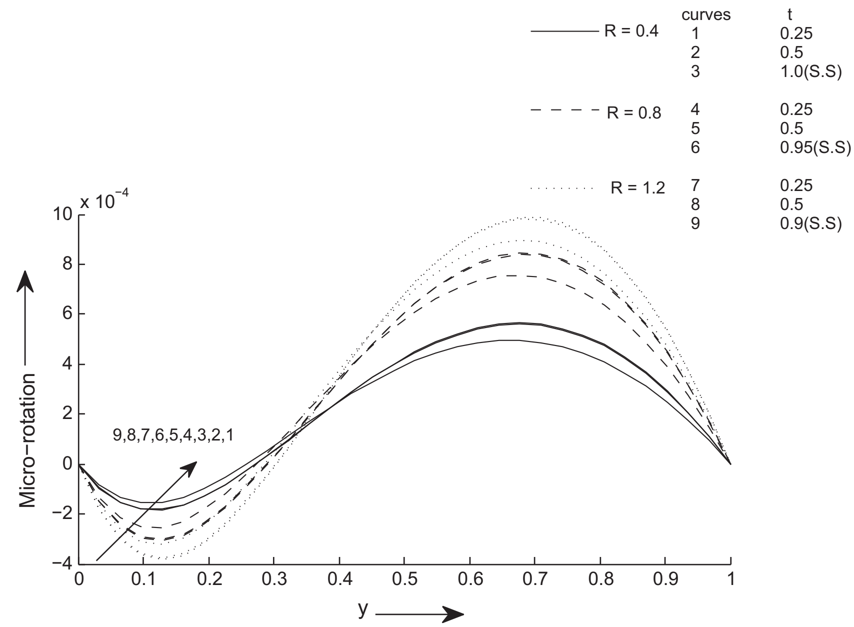

Figure 13 Micro-rotation profile w for different values of y at $m = 0$, $b = 0.1$, $Pr = 1$ and $M = 5$.

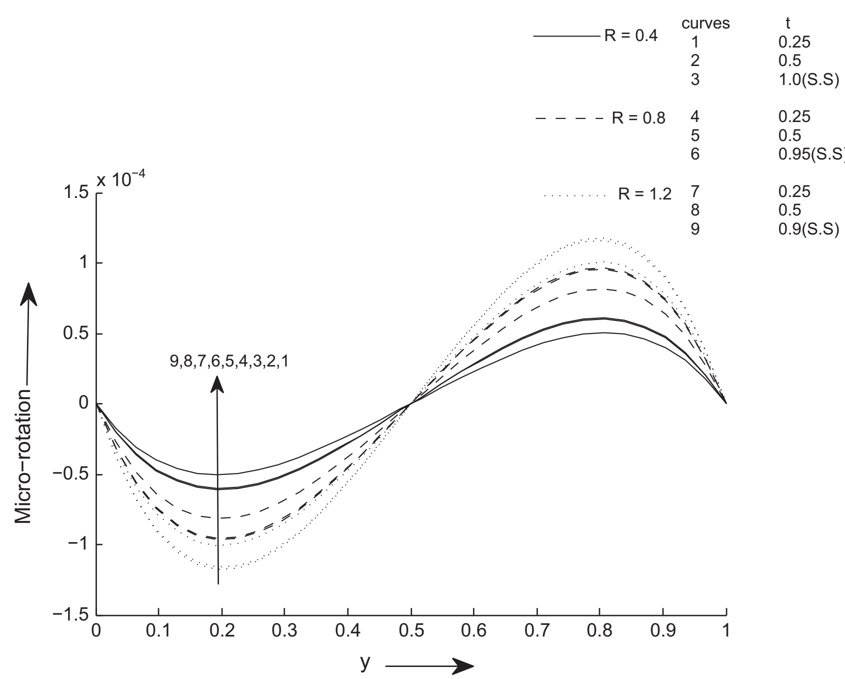

Figure 14 Micro-rotation profile w for different values of y at $m = 1$, $b = 0.1$, $Pr = 1$ and $M = 5$.

Case-II

$\boldsymbol{a_1 = 0,\ a_2 \neq 0}$

$$w = e^{\sqrt{\frac{a_2}{2}}y}\left((c_{29} + c_{30}y)\cos\sqrt{\frac{a_2}{2}}y + (c_{31} + c_{32}y)\sin\sqrt{\frac{a_2}{2}}y\right) + \frac{a_3}{a_2} \tag{24}$$

$$u = (b_{82}y + b_{83})e^{q_2y}\cos q_2y + (b_{84}y + b_{85})e^{q_2y}\sin q_2y + A_6y + A_7 \tag{25}$$

Case-III

$\boldsymbol{a_1 \neq 0,\ a_2 = 0}$

$$w = c_{33}e^{\sqrt{a_1}y} + c_{34}e^{-\sqrt{a_1}y} + c_{35}\cos\sqrt{a_1}y + c_{36}\sin\sqrt{a_1}y - \frac{y^2a_3}{2a_1} \tag{26}$$

$$u = (b_{91} - c_{35})e^{ry} + b_{91}e^{-ry} + b_{92}\sin ry + b_{93}\cos ry + b_{94}y + A_7 \tag{27}$$

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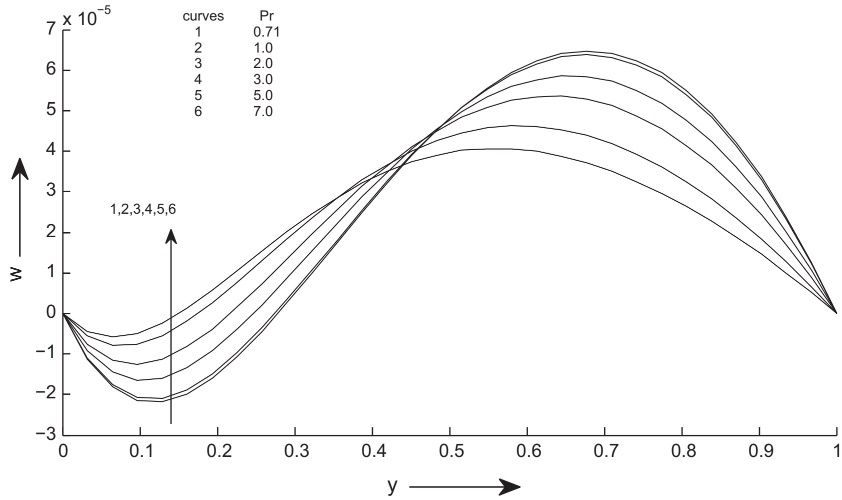

Figure 15 Micro-rotation profile w for different values of y at $m = 0$, $b = 0.1$, $R = 0.5$ and $M = 5$.

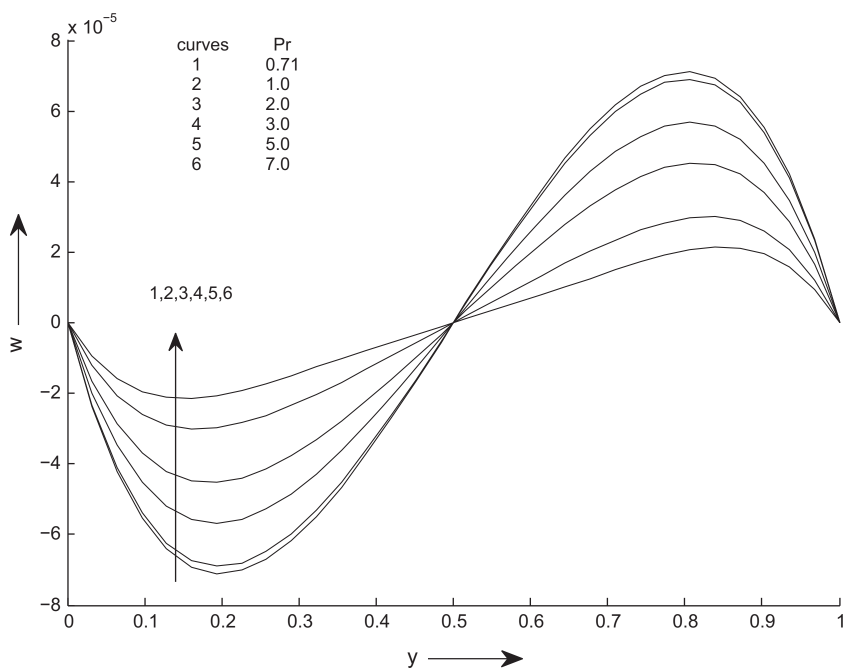

Figure 16 Micro-rotation profile w for different values of y at $m = 1$, $b = 0.1$, $R = 0.5$ and $M = 5$.

Case-IV
$\boldsymbol{a_1 = 0,\ a_2 = 0}$

$$w = c_{37} + c_{38}y + c_{39}y^2 + c_{40}y^3 + \frac{y^4 a_3}{24} \quad (28)$$

$$u = b_{95}y^3 + b_{96}y^2 + b_{97}y + b_{99} \quad (29)$$

The steady state solution of Eq. (8) is

$$\theta = 1 + (m - 1)y \quad (30)$$

In Table 1, we have compared numerical and analytical solutions of steady-state velocity, micro rotation and Temperature profiles for different values of the magnetic parameter M, vortex viscosity R, material parameter b and Prandtl number Pr for both the cases asymmetric and symmetric. We can see that the numerical and analytic results agree very well. Tables 2 and 3 verify that our solution is independent of step size for asymmetric and symmetric cases respectively.

5. Result and discussion

The physical parameters appearing in the model are vortex viscosity parameter R, material parameter b, Prandtl number Pr and temperature ratio m having values 0 and 1 for asymmetric and symmetric heating respectively. Several researchers have

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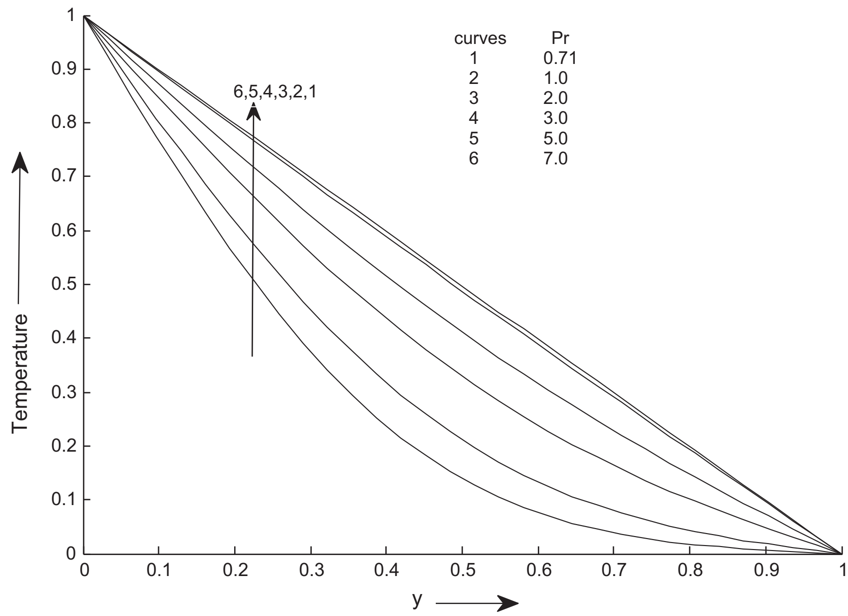

Figure 17 Temperature profile θ for different values of y at $m = 0$.

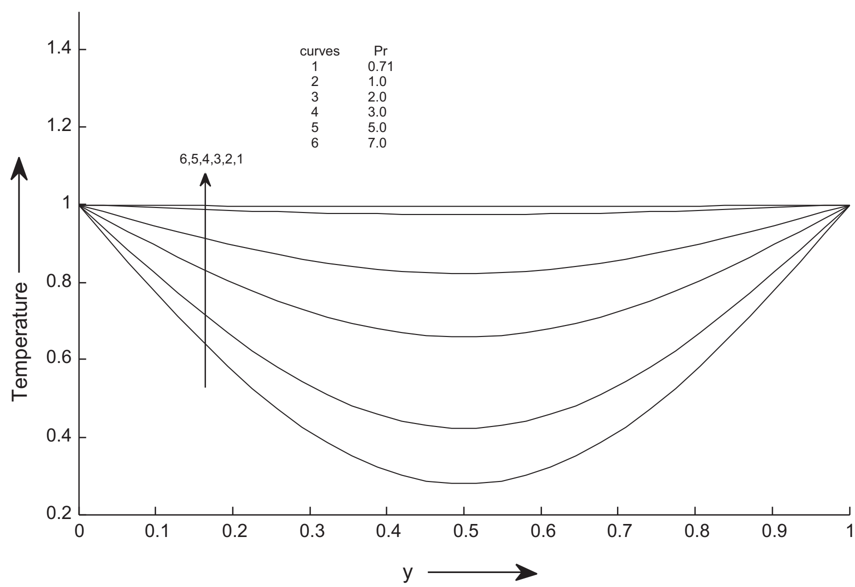

Figure 18 Temperature profile θ for different values of y at $m = 1$.

studied effects of physical parameters R, b and Pr on the velocity as well as micro-rotation profiles under the asymmetric/symmetric heating thermal condition on the vertical walls. In this study, we have focused on the physical parameters M and presented their influences through Figs. 1–20 on the velocity, micro-rotation temperature and velocity vector profiles for asymmetric and symmetric heating of the vertical walls. Odd numbered figures are for $m = 0$ and rest are for $m = 1$.

Figs. 1 and 2 display the effect of magnetic parameter M on the velocity profiles. It is observed that the amplitude of the velocity as well as the boundary layer thickness decreases when M is increased. Physically, it may also be expected due to the fact that the application of a transverse magnetic field results in a resistive type force (called Lorentz force) similar to the drag force, and upon increasing the values of M, the drag force increases which leads to the deceleration of the flow.

The influence of material parameter b on velocity profiles is shown in Figs. 3 and 4. In both thermal cases, we can observe that the velocity of fluid has attained steady state by increasing with the time.

The graphical results for vortex viscosity R are shown in Figs. 5 and 6. For both thermal cases we observed that the vortex viscosity parameter has decreasing tendency on the velocity profiles as well as on the steady state time.

Figs. 7 and 8 exhibit the velocity profiles for different values of Prandtl number Pr, when the other parameters are fixed. It is observed that velocity of the fluid decreases with increasing Prandtl number.

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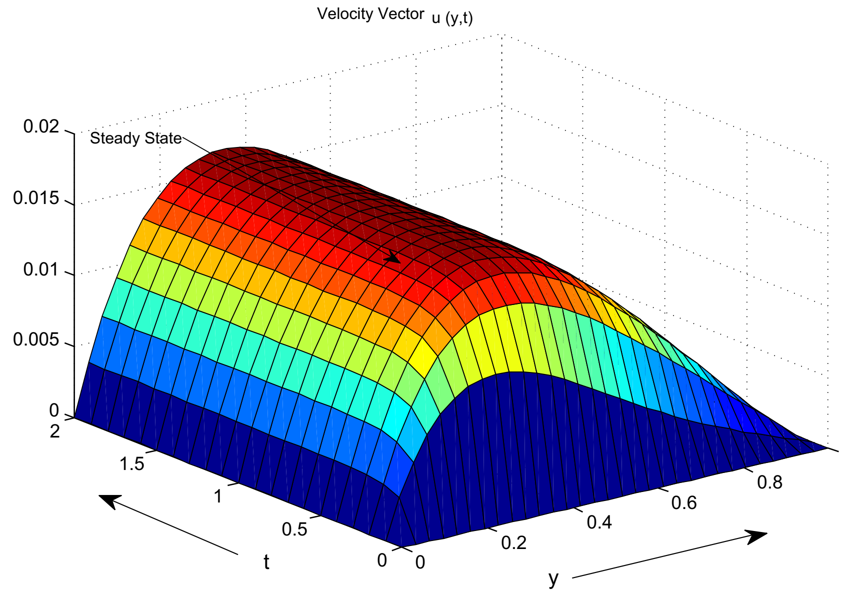

Figure 19 velocity vector $u(y, t)$ for different values of y & t at $m = 0$, $R = 0.5$, $b = 0.1$, $M = 5$ and $Pr = 1$.

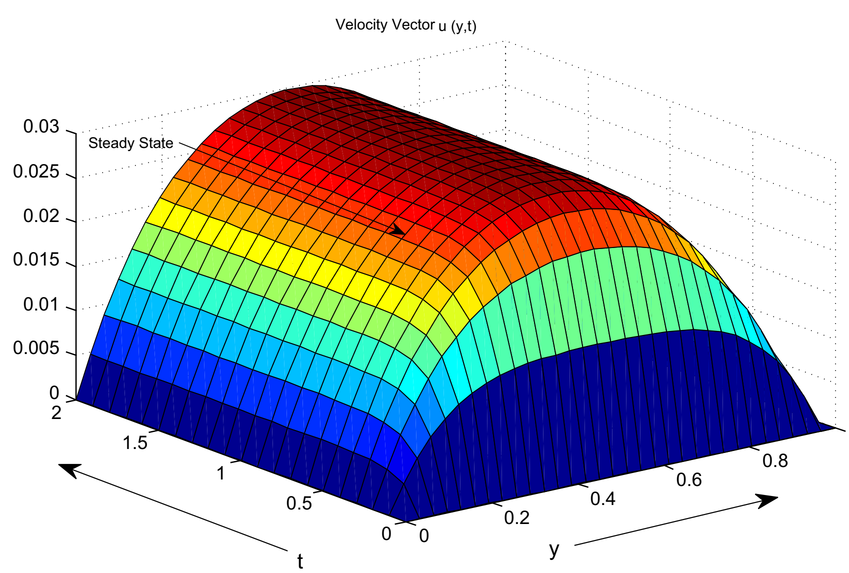

Figure 20 velocity vector $u(y, t)$ for different values of y & t at $m = 1$, $R = 0.5$, $b = 0.1$, $M = 5$ and $Pr = 1$.

Figs. 9 and 10 are plotted to show the effects of magnetic parameter M on the micro-rotation profiles. In both thermal cases, the steady state time and the magnitude of the micro-rotation have increasing tendency with the material parameter.

Figs. 11 and 12 display the effect of material parameter b on the micro-rotation profiles. It is observed that the magnitude of the micro-rotation increases with increase in b.

The influence of vortex viscosity R on micro rotation profiles is shown in Figs. 13 and 14. The magnitudes of the micro-rotation profiles show the increasing tendency with R.

Figs. 15 and 16 reveal that magnitude of micro rotation decreases with increasing Prandtl number.

It is depicted from Figs. 17 and 18 that the temperature decreases as the Prandtl number Pr increases. It is justified due to the fact that thermal conductivity of the fluid decreases with increasing Prandtl number Pr and hence decreases the thermal boundary layer thickness.

Figs. 19 and 20 display the velocity vectors for different values of t & y at $M = 0.1$, $b = 0.1$, $R = 0.5$ and $Pr = 1$ for asymmetric and symmetric cases.

A close study of these figures reveals that the steady state time in case of asymmetric heating is more than the symmetric heating.

6. Conclusion

We investigated the effect of magnetic, material and viscosity parameters on natural convective flow along vertical walls in case of both asymmetric and symmetric heating (and cooling) of walls. An exact analysis is performed to investigate the

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steady boundary layer momentum, angular momentum and energy equations of flow of an incompressible micropolar fluid under uniform magnetic field between vertical walls. The governing unsteady boundary layer problem is solved numerically.

The main conclusions of this study are as follows:

1. Velocity of the fluid decreases with increasing Prandtl number *Pr*.
2. The amplitude of the velocity as well as the boundary layer thickness decreases when magnetic parameter *M* is increased.
3. Velocity is an increasing function of time *t*.
4. Temperature decreases as the Prandtl number *Pr* increases.
5. Temperature increases with increasing time *t*.
6. Magnitude of the micro-rotation has increasing tendency with the material parameter *M*, material parameter *b* and vortex viscosity *R* while decreases with increase in Prandtl number *Pr*.
7. The steady state time of fluid velocity as well as micro-rotation is more for symmetric cases compared to asymmetric cases.
8. The velocity and micro-rotation profiles of fluid decrease at any point of fluid regime with magnetic parameter.
9. The velocity decreases and micro rotation profile of fluid increases at any point of fluid regime with vortex viscosity parameter.
10. The steady state time of velocity profile and micro-rotation have decreasing tendency with material parameter.

Appendix A.

$A_4 = \frac{(1+R)(1+0.5R)}{RbM^2}$	$A_5 = \frac{-(2+R)}{M^2}$	$A_6 = \frac{(1-m)}{M^2}$
$A_7 = \frac{-1}{M^2}$	$A' = A_6 + A_7$	$a_1 = \frac{-RbR}{(1+R)(1+0.5R)} + \frac{2Rb}{(1+0.5R)} + \frac{M^2}{(1+R)}$
$a_2 = \frac{2M^2Rb}{(1+R)(1+0.5R)}$	$a_3 = \frac{(1-m)Rb}{(1+R)(1+0.5R)}$	$k = p^2 = a_1^2 - 4a^2$
$A = \frac{a_3}{a_2}$	$p_1 = \frac{a1+p}{2}$	$p_2 = \frac{a1-p}{2}$
$q_1 = \sqrt{\frac{a_1}{2}}$	$q_2 = \sqrt{\frac{a_2}{2}}$	$r = \sqrt{a_1}$
$v_1 = \sqrt{\frac{\sqrt{\frac{a_1^2}{4}+\frac{p^2}{4}}+\frac{a_1}{2}}{2}}$	$v_2 = \sqrt{\frac{\sqrt{\frac{a_1^2}{4}+\frac{p^2}{4}}-\frac{a_1}{2}}{2}}$	$b_1 = p_1^3 A_4$
$b_2 = p_2^3 A_4$	$b_3 = p_1 A_5$	$b_4 = A_5 p_2$
$b_5 = b_1 + b_3$	$b_6 = b_2 + b_4$	$b_7 = b_5(-A_1 - 1)$
$b_8 = b_5(-A_2 - 1)$	$b_9 = b_5(-A_3 - A)$	$b_{10} = c_3 b_7 + c_4 b_8 + b_9$
$b_{11} = A_1 c_3 b_5 + A_2 c_4 b_5 - A_3 b_5$	$A_1 = \frac{e^{p_1} - e^{p_2}}{e^{-p_1} - e^{p_1}}$	$A_2 = \frac{e^{p_1} - e^{-p_2}}{e^{-p_1} - e^{p_1}}$
$A_3 = \frac{A(e^{p_1} - 1)}{e^{-p_1} - e^{p_1}}$	$A_8 = -2b_5 A_1 - b_5 + b_6$	$A_9 = -2b_5 A_2 - b_5 - b_6$
$A_{10} = -2b_5 A_3 - Ab_5 + A_7$	$A_{11} = b_7 e^{p_1} - A_1 b_5 e^{-p_1} + b_6 e^{p_2}$	$A_{12} = b_8 e^{p_1} - A_2 b_5 e^{-p_1} - b_6 e^{-p_2}$
$A_{13} = b_9 e^{p_1} - A_3 b_5 e^{-p_1} + A'$	$c_4 = \frac{A_{13}A_8 - A_{10}A_{11}}{A_{11}A_9 - A_8 A_{12}}$	$c_3 = \frac{A_{13}A_9 - A_{10}A_{12}}{A_{11}A_9 - A_8 A_{12}}$
$c_2 = A_1 c_3 + c_4 A_2 + A_3$	$c_1 = -c_2 - c_3 - c_4 - A$	$b_{12} = b_2 - b_4$
$b_{13} = b_5(-B_1 - 1)$	$b_{14} = b_5(B_3 - A)$	$b_{15} = c_7 b_{13} + c_8 B_2 b_5 + b_{14}$
$b_{16} = c_7 b_5 B_1 + c_8 b_5 B_2 + b_5 B_3$	$B_1 = \frac{e^{p_1} - cosp_2}{e^{-p_1} - e^{p_1}}$	$B_2 = \frac{-sinp_2}{e^{-p_1} - e^{p_1}}$
$B_3 = \frac{Ae^{p_1} - A}{e^{-p_1} - e^{p_1}}$	$B_4 = -2b_5 B_1 - b_5$	$B_5 = -b_2 + b_4$
$B_6 = -Ab_5 + A_7$	$B_7 = b_{13} e^{p_1} - b_5 B_1 e^{-p_1} + b_{12} \sin p_2$	$B_8 = b_5 e^{p_1} B_2 - b_5 B_2 e^{-p_1} - b_{12} \cos p_2$
$B_9 = b_{14} e^{p_1} - b_5 B_3 e^{-p_1} + A_6 + A_7$	$c_8 = \frac{B_4 B_9 - B_6 B_7}{B_5 B_7 - B_8 B_4}$	$c_7 = \frac{B_5 B_9 - B_6 B_8}{B_5 B_7 - B_8 B_4}$
$c_6 = B_1 c_7 + c_8 B_2 + B_3$	$c_5 = -c_6 - c_7 - A$	$b_{17} = b_1 - b_3$
$b_{18} = (-c_{11} - c_{12} - A) b_{17}$	$b_{19} = b_{17}(-c_{11} B_{11} - c_{12} B_{12} - B_{13})$	$B_{11} = \frac{1 - e^{p_2}}{\sin p_1}$
$B_{12} = \frac{1 - e^{-p_2}}{\sin p_1}$	$B_{13} = \frac{A(\cos p_1 - 1)}{\sin p_1}$	$B_{14} = -B_{11} b_{17} + b_6$
$B_{15} = -B_{12} b_{17} - b_6$	$B_{16} = -B_{13} b_{17} + A_7$	$B_{17} = (-\sin p_1 - B_{11} \cos p_1) b_{17} + b_6 e^{p_2}$
$B_{18} = (-\sin p_1 - A_{12} \cos p_1) b_{17} - b_6 e^{-p_2}$	$B_{19} = (-A \sin p_1 - A_{13} \cos p_1) b_{17} + A'$	$c_{12} = \frac{B_{19} B_{15} - B_{16} B_{18}}{B_{14} B_{18} - B_{17} B_{15}}$
$c_{11} = \frac{B_{19} B_{15} - B_{16} B_{18}}{B_{15} B_{17} - B_{18} B_{14}}$	$c_{10} = B_{11} c_{11} + c_{12} B_{12} + B_{13}$	$c_9 = -c_{11} - c_{12} - A$
$H_1 = \frac{cosp_1 - cosp_2}{\sin p_1}$	$H_2 = \frac{-sinp_2}{\sin p_1}$	$H_3 = \frac{Acosp_1 - A}{\sin p_1}$
$b_{20} = (-c_{15} - A) b_{17}$	$b_{21} = b_{17}(-c_{15} H_1 - H_3)$	$H_4 = -H_1 b_{17}$
$H_5 = H_2 b_{17} - b_{12}$	$H_6 = H_3 b_5 + A_7$	$H_7 = (-\sin p_1 - H_1 \cos p_1) b_{17} + b_{12} \sin p_2$
$H_8 = H_2 b_{17} \cos p_1 - b_{12} \cos p_2$	$H_9 = -Ab_{17} \sin p_1 - b_{17} H_3 \cos p_1 + A'$	$c_{16} = \frac{H_9 H_4 - H_6 H_7}{H_5 H_7 - H_8 H_4}$
$c_{15} = \frac{H_9 H_5 - H_6 H_8}{H_4 H_8 - H_7 H_5}$	$c_{14} = H_1 c_{15} - c_{16} H_2 + H_3$	$c_{13} = -c_{15} - A$
$I_1 = \frac{e^{q_1} - e^{-q_1}}{e^{q_1}}$	$I_2 = \frac{e^{-q_1}}{e^{q_1}}$	$I_3 = \frac{A(e^{q_1} - 1)}{e^{q_1}}$

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$b_{22} = q_1^3 A_4$	$b_{23} = q_1^2 A_4$	$b_{24} = q_1 A_5$
$b_{25} = b_{22} + b_{24}$	$b_{26} = 3b_{23} + A_5$	$b_{27} = b_{25} + b_{26}$
$b_{28} = b_{25} - b_{26}$	$b'_{29} = c_{19} I_1 - c_{20} I_2 + I_3$	$b_{29} = b_{25}(-c_{19} - A) + b_{26} b'_{29}$
$b_{30} = b_{25} b'_{29}$	$b''_{29} = -b_{25} c_{19} + b_{26}$	$b''_{30} = -c_{20} b_{25}$
$I_4 = -2b_{25} + I_1 b_{26}$	$I_5 = -I_2 b_{26} + b_{26}$	$I_6 = -Ab_{22} + I_3 b_{26} - Ab_{24} + A_7$
$I_7 = -b_{25}(e^{q_1} + e^{-q_1}) + I_1 e^{q_1} b_{27}$	$I_8 = -I_2 e^{q_1} b_{27} - e^{-q_1} b_{28}$	$I_9 = -Ae^{q_1} b_{25} + I_3 e^{q_1} b_{27} + A_6 + A_7$
$c_{20} = \frac{I_9 I_4 - I_6 I_7}{I_5 I_7 - I_8 I_4}$	$c_{19} = \frac{I_9 I_5 - I_6 I_8}{I_4 I_8 - I_7 I_5}$	$c_{18} = I_1 c_{19} - c_{20} I_2 + I_3$
$c_{17} = -c_{19} - A$	$J_1 = -tanq_1$	$J_2 = -tanq_1$
$J_3 = \frac{A(\cos q_1 - 1)}{\cos q_1}$	$b_{31} = 3b_{23} - A_5$	$b_{32} = b_{22} - b_{24}$
$b_{33} = c_{23} J_1 + c_{24} J_2 + J_3$	$b_{34} = -Ab_{32}$	$b_{35} = b_{32} b_{33}$
$b_{36} = -b_{31} b_{33} - c_{23} b_{32}$	$b_{37} = -c_{24} b_{32}$	$J_4 = -J_1 b_{31} - b_{32}$
$J_5 = -J_2 b_{31}$	$J_6 = -J_3 b_{31} + A_7$	$J'_6 = J_1 b_{31} - b_{32}, \quad J'_7 = J_2 b_{32} - b_{31}$
$J'_8 = J_2 b_{31} - b_{32}$	$J'_9 = -Ab_{32} + J_3 b_{31}$	$J_7 = \sin q_1 J_1 b_{32} - \cos q_1 J'_6$
$J_8 = \sin q_1 J'_7 - \cos q_1 J'_8$	$J_9 = \sin q_1 J'_9 - J_3 \cos q_1 b_{31} + A'$	$c_{24} = \frac{J_9 J_4 - J_6 J_7}{J_5 J_7 - J_8 J_4}$
$c_{23} = \frac{J_9 J_5 - J_6 J_8}{J_4 J_8 - J_7 J_5}$	$c_{22} = J_1 c_{23} - c_{24} J_2 + J_3$	$c_{21} = -A$
$M_1 = -\tan v_1$	$M_2 = -\tan v_1$	$M_3 = \frac{A(\cos v_1 e^{v_2} - 1)}{\cos v_1 e^{v_2}}$
$b_{38} = v_2^2 A_4$	$b_{39} = v_2^2 v_1 A_4$	$b_{40} = v_1^2 A_4$
$b_{41} = v_1^3 A_4$	$b_{42} = v_1 v_2 A_4$	$b_{43} = v_1 v_2^3 A_4$
$b_{44} = v_2 v_1^2 A_4$	$b_{45} = v_1 A_5$	$b_{46} = v_2 A_5$
$b'_{46} = v_2^3 A_4$	$b_{47} = b_{38} - b_{40}$	$b_{48} = 3b_{39} - b_{41}$
$b_{49} = 3b_{44} + b_{46}$	$b_{50} = A(b'_{46} + b_{49})$	$b_{51} = b_{41} - b_{45}$
$b_{52} = 3b_{47} - b_{50} + b'_{46} + A_5$	$b_{53} = -b_{48} - 6b_{42} - b_{45}$	$b_{54} = -b_{51} + 3b_{39}$
$b_{55} = b'_{46} + b_{50}$	$b_{56} = M_1 b_{52} + b_{54}$	$b_{57} = M_1 b_{53} + b_{55}$
$b_{58} = b_{54} + b_{42}$	$b_{59} = b_{55} + 3b_{38} - 2b_{40} + A_5$	$b_{60} = M_2 b_{52} + b_{58}$
$b_{61} = M_2 b_{53} + b_{59}$	$b_{62} = b_{55} + (b'_{46} + A_5)$	$b_{63} = Ab_{50} - Ab'_{46}$
$b_{64} = M_3 b_{62} + b_{63}$	$b_{65} = M_3 b_{53} + Ab_{54}$	$b_{66} = 3b_{47} + A_5$
$b_{67} = 3b_{38} - 2b_{40} + A_5$	$b_{68} = b_{48} + b_{45}$	$b_{69} = M_1 c_{27} + M_2 c_{28} + M_3$
$b_{70} = b_{54} + b_{42} - Ab_{55}$	$b_{71} = b_{55} + b_{67} + Ab_{68}$	$b_{72} = b_{66} b_{69} + b_{70}$
$b_{73} = b_{55} b_{69} + b_{68}$	$b_{74} = -b_{42} b_{69} + b_{71}$	$b_{75} = -b_{68} b_{69} + b_{55}$
$M_4 = (3b_{47} + A_5)M_1 + b_{48} + b_{45}$	$M_5 = (3b_{47} + A_5)M_2 + 6b_{42}$	$M_6 = (3b_{47} + A_5)M_3 - b_{50} + A_7$
$M_7 = (\cos v_1 b_{56} + \sin v_1 b_{57})e^{v_2}$	$M_8 = e^{v_2}(\cos v_1 b_{60} + \sin v_1 b_{61})$	$M_9 = e^{v_2}(\cos v_1 b_{64} + \sin v_1 b_{65})$
$c_{28} = \frac{M_9 M_4 - M_6 M_7}{M_5 M_7 - M_8 M_4}$	$c_{27} = \frac{M_9 M_5 - M_6 M_8}{M_4 M_8 - M_7 M_5}$	$c_{26} = M_1 c_{27} + c_{28} M_2 + M_3$
$c_{25} = -A$	$L_1 = -\tan q_2$	$L_2 = \frac{A(\cos q_2 e^{q_2} - 1)}{\cos q_2 e^{q_2}}$
$b_{77} = q_2^3 A_4$	$b_{78} = q_2 A_5$	$b_{79} = -2b_{77} + b_{78}$
$b_{80} = 2b_{77} + b_{78}$	$b_{81} = L_1 b_{79} + L_1 A_5$	$b'_{81} = b_{81} + b_{80}$
$b'_{82} = b_{82} + b_{79}$	$b'_{83} = b'_{81} + 6b_{76}$	$b'_{84} = b'_{82} + A_5$
$b'_{85} = b_{81} - Ab_{79}$	$b'_{86} = b_{82} + Ab_{80}$	$b'_{87} = c_{31} + c_{32}$
$b'_{87} = A(A_5 - b_{79})$	$b'_{88} = -6b'_{87} L_1 - 6A$	$b_{82} = -L_1 b_{80} - 6L_1 b_{76}$
$b_{83} = c_{31} b_{80} + L_1 A_5 b'_{87} + 6c_{32} b_{76} + b'_{87}$	$b_{84} = -b_{80} b_{81} + b_{79}$	$b_{85} = c_{31} b_{79} - b_{76} b'_{88} + c_{32} A_{75} L_1$
$L_4 = 2b_{77} + A_5 L_1 + b_{79}$	$L_5 = 6b_{76} + A_5 L_1$	$L_6 = 2Ab_{77} - Ab_{79} + A_5 L_2 + A_7$
$L_7 = (\cos q_2 b'_{81} + \sin q_2 b'_{82})e^{q_2}$	$L_8 = (\cos q_2 b'_{83} + \sin q_2 b'_{84})e^{q_2}$	$L_9 = (\cos q_2 b'_{85} + \sin q_2 b'_{86})e^{q_2} + A'$
$c_{32} = \frac{L_9 L_4 - L_6 L_7}{L_5 L_7 - L_8 L_4}$	$c_{31} = \frac{L_9 L_5 - L_6 L_8}{L_4 L_8 - L_7 L_5}$	$c_{30} = L_1 c_{31} + c_{32} L_1 + L_2$
$c_{29} = -A$	$G_1 = \frac{\cos r - e^r}{e^r - e^{-r}}$	$G_2 = \frac{\sin r}{e^r - e^{-r}}$
$G_3 = \frac{(a_3/2a_1)}{e^r - e^{-r}}$	$b_{86} = r^3 A_4$	$b_{87} = A_5 r$
$b_{88} = b_{86} + b_{87}$	$b_{89} = b_{86} - b_{87}$	$b_{90} = -G_1 c_{35} - G_2 c_{36} + G_3$
$b_{91} = b_{88} b_{90}$	$b_{92} = c_{35} b_{89}$	$b_{93} = c_{36} b_{89}$
$b_{94} = A_6 - \frac{a_3}{a_1}$	$G_4 = (-2G_1 - 1)b_{88}$	$G_5 = -2G_2 b_{88} - b_{89}$
$G_6 = 2G_3 b_{88} + A_7$	$G_7 = b_{88}(G_1(-e^r - e^{-r}) - e^r) + \sin r b_{89}$	$G_8 = b_{88} G_2(-e^r - e^{-r}) + \cos r b_{89}$
$G_9 = b_{88} G_3(-e^r - e^{-r}) - \frac{a_3}{a_1} + A'$	$c_{36} = \frac{G_9 G_4 - G_6 G_7}{G_5 G_7 - G_8 G_4}$	$c_{35} = \frac{G_9 G_5 - G_6 G_8}{G_4 G_8 - G_7 G_5}$
$c_{34} = G_1 c_{35} + c_{36} G_2 - G_3$	$c_{33} = -c_{34} - c_{35}$	$b_{95} = \frac{a_3}{6}$
$b_{96} = 3c_{40}$	$b_{97} = 2c_{39} + A_6 + A_4 a_3$	$b_{98} = c_{39} + c_{40} + \frac{a_3}{24}$
$b_{99} = 6c_{40} A_4 - b_{98} A_5 + A_7$	$D_1 = -A_5,$	$D_2 = -6A_4$
$D_3 = \frac{-25a_3}{24} + A_7$	$D_4 = D_1 + 2$	$D_5 = D_2 + D_1 + 3$
$D_6 = A_4 c_{39} - \frac{A_5 a_3}{24} + b_{95} + A_6 + A_7$	$c_{40} = \frac{D_6 D_1 - D_3 D_4}{D_2 D_4 - D_5 D_1}$	$c_{39} = \frac{D_6 D_2 - D_3 D_5}{D_1 D_5 - D_4 D_2}$
$c_{38} = -c_{39} - c_{40} - \frac{a_3}{24}$	$c_{37} = 0$	

Please cite this article in press as: Kataria HR et al., Effect of magnetic field on unsteady natural convective flow of a micropolar fluid between two vertical walls, Ain Shams Eng J (2015), http://dx.doi.org/10.1016/j.asej.2015.08.013

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